Phase-controlled gain, phase shift, and group velocity using a room-temperature active-Raman-gain medium

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A four-level tripod active-Raman-gain scheme is analyzed for obtaining phase-controlled gain, phase shift, and group velocity at room temperature. The scheme can be used to eliminate significant probe field attenuation or distortion which is unavoidable in the scheme based on electromagnetically induced transparency. It is shown that the intensity gain, phase shift, and group velocity of a probe field can be simultaneously manipulated by changing the relative phase of two pump fields. The scheme is also different from that proposed recently by Deng *et al*. where a probe-field gain always exists. New features of the scheme presented here raise the possibility of designing rapidly responding optical switches and gates for optical information processing.

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Control of intensity, phase, and propagating velocity of electromagnetic fields has great technological importance in the field of information science. In a solid state medium such as an optical fiber[1], the requirement of low light absorption necessarily implies the system must work in off-resonance regime, which means that an active control is difficult due to the lack of distinctive energy levels and transition selection rules. In recent years, much effort has been made to study the optical property of active media via electromagnetically induced transparency (EIT)[2], in which an on-resonance excitation scheme is employed. Due to the quantum interference effect induced by a control field, the light wave propagation in such media displays many striking features, including a large suppression of optical absorption, a significant reduction of group velocity, and a great enhancement of nonlinear Kerr effects, which are promising for many practical applications in optical information processing[3−6].

However, a weakly driven EIT-based scheme has still severe problems. One of them is the strong probe-field attenuation and spreading at room temperature. Another one is the very long response time due to the ultraslow propagating velocity. In this letter, we propose a new scheme to overcome these drawbacks. The scheme is a modified form of the active-Raman-gain (ARG) configuration, suggested recently by several authors[7−12]. Contrary to EIT-based schemes where the probe field operates in an absorption mode, the central idea of the ARG-based scheme is that the probe field operates in a stimulated Raman emission mode. We show that, using our scheme a probe field can acquire phase-controlled intensity gain, phase shift, and superluminal group velocity, suffering no attenuation or distortion. New features of our scheme raise the possibility of rapidly responding optical switches and gates for information science.

We consider a four-state tripod atomic system (Fig. 1(a)) interacting with two strong, continuous-wave (CW) pump fields of angular frequencies ω_{L1} ($|1\rangle \leftrightarrow |4\rangle$), ω_{L2} ($|2\rangle \leftrightarrow |4\rangle$) and a weak, pulsed probe field of center angular frequency $\omega_{\rm p}$ ($|4\rangle \leftrightarrow |3\rangle$). Under a rotating-wave approximation, the Schrödinger equation controlling atomic response reads

$$\mathrm{i}\frac{\partial}{\partial t}A_1+\Omega_{L1}^{*}A_4=0, \tag{1a}$$

$$\mathrm{i}\frac{\partial}{\partial t}A_2+\Omega_{L2}^{*}A_4=0, \tag{1b}$$

$$\left(\mathrm{i}\frac{\partial}{\partial t}+d_3\right)A_3+\Omega_{\rm p}^{*}A_4=0, \tag{1c}$$

$$\left(\mathrm{i}\frac{\partial}{\partial t}+d_4\right)A_4+\Omega_{L1}A_1+\Omega_{L2}A_2+\Omega_{\rm p}A_3=0, \tag{1d}$$

where A_j (j = 1, 2, 3, 4) is the probability amplitude in an interaction picture of the bare atomic state $|j\rangle$ (with eigenenergy E_j). The states $|1\rangle$ and $|2\rangle$ are hyperfine splitting of the ground state. $d_4=\Delta_4+\mathrm{i}\gamma_4$ with $\Delta_4=\omega_{L1}-(E_4-E_1)/\hbar=\omega_{L2}-(E_4-E_2)/\hbar$ being the one-photon detuning and γ_4 the decay rate of the state $|4\rangle$. $d_3=\Delta_3+\mathrm{i}\gamma_3$ with $\Delta_3=\omega_{L1}-\omega_{\rm p}-(E_3-E_1)/\hbar=\omega_{L2}-\omega_{\rm p}-(E_3-E_2)/\hbar$ being the two-photon detuning

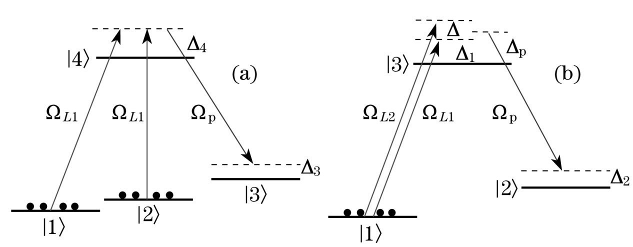

Fig. 1. Tripod four-state ARG system interacting with two strong, CW pump fields of Rabi frequencies Ω_{L1}, Ω_{L2} and a weak, pulsed probe field of Rabi frequency $\Omega_{\rm p}$. Δ_j are detunings of energy state $|j\rangle$ ($j=3,4$). (b) Three-state ARG system used in Ref [12].

and γ_3 the decay rate of the state $|3\rangle$. $2\Omega_x$ ($x = L1$, $L2$, p) is the Rabi frequency associated with the relevant laser-driven atomic transition.

In order to predict the propagation of the probe field, Eq. (1) must be solved simultaneously with Maxwell equation for the probe field, which is reduced to the form

$$\mathrm{i}\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_{\mathrm{p}}+\kappa A_4 A_3^*=0 \tag{2}$$

under a slowly-varying envelope approximation, where c is the light speed in vacuum and $\kappa=\mathcal{N}_{\mathrm{a}}\omega_{\mathrm{p}}|\mathbf{p}_{23}|^2/(2\epsilon_0 c\hbar)$ with $\mathcal{N}_{\mathrm{a}}$ being the atomic density, $\mathbf{p}_{34}$ the electric dipole matrix element for the transition from $|3\rangle$ to $|4\rangle$.

We assume that atoms are initially populated in the states $|1\rangle$ and $|2\rangle$. Under the action of the strong CW pump field, a significant transfer of the population from ground states $|1\rangle$ and $|2\rangle$ to the excited state $|4\rangle$ can occur. Furthermore, Doppler effect is significant for the warm atomic vapor at room temperature. In order to suppress a large gain and the Doppler effect, we assume that the one-photon detuning Δ_4 is large enough. In addition, large Δ_4 and active gain scheme can make the probe field maintain or gain its amplitude, resulting in excellent signal-to-noise ratio. Since $\Omega_{L1,L2}\gg\Omega_{\mathrm{p}}$ and $A_{1,2}\gg \mathrm{A}_3$, Eq. (1d) can be solved under adiabatical approximation. We obtain

$$A_4=-\frac{\Omega_{L1}}{d_4}A_1-\frac{\Omega_{L2}}{d_4}A_2. \tag{3}$$

Substituting Eq. (3) into Eqs. (1a) and (1b), we obtain

$$\left(\mathrm{i}\frac{\partial}{\partial t}-\frac{|\Omega_{L1}|^2}{d_4}\right)A_1-\frac{\Omega_{L1}^*\Omega_{L2}}{d_4}A_2=0, \tag{4a}$$

$$\left(\mathrm{i}\frac{\partial}{\partial t}-\frac{|\Omega_{L2}|^2}{d_4}\right)A_2-\frac{\Omega_{L1}\Omega_{L2}^*}{d_4}A_1=0. \tag{4b}$$

It is easy to get the solutions of $A_1 = A_1(0)\exp\{-\mathrm{i}[(|\Omega_{L1}|^2+|\Omega_{L2}|^2)/d_4]t\}$ and $A_2 = A_2(0)\exp\{-\mathrm{i}[(|\Omega_{L1}|^2+|\Omega_{L2}|^2)/d_4]t\}$. Substituting Eq. (3) into Eqs. (1c) and (2), and using the solutions of Eq. (4), we obtain

$$\left(\mathrm{i}\frac{\partial}{\partial t}+d_3\right)A_3 -\left[\frac{\Omega_{L1}}{d_4}A_1(0)+\frac{\Omega_{L2}}{d_4}A_2(0)\right]\mathrm{e}^{-\mathrm{i}\beta t}\Omega_{\mathrm{p}}^*=0, \tag{5a}$$

$$\mathrm{i}\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_{\mathrm{p}}^* +\kappa\left[\frac{\Omega_{L1}^*}{d_4^*}A_1(0)+\frac{\Omega_{L2}^*}{d_4^*}A_2(0)\right]\mathrm{e}^{\mathrm{i}\beta^* t}A_3=0, \tag{5b}$$

where $\beta=(|\Omega_{L1}|^2+|\Omega_{L2}|^2)/d_4$. The exponential function in the above equations can be further eliminated by introducing $A_3 = A_3'\exp[-\mathrm{i}\mathrm{Re}(\beta)]$ under the condition that $\mathrm{Re}[\beta]\gg\mathrm{Im}[\beta]$.

By using Fourier transform, Eq. (5) is converted into

$$\left(\tilde{\beta}-\omega+d_3\right)a_3' -\left[\frac{\Omega_{L1}}{d_4}A_1(0)+\frac{\Omega_{L2}}{d_4}A_2(0)\right]\Lambda_{\mathrm{p}}^*=0, \tag{6a}$$

$$\mathrm{i}\left(\frac{\partial}{\partial z}+\frac{\mathrm{i}}{c}\omega\right)\Lambda_{\mathrm{p}}^* +\kappa\left[\frac{\Omega_{L1}^*}{d_4^*}A_1(0)+\frac{\Omega_{L2}^*}{d_4^*}A_2(0)\right]a_3'=0, \tag{6b}$$

where $\tilde{\beta}=(|\Omega_{L1}|^2+|\Omega_{L2}|^2)\Delta_4/(\Delta_4^2+\gamma_4^2)$, a_3' and Λ_{p} are the Fourier transforms of A_3' and Ω_{p}, respectively. Solving Eq. (6) yields

$$\Lambda_{\mathrm{p}}(z,\omega)=\Lambda_{\mathrm{p}}(0,\omega)\mathrm{e}^{\mathrm{i}K(\omega)z}, \tag{7}$$

where

$$K(\omega)=\frac{\omega}{c}-\kappa\frac{|\Omega_{L1}+\Omega_{L2}|^2}{2(\tilde{\beta}-\omega+d_3^*)|d_4|^2} \tag{8}$$

is the linear dispersion relation of the system. When obtaining Eq. (8), $A_1(0)=A_2(0)=\sqrt{2}/2$ is assumed, i.e., the atoms are initially populated equally in the states $|1\rangle$ and $|2\rangle$.

In most operation conditions, $K(\omega)$ can be expanded into a rapid conversion power series around the center frequency ω_{p} of the probe field ($\omega=0$). We thus have $K(\omega)= K_0+K_1\omega+\frac{1}{2}K_2\omega^2+\cdots$, with $K_j = [\partial^j K(\omega)/\partial\omega^j]|_{\omega=0}$ ($j=0, 1, 2, \cdots$). With Eq. (8), the dispersion coefficients K_j can be obtain analytically and their physical interpretation is rather clear. $K_0 = \phi+\mathrm{i}\alpha/2$ describes the phase shift ϕ per unit length and absorption (gain) coefficient α of the probe field with

$$\phi=-\kappa\frac{|\Omega_{L1}+\Omega_{L2}|^2\Delta_3}{2[(\tilde{\beta}+\Delta_3)^2+\gamma_3^2](\Delta_4^2+\gamma_4^2)}, \tag{9a}$$

$$\alpha=-\kappa\frac{|\Omega_{L1}+\Omega_{L2}|^2\gamma_3}{[(\tilde{\beta}+\Delta_3)^2+\gamma_3^2](\Delta_4^2+\gamma_4^2)}. \tag{9b}$$

We see that since $\alpha<0$, the probe field acquires a gain during the propagation, which is different to EIT-based schemes where a probe field always suffers an attenuation. Furthermore, both the phase shift and intensity gain of the probe field are proportional to $|\Omega_{L1}+\Omega_{L2}|^2$, and hence we can manipulate the relative phase of the two pump fields to implement an effective control over them. Particularly, if the two pump fields satisfy $\Omega_{L1} = \Omega_L\exp(\mathrm{i}\theta)$ and $\Omega_{L2}=\Omega_L\exp[\mathrm{i}(\theta+\pi)]$, a destructive interference happens and hence $\phi=\alpha=0$. Consequently, the static intensity gain and phase shift can be eliminated simultaneously in this order. This behavior is very different from that in the system with a two-mode ARG core (Fig. 1(b)) where a gain of the probe field is always present even a static phase shift is eliminated[12].

The linear coefficient K_1 determines the group velocity of the probe field given by $V_{\mathrm{g}}=1/\mathrm{Re}(K_1)$. From Eq. (8), we obtain

$$V_{\mathrm{g}}=\left\{\frac{1}{c}-\frac{\kappa}{2}\frac{|\Omega_{L1}+\Omega_{L2}|^2[(\tilde{\beta}+\Delta_3)^2-\gamma_3^2]}{[(\tilde{\beta}+\Delta_3)^2+\gamma_3^2]^2(\Delta_4^2+\gamma_4^2)}\right\}^{-1}. \tag{10}$$

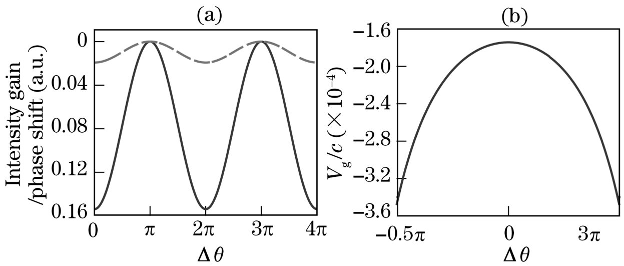

Fig. 2. (a) Intensity gain α (dashed line) and phase shift ϕ (solid line) of the probe field as functions of the relative phase $\Delta\theta = \theta_1 - \theta_2$ between the two pump fields. (b) Group velocity of the probe field $V_{\rm g}/c$ as a function of the relative phase $\Delta\theta$.

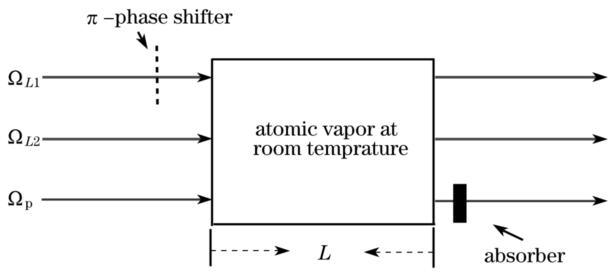

Fig. 3. Possible design of a gain switch based on the present ARG scheme. $\Omega_{L1,L2}$: pump fields; $\Omega_{\rm p}$: probe field. L: length of the gas cell.

It is easy to show that the probe field can be either superluminal or subluminal, depending on the system parameters. However, if $\Omega_{L1} = \Omega_L \exp({\rm i}\theta)$ and $\Omega_{L2} = \Omega_L \exp[{\rm i}(\theta + \pi)]$, the probe field travels with c.

The group velocity dispersion can be characterized by the quadratic coefficient $K_2 = -\kappa|\Omega_{L1} + \Omega_{L2}|^2/[(\tilde{\beta} + d_3^*)^3|d_4|^2]$. For a Gaussian input of the probe field, i.e., $\Omega_{\rm p}(0,t) = \Omega_{\rm p}(0,0)\exp(-t^2/\tau_0^2)$ with the pulse length τ_0, we obtain

$$\Omega_{\rm p}(z,t) = \frac{\Omega_{\rm p}(0,0)}{\sqrt{b_1(z) - {\rm i}b_2(z)}} \times \exp\left[{\rm i}K_0 z - \frac{(t - K_1 z)^2}{[b_1(z) - {\rm i}b_2(z)]\tau_0^2}\right], \quad (11)$$

where $b_1(z) = 1 + 2z{\rm Im}(K_2)/\tau_0^2$ and $b_2(z) = 2z{\rm Re}(K_2)/\tau_0^2$. Equation (11) shows that linear and quadratic dispersion effects contribute to the probe field attenuation (gain), phase shift, group velocity, and propagation-dependent pulse spreading. These effects can be effectively controlled by the relative phase of two pump fields since the coefficients characterizing these effects are both proportional to $|\Omega_{L1} + \Omega_{L2}|^2$.

Now we consider an atomic system that can be realized by a room-temperature alkali atomic vapor. Physical parameters suitable to this system are given as $2\gamma_3 = 1.0\times10^5$ Hz and $2\gamma_4 = 500$ MHz. We take $\kappa = 1.0 \times 10^9$ cm$^{-1}\cdot$ s^{-1}, $\Omega_{L1} = \Omega_{L2} = 1.0 \times 10^7$ s^{-1}, $\Delta_3 = 5.0 \times 10^8$ s^{-1}, and $\Delta_4 = 1.0 \times 10^9$ s^{-1}. With these parameters, we get $K_0 = -(1.90 + {\rm i}0.10) \times 10^{-1}$ cm^{-1}, $K_1 = -(1.91 + {\rm i}0.19) \times 10^{-7}$ cm$^{-1}\cdot$ s, and $K_2 = -(3.84 + {\rm i}0.59) \times 10^{-13}$ cm$^{-1}\cdot$ s^2. We see that the imaginary part of the coefficients are much smaller than their corresponding real part. The ratio of the group velocity to the light speed in vacuum $V_{\rm g}/c = -1.7 \times 10^{-4}$ and hence the probe field travels with a superluminal propagating velocity. If taking $\Omega_{L1} = 1.0 \times 10^7$ s^{-1} and $\Omega_{L2} = 1.0 \times 10^7 \exp({\rm i}\pi)$ s^{-1} without changing any other parameters, we get $K_0 = K_2 = 0$ and $K_1 = 0.33 \times 10^{-10}$ cm$^{-1}\cdot$ s, and hence the ratio $V_{\rm g}/c = 1.0$.

Figure 2(a) shows the curves of intensity gain α and phase shift ϕ of the probe field versus the relative phase $\Delta\theta = \theta_1 - \theta_2$ between the two pump fields. Figure 2(b) shows the curve of the ratio of group velocity of the probe field to the light speed in vacuum, $V_{\rm g}/c$, versus $\Delta\theta$. When plotting the figure, we have taken $\Omega_{L1} = 1.0 \times 10^7 \exp({\rm i}\theta_1)$ s^{-1}, $\Omega_{L2} = 1.0 \times 10^7 \exp({\rm i}\theta_2)$ s^{-1} and kept the other parameters as given above. We see that the gain, phase shift, and group velocity of the probe field can be tuned through changing the relative phase of the two pump fields. The phase shift is the maximum when taking $\Delta\theta = 2\pi$. The negative group velocity of the probe field demonstrates the superluminal characteristic of the system.

The interesting feature of our scheme presented above allows us to implement possible rapidly responding optical switches and logic gates. As an example, we construct a gain switch by means of the controllable intensity gain. As we have mentioned, when $\Delta\theta = 0$ two pump fields constructively interfere, the probe field intensity acquires a maximum gain. However, when $\Delta\theta = \pi$ the two pump fields destructively interfere, there is no gain acquired by the probe field. If taking $\Omega_{\rm p} = 1.0 \times 10^6$ s^{-1}, the length of the atomic gas cell $L = 10$ cm , and other parameters being the same as given above, we obtain the intensity gain $|\alpha| = 0.19$ and the probe intensity grows about 21% resulting from the constructive interference of the two pump fields. The possible design of such a gain switch is shown in Fig. 3. The two pump fields Ω_{L1} and Ω_{L2} are produced by the same laser source. If we put a π-phase shifter, the two pump fields will destructively interfere and the output of the probe field will be totally absorbed by the absorber. If we take away the π-phase shifter, the two pump fields will constructively interfere and the output of the probe field will not be totally absorbed due to the acquired intensity gain. The unabsorbed intensity of the probe field is about 7.0×10^{-2} mW/cm^2. The response time of the gain switch in the present ARG scheme is faster than EIT schemes because of the superluminal characteristic of the system.

In conclusion, we have proposed a four-level tripod scheme with an ARG for obtaining phase-controlled gain, phase shift, and group velocity at room temperature. We have shown that the new scheme can be used to eliminate significant probe field attenuation and distortion unavoidable in EIT-based schemes. The intensity gain, phase shift, and group velocity of a probe field can be simultaneously manipulated and controlled, which can be applied to the design of rapidly responding optical switches and gates for information science.

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