Highly efficient four-wave mixing in a coherent six-level system in ultraslow propagation regime

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(Received 30 May 2007; published 10 October 2007)

We propose a scheme to obtain a highly efficient four-wave mixing (FWM) in a coherent six-level tripod system by using a double-dark resonance and multiphoton destructive interference induced transparency. We show both analytically and numerically that the optical-wave mixing in such a system with a pump wave and an internally generated FWM wave mediated by electromagnetically induced transparency and hence propagating with matched ultraslow group velocities can result in a significant enhancement of FWM conversion efficiency. This interesting scheme may be used to generate coherent short-wavelength radiation in dense optical media with very low pump-wave intensity.

DOI: 10.1103/PhysRevA.76.043809 PACS number(s): 42.50.Gy, 42.65.Ky, 42.50.Hz

I. INTRODUCTION

It is well known that nonlinear optical processes can be largely enhanced by utilizing resonant atomic and molecular systems [1] and hence it is hopefully possible to obtain highly efficient frequency conversions. Unfortunately, attempts to use such resonance enhancement have long been frustrated by the problem associated with serious optical absorption. However, this paradigm has been challenged by recent theoretical and experimental studies on electromagnetically induced transparency (EIT) [2,3]. By means of the quantum interference effect induced by control laser fields, the absorbtion of a probe laser field tuned to a strong one-photon resonance can be largely suppressed and hence an initially highly opaque optical medium becomes transparent. The wave propagation in such EIT media displays many striking features, including the drastic reduction of group velocity and the significant enhancement of Kerr nonlinearity, which are beneficial to certain important nonlinear optical processes under weak driving conditions [2,3]. Based on these features, it has been shown recently that optical solitons with ultraslow propagating velocity are possible [4–8]. On the other hand, high-efficiency multiwave mixing or large parametric gain can also be achieved in such highly resonant, phase-coherent optical media [3].

The motivation for the study of four-wave mixing (FWM) in EIT-related media lies in its interesting physical properties and promising applications in high-efficiency generation of short-wavelength coherent radiation at pump intensity approaching the single-photon level, nonlinear spectroscopy at very low light intensity, quantum single-photon nonlinear optics, and quantum information science. The FWM in EIT media in the ultraslow propagation regime has been the focus of several recent studies [9–17]. In Ref. [12], Deng *et al.* proposed a novel channel opening technique of FWM in a four-level system based on EIT and associated ultraslow propagation and showed that nonexisting wave-mixing channels may be opened deeply inside the medium and the efficiency of the FWM can be significantly enhanced. Later on two generalizations of Ref. [12] to five-level systems appeared by replacing a two-photon transition into two one-photon transitions in upper levels (Ref. [15]) or by using a two-photon transition in upper levels but a tripod configuration in lower levels [17]. However, in all these schemes the FWM conversion efficiency is still low.

In the present work, we propose a six-level tripod scheme to achieve a much higher FWM conversion efficiency than that obtained in Refs. [15,17]. Different from the configuration in Ref. [17], we use two one-photon transitions in upper levels and show both analytically and numerically that with four continuous-wave (cw) control laser fields and a weak, pulsed pump field, a pulsed FWM field can be effectively generated. This different scheme may be used to generate coherent short-wavelength radiation in dense optical media with very low pump-wave intensity. The paper is arranged as follows. In Sec. II we introduce the theoretical model of the six-level tripod system. The analytical solution of this model for the FWM is presented in Sec. III. In Sec. IV we discuss the FWM conversion efficiency and make a detailed comparison between our result and those obtained by using different schemes. Finally, in the last section we provide a numerical simulation of the FWM generation, compare the numerical and the approximated analytical solutions, and present a summary of the main results of this work.

II. MODEL

Consider a six-level tripod atomic system interacting with one weak, pulsed pump field and four strong, cw laser fields (see Fig. 1). The pump field with center frequency ω_p (half Rabi frequency Ω_p, turned to $|0\rangle \rightarrow |3\rangle$ resonance) serves as the first step of the three-photon excitation of state $|1\rangle$. Two cw fields ω_a (half Rabi frequency Ω_a) and ω_b (half Rabi frequency Ω_b) provide, respectively, two one-photon transitions from $|3\rangle$ to $|2\rangle$ and $|2\rangle$ to $|1\rangle$ to complete the three-photon excitation from state $|0\rangle$ to state $|1\rangle$. Two additional cw fields, with frequencies ω_c (half Rabi frequency Ω_c) and

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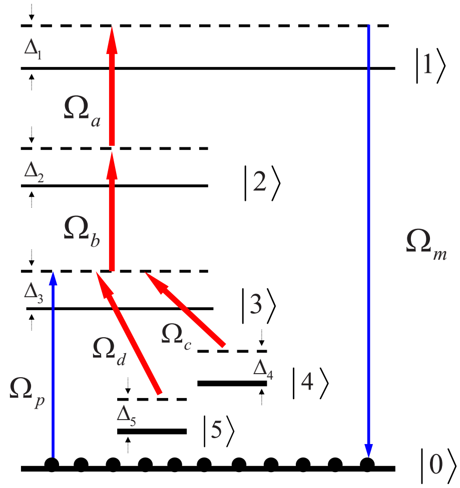

FIG. 1. (Color online) The energy-level diagram and excitation scheme of a lifetime broadened six-state system interacting with four cw control fields ($\Omega_{a,b,c,d}$), and a weak, pulsed pump field (Ω_p) to generate a pulsed FWM field (Ω_m). Δ_j (j=1 to 5) are detunings for relevant transitions.

ω_d (half Rabi frequency Ω_d), respectively, form a tripod configuration by the states $|0\rangle$, $|3\rangle$, $|4\rangle$, and $|5\rangle$, providing a modified EIT configuration and thus a dispersion manipulation of the state $|3\rangle$. Δ_j ($j=1-5$) are detunings in relevant optical transitions. It is expected that an internally generated pulsed field with center frequency ω_m (half Rabi frequency Ω_m, turned to $|1\rangle \to |0\rangle$ resonance) will appear through the FWM process $|0\rangle \to |3\rangle \to |2\rangle \to |1\rangle \to |0\rangle$ with $\omega_m = \omega_p + \omega_a + \omega_b$. The electric-field vector of the system is $\mathbf{E} = \sum_{l=p,m,a,b,c,d} \mathbf{e}_l \mathcal{E}_l \exp[i(k_l z - \omega_l t)] + \text{c.c.}$, where $\mathbf{e}_l$ is the unit vector of the polarization component of the electric field with the envelope $\mathcal{E}_l$. The Rabi frequencies are defined, respectively, by $\Omega_p = (\mathbf{e}_p \cdot \mathbf{p}_{03})\mathcal{E}_p/\hbar$, $\Omega_m = (\mathbf{e}_m \cdot \mathbf{p}_{01})\mathcal{E}_m/\hbar$, $\Omega_a = (\mathbf{e}_a \cdot \mathbf{p}_{21})\mathcal{E}_a/\hbar$, $\Omega_b = (\mathbf{e}_b \cdot \mathbf{p}_{32})\mathcal{E}_b/\hbar$, $\Omega_c = (\mathbf{e}_c \cdot \mathbf{p}_{43})\mathcal{E}_c/\hbar$, and $\Omega_d = (\mathbf{e}_d \cdot \mathbf{p}_{53})\mathcal{E}_d/\hbar$, where $\mathbf{p}_{ij}$ is the electric dipole matrix element associated with the transition from $|i\rangle$ to $|j\rangle$. In an interaction picture and under rotating-wave and slowly varying envelope approximations, the dynamics of atomic response and the optical field is governed by the Maxwell-Schrödinger (MB) equations

$$i\frac{\partial a_0}{\partial t} + \Omega_p^* a_3 + \Omega_m^* a_1 = 0, \tag{1a}$$

$$\left(i\frac{\partial}{\partial t} + d_1\right)a_1 + \Omega_a a_2 + \Omega_m a_0 = 0, \tag{1b}$$

$$\left(i\frac{\partial}{\partial t} + d_2\right)a_2 + \Omega_a^* a_1 + \Omega_b a_3 = 0, \tag{1c}$$

$$\left(i\frac{\partial}{\partial t} + d_3\right)a_3 + \Omega_b^* a_2 + \Omega_p a_0 + \Omega_d a_5 + \Omega_c a_4 = 0, \tag{1d}$$

$$\left(i\frac{\partial}{\partial t} + d_{4(5)}\right)a_{4(5)} + \Omega_{c(d)}^* a_3 = 0, \tag{1e}$$

$$i\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_{p(m)} + \kappa_{03(01)} a_{3(1)} a_0^* = 0, \tag{1f}$$

where $d_j = \Delta_j + i\gamma_j$ with Δ_j being the detuning, a_j and γ_j are, respectively, the probability amplitude and the decay rate of the atomic state $|j\rangle$ ($j=0-5$). The coupling constants in Eq. (1f) are given by $\kappa_{03(01)} = \mathcal{N}_a \omega_{p(m)} |\mathbf{p}_{03(01)}|^2 / (2\epsilon_0 c\hbar)$, with $\mathcal{N}_a$ being the atomic density. In addition, we have the relations $\Delta_1 = \omega_m - \omega_{10}$, $\Delta_2 = \omega_m - \omega_a - \omega_{20}$, $\Delta_3 = \omega_p - \omega_{30}$, and $\Delta_{4(5)} = \omega_p - \omega_{c(d)} - \omega_{40(50)}$ with $\omega_{ij} = \omega_i - \omega_j$.

III. ANALYTICAL SOLUTION FOR A FWM

We now study the solution of a FWM problem based on the MB equations (1a)–(1f). Because the pulsed pump field is assumed very weak, the atomic ground state $|0\rangle$ is not depleted, i.e., $a_0 \approx 1$. In this situation Eqs. (1a)–(1f) become linear and hence can be solved analytically. A plane-wave solution takes the form

$$a_j = A_j \exp\{i[K(\omega)z - \omega t]\} \quad (j = 1 \text{ to } 5) \tag{2a}$$

$$\Omega_{p(m)} = \Lambda_{p(m)} \exp\{i[K(\omega)z - \omega t]\}. \tag{2b}$$

It is readily to obtain the following relations

$$A_1 = \frac{\Omega_a \Omega_b}{D} L_4 L_5 \Lambda_p + \frac{D_1}{D}\Lambda_m, \tag{3a}$$

$$A_3 = \frac{D_3}{D} L_4 L_5 \Lambda_p + \frac{\Omega_a^* \Omega_b^*}{D} L_4 L_5 \Lambda_m, \tag{3b}$$

where we have defined $D = L_1 L_2 L_5 |\Omega_c|^2 + L_1 L_2 L_4 |\Omega_d|^2 + L_1 L_4 L_5 |\Omega_b|^2 + L_3 L_4 L_5 |\Omega_a|^2 - L_5 |\Omega_a|^2 |\Omega_c|^2 - L_4 |\Omega_a|^2 |\Omega_d|^2 - L_1 L_2 L_3 L_4 L_5$, $D_1 = L_2 L_3 L_4 L_5 - L_2 L_5 |\Omega_c|^2 - L_2 L_4 |\Omega_d|^2 - L_4 L_5 |\Omega_b|^2$, and $D_3 = L_1 L_2 - |\Omega_a|^2$, with $L_j = \omega + d_j$ ($j=1-5$). The dispersion relation of the system is given by

$$K_{\pm} = \frac{\omega}{c} + \frac{\kappa_{01} D_1 + \kappa_{03} L_4 L_5 D_3 \pm G}{2D}, \tag{4}$$

with $G = [(\kappa_{01} D_1 - \kappa_{03} L_4 L_5 D_3)^2 + 4\kappa_{01}\kappa_{03} L_4^2 L_5^2 |\Omega_a|^2 |\Omega_b|^2]^{1/2}$. Thus the system allows two branches of dispersion curves K_+ and K_-.

In Fig. 2 we have plotted the imaginary part of K_+ (solid line) and K_- (dashed line) as functions of dimensionless frequency $\omega\tau$, where τ is the pulse length of the pump field. Parameters are chosen as $\tau = 1.0 \times 10^{-6}$ s, $\kappa_{01} = 10^9$ (s m)$^{-1}$, $\kappa_{03} = 10^{11}$ (s m)$^{-1}$, $\Delta_1\tau = 3.0$, $\Delta_2\tau = 5.0$, $\Delta_3\tau = -1.0$, $\Delta_4\tau = 5.0$, $\Delta_5\tau = -1.0$, $\gamma_1\tau = 0.09$, $\gamma_2\tau = 0.8$, $\gamma_3\tau = 5.9$, $\gamma_4\tau = \gamma_5\tau = 0$, $\Omega_a\tau = \sqrt{10}$, $\Omega_b\tau = 15\sqrt{2}$, $\Omega_c\tau = 4.0$, and $\Omega_d\tau = 12\sqrt{2}$. A realistic experimental candidate for such system is ^{85}Rb atoms [15,16]. We see that each branch of the dispersion curves has two transparency windows for wave propagation. This interesting property is due to a double dark resonance of the system which has received considerable attention in recent years

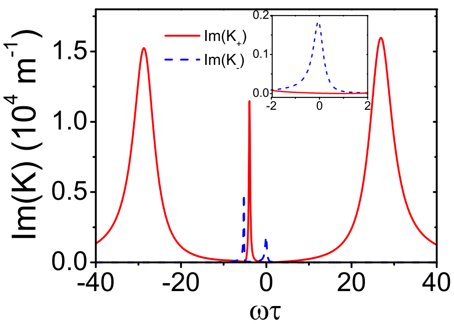

FIG. 2. (Color online) The imaginary part of the dispersion relations Im(K_+) (solid line) and Im(K_-) (dashed line) as functions of $\omega\tau$ of the six-level tripod system. The parameters of the system have been given in the text. The inset shows the detail of Im $K_\pm$ near $\omega=0$.

[18–21]. Notice that the transparency windows are very different for the K_+ and K_- modes. It is expected that around $\omega=0$ the absorption of the K_+ mode is much less than that of the K_- mode for an optical thick medium [22].

The general solution of Eqs. (1) can be obtained by the Fourier superposition of the solution (2):

$$\Omega_p(z,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega[F_p^+\exp(i\theta_+) + F_p^-\exp(i\theta_-)], \quad (5a)$$

$$\Omega_m(z,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\frac{D}{\kappa_{03}\Omega_a^*\Omega_b^* L_4L_5}\times\left[\left(K_+ - \frac{\omega}{c} - \kappa_{03}\frac{D_3}{D}L_4L_5\right)F_p^+\exp(i\theta_+) + \left(K_- - \frac{\omega}{c} - \kappa_{03}\frac{D_3}{D}L_4L_5\right)F_p^-\exp(i\theta_-)\right], \quad (5b)$$

with $\theta_\pm = K_\pm(\omega)z - \omega t$, where $F_p^\pm$ is determined by a given boundary condition. For a FWM problem the boundary condition is

$$\Omega_p(0,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\exp(-i\omega t)\Lambda_p(0,\omega), \quad (6a)$$

$$\Omega_m(0,t) = 0, \quad (6b)$$

with $\Lambda_p(0,\omega) = \int_{-\infty}^{+\infty} dt\exp(i\omega t)\Omega_p(0,t)$. Then we obtain the FWM solution

$$\Omega_p(z,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\frac{U_+\exp(i\theta_-) - U_-\exp(i\theta_+)}{U_+ - U_-}\Lambda_p(0,\omega), \quad (7a)$$

$$\Omega_m(z,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\frac{U_+U_-}{U_+ - U_-}[\exp(i\theta_-) - \exp(i\theta_+)]\Lambda_p(0,\omega), \quad (7b)$$

with $U_\pm = (\kappa_{01}D_1 - \kappa_{03}D_3L_4L_5 \pm G)/(2\kappa_{03}\Omega_a^*\Omega_b^*L_4L_5)$.

IV. FWM CONVERSION EFFICIENCY

The general expression of the FWM solution given by Eqs. (7a) and (7b) can be simplified under suitable and realistic physical conditions. As in Ref. [16] we focus on the adiabatic regime where the power series of $K_\pm$ and $U_\pm$ on ω converge rapidly. By taking $K_\pm = K_\pm(0) + \omega/V_{g_\pm} + O(\omega^2)$, $U_\pm = W_\pm(0) + O(\omega)$, we obtain

$$\Omega_p = \frac{W_+\exp(iK_-z)\Omega_p(\eta_-) - W_-\exp(iK_+z)\Omega_p(\eta_+)}{W_+ - W_-}, \quad (8a)$$

$$\Omega_m = \frac{W_+W_-}{W_+ - W_-}[\exp(iK_-z)\Omega_p(\eta_-) - \exp(iK_+z)\Omega_p(\eta_+)], \quad (8b)$$

where $\eta_\pm = t - z/V_{g_\pm}$ with $V_{g_\pm} = 1/\mathrm{Re}(\partial K_\pm/\partial\omega|_{\omega=0})$ being the group velocity of the $K_\pm$ mode.

We are interested in the FWM problem for an optically thick medium. From Fig. 2 we see that both Im K_+ and Im K_- arse positive. However, around the center frequency of the pump and FWM waves, i.e., at $\omega=0$, one has Im $K_- \gg$ Im K_+ [22]. Thus after propagating for a longer optical depth the amplitude of the K_- mode decays away rapidly and hence the K_- mode can be neglected. Then Eqs. (8a) and (8b) are simplified as

$$\Omega_p(z,t) = \frac{W_-}{W_- - W_+}\exp(iK_+z)\Omega_p(\eta_+), \quad (9a)$$

$$\Omega_m(z,t) = \frac{W_+W_-}{W_- - W_+}\exp(iK_+z)\Omega(\eta_+). \quad (9b)$$

We see that both the pump and the FWM fields have a matched group velocity V_{g_+}. The FWM conversion efficiency is given by [16]

$$\eta = \frac{|W_+W_-|^2}{|W_+ - W_-|^2}|\exp(iK_+z)|^2. \quad (10)$$

The solid line shown in Fig. 3 is the result of η as a function of dimensionless optical depth $\mathcal{N}_a\sigma_{01}z$, where $\sigma_{01} = \mathbf{p}_{01}^2\omega_m/(c\epsilon_0\hbar\gamma_1)$ is absorption cross section related to the FWM field. The parameters are chosen as $\mathcal{N}_a = 6.2\times10^8$ cm^{-3}, $|\mathbf{p}_{01}| = 1.2\times10^{-27}$ cm C, $\lambda_m = 299$ nm. The other parameters are the same as in Fig. 2. With these parameters the optical depth from 0 to 4.4×10^2 corresponds to propagating distance z from 0 to 2 cm.

For comparison we have also calculated the FWM conversion efficiency in the case of no EIT effect (i.e., $\Omega_c = \Omega_d = 0$ [23]). The result is shown by the dashed line in Fig. 3. We see that only for very small optical depth the FWM conversion efficiency without EIT can be a little larger than that with EIT (i.e., nonvanishing Ω_c and Ω_d). However, as the optical depth increases it decreases rapidly. In contrast, the conversion efficiency with EIT has only a very slow decay. Consequently, to increase the useful flux we need to work in large optical depth regime where EIT is important so that the system has a large FWM conversion efficiency. This is the

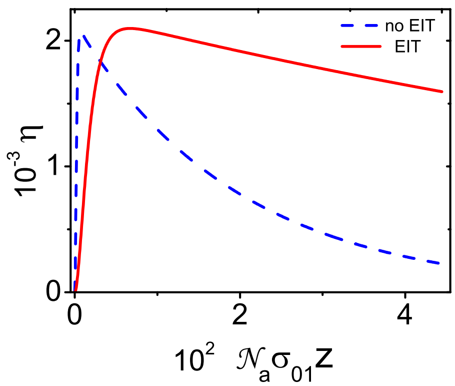

FIG. 3. (Color online) The FWM conversion efficiency η as a function of the dimensionless optical depth $\mathcal{N}_a\sigma_{01}z$. The solid (dashed) line is the result of with (without) EIT effect.

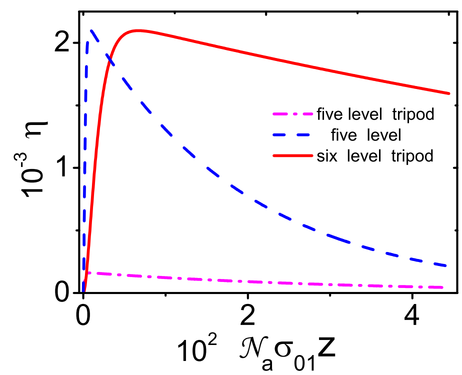

FIG. 4. (Color online) The FWM conversion efficiencies η as functions of the dimensionless optical depth $\mathcal{N}_a\sigma_{01}z$ for different schemes. Dashed line: five-level system (Ref. [15]). Dot-dashed line: five-level tripod system (Ref. [17]). For illustration, the η value for the five-level tripod system (Ref. [17]) has been amplified 10^3 times. Parameters used are the same as in Fig. 3.

key idea of channel opening technique [12,16]. The physical reason for the above result is basically the same as that illustrated in Ref. [16] and can be understood as the outcome of multiphoton destructive interference induced transparency. When the FWM field becomes sufficiently intense, an additional excitation channel to the state $|3\rangle$, i.e., $|0\rangle \to |3\rangle$ via $\Omega_m+\Omega_a^*+\Omega_b^*$, opens and becomes important. This efficient backcoupling pathway leads to competitive multiphoton excitations of the FWM generating state via five supplied and one internally generated fields. This competition is destructive in nature, resulting in the suppression of the amplitudes of the states $|j\rangle$ $(j=1-3)$ and hence the transparency of the pump and the FWM fields. In addition, with the parameters used above, we obtain $V_{g_+} \approx 1.2\times 10^{-5}c$, much less than the light speed in vacuum. This matched, ultraslow propagating velocity of both the pump and FWM waves increases the field-medium interaction time and also results in an increase of the FWM conversion efficiency of the system.

In order to demonstrate the advantage of our scheme, we have made additional calculations on the FWM conversion efficiencies relevant to different systems reported in the literature. The calculating result of η as functions of the dimensionless optical depth $\mathcal{N}_a\sigma_{01}z$ for different schemes is shown in Fig. 4. In the figure, the dashed (dot-dashed) line is obtained by taking $\Omega_d\tau=0$ $(|\Delta_2|\tau=10^3 \gg 1)$, corresponding to the five-level system used in Ref. [15] (Ref. [17]). The result based on our six-level tripod system is shown by the solid line. The parameters are the same as used in Fig. 3. Notice that for illustration the η value of the five-level tripod system used by Ref. [17] has been amplified 10^3 times.

From Fig. 4 we see that for an optical thick medium the FWM conversion efficiency of our six-level tripod scheme (solid line) is higher than that obtained by using the scheme in Ref. [15] (dashed line). The reasons are the following. The main difference between our scheme and the one in Ref. [15] is that we have utilized the double-dark resonance induced by Ω_c and Ω_d while in Ref. [15] only a single-dark resonance (i.e., conventional EIT) is used. The double-dark resonance scheme has several advantages for enhancing the FWM conversion efficiency in comparison with the single-dark resonance one. For example, the double-dark resonance provides two transparency windows, and in different windows the FWM conversion efficiency are different. Due to quantum interference effect there is one window in which the conversion efficiency is bigger than the other. In addition, in the double-dark resonance scheme more physical parameters (e.g., the control fields Ω_c and Ω_d) can be manipulated and hence one can select suitable parameters to make the conversion efficiency higher than that in the single-dark resonance case.

We see from Fig. 4 that, in comparison with the result obtained by the five-level tripod scheme used in Ref. [17] where the maximum FWM conversion efficiency is $\eta_{max} = 1.6\times 10^{-7}$ (dot-dashed line), there is an increase of four orders of magnitude by using our present six-level tripod scheme (solid line with $\eta_{max}=2.1\times 10^{-3}$). This giant increase of η is due to the replacement of a two-photon transition (used in Ref. [17]) by two single-photon transitions (used in our scheme). Because the transition probability of the single-photon transitions is much bigger than the two-photon transition, the FWM conversion efficiency is hence largely enhanced. In fact, this idea has been used by the authors of Ref. [15], in which two single-photon transitions are employed to replace the two-photon transition used in Ref. [12] and thus a larger enhancement of the FWM conversion efficiency can be obtained.

Last, let us make also a simple comparison on the FWM conversion efficiency given by the dashed line in Fig. 3 (corresponding to four-level ladder-type scheme used in Ref. [16]) and the dashed line in Fig. 4 (corresponding to the five-level scheme used in Ref. [15]). Since both results look exactly the same, it seems that the addition of one more level and one more field to the four-level ladder-type system in the way as in Ref. [15] gives no contribution to the FWM enhancement. However, we must point out that such a conclusion is valid only for the system parameters chosen above. For different system parameters the FWM conversion efficiency will be different. Figure 5 shows the result of η as a function of the dimensionless optical depth by choosing different Ω_c and Δ_4 but keeping other parameters the same as those used in Fig. 4. We see that in this case the FWM

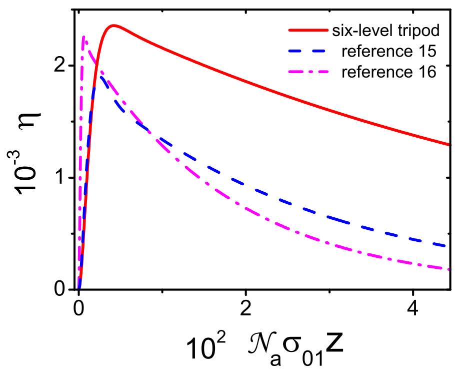

FIG. 5. (Color online) The FWM conversion efficiencies η as functions of the dimensionless optical depth $\mathcal{N}_a\sigma_{01}z$ for different schemes. The dashed line is for a five-level system (corresponding to Ref. [15]). The dot-dashed line is for a four-level ladder system (corresponding to Ref. [16]). The solid line is the result for our six-level tripod system. Here we have chosen $\Omega_c\tau=12\sqrt{2}$, $\Delta_4\tau=3.0$, and other parameters are kept the same as those used in Fig. 4.

conversion efficiency of five-level scheme is higher than that of the four-level ladder-type one.

V. DISCUSSION AND SUMMARY

To demonstrate the generating process of the FWM in our six-level tripod system, we have made a detailed numerical

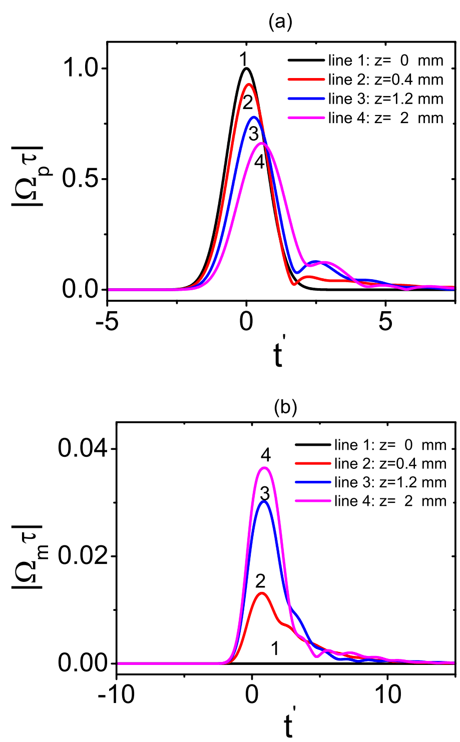

FIG. 6. (Color online) Numerical simulation of the FWM generation in the six-level tripod system. Panel (a) [panel (b)] is dimensionless pump wave $\Omega_p\tau$ (FWM wave $\Omega_m\tau$) for different propagation distance z. Lines 1, 2, 3, 4 are the pulse shape for $z=0$, 0.4, 1.2, and 2 mm, respectively. Here $t'=t/\tau$ and the parameters used are the same as in Fig. 2.

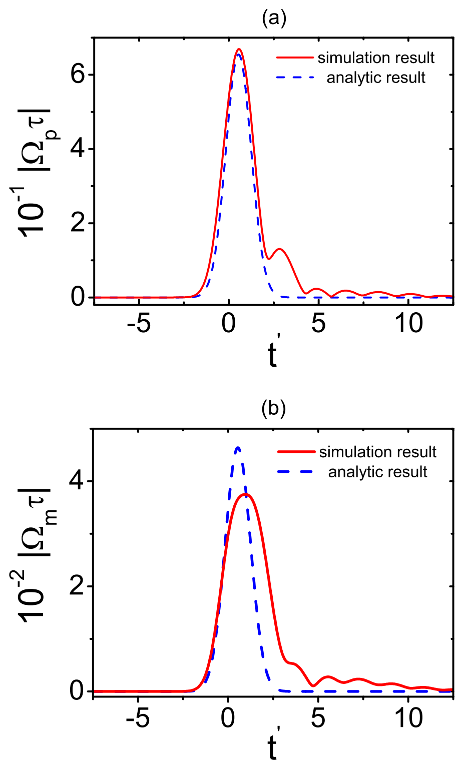

FIG. 7. (Color online) Numerical solutions of the pump and FWM fields by directly integrating Eqs. (1a)–(1f). In both panels of the figure, the dashed (solid) line is the analytical (numerical) result after the pump wave [$\Omega_p\tau$, panel (a)] and the FWM wave [$\Omega_m\tau$, panel (b)] propagate to the distance $z=2$ mm. Here $t'=t/\tau$ and the parameters used are the same as in Fig. 2.

simulation starting directly from Eqs. (1a)–(1f) by a fourth-order Runge-Kutta method combining a finite difference approach. In Fig. 6 we have plotted the result of the simulation for the dimensionless pump wave $\Omega_p\tau$ and the FWM wave $\Omega_m\tau$ for different propagation distance z, with the initial condition $\Omega_p(0,t)=1.0\exp[-(t/\tau)^2]$, $\Omega_m(0,t)=0$, and the other system parameters being the same as in Fig. 2. Lines 1, 2, 3, 4 in the figure denote the pulse shape for $z=0$, 0.4, 1.2, and 2 mm, respectively. From the figure we see clearly that as z increases an energy transfer from the pump wave to the FWM wave occurs. Due to the dispersion effect of the system there are some small radiations appearing in the tails of both waves.

To test our approximated analytical calculation presented in Sec. III, we have made an additional numerical simulation for the FWM solution under the conditions specified in the last section. As in Fig. 6 we assume the envelope of input pump wave is a Gaussian wave packet.

The calculating result of the waveforms of the pump and the FWM fields are shown, respectively, in Figs. 7(a) and 7(b), with the system parameters the same as in Fig. 6. In both panels of the figure, the dashed (solid) line is the analytical (numerical) solution after the wave propagates to the distance $z=2$ mm. We see that the analytical solution agrees fairly with the numerical one except for some radiations appearing in the tails of the numerical solution, which originate from the high-order nonadiabatic (dispersion) effects neglected in the analytical approach.

In summary, in this work we have proposed a scheme for obtaining a high-efficiency FWM in an ultraslow propagation regime in a six-level tripod system by using a double-dark resonance and multiphoton destructive interference induced transparency. We have demonstrated both analytically and numerically that the wave mixing process with a weak pump wave Ω_p mediated by electromagnetically induced transparency and thereby propagating with an extremely slow group velocity may lead to a significant enhancement of the internally generated FWM wave Ω_m, which has a matched group velocity with the pump wave when propagating deeply inside the medium. We have shown that the FWM conversion efficiency in such a system is much higher than those obtained from the five-level and five-level-tripod schemes reported recently in the literature. Practically, it is hopeful to use this scheme to generate coherent short-wavelength radiation in optical thick media at very low pump intensity.

ACKNOWLEDGMENTS

We thank L. Deng for many useful discussions and suggestions. This work was supported by the Key Development Program for Basic Research of China under Grants No. 2005CB724508 and No. 2006CB921104, by the NSF China under Grants No. 10434060, No. 90403008, and No. 10674060, and by the Program for Changjiang Scholars and Innovative Research Team of Chinese Ministry of Education.

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