Depletion of control field during the propagation of ultraslow optical solitons

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We study nonlinear excitations in a lifetime-broadened Λ-type three-level atomic system with a configuration of electromagnetically induced transparency. Different from previous works, we show that a significant depletion of the control field may occur during the formation and propagation of ultraslow optical solitons for the probe field. We demonstrate that ultraslow optical solitons predicted in previous works correspond to the limits of weak dispersion and weak nonlinearity, adiabatons correspond to the limits of stronger dispersion and stronger nonlinearity, and simultons correspond to the limits of strong dispersion and strong nonlinearity. Between these different limits the system also yields solitonlike nonlinear excitations with different levels of depletion of the control field. The results provided here are useful not only for a deep understanding of the interrelation between ultraslow optical solitons, adiabatons, and simultons, but also for potential applications in optical information processing and transmission. © 2009 Optical Society of America

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1. INTRODUCTION

In recent years, much attention has been paid to the study of wave propagation in lifetime-broadened atomic systems via electromagnetically induced transparency (EIT) [1]. By means of the quantum interference effect induced by a controlling field, the absorption of a probe laser field tuned to a strong one-photon resonance can be largely suppressed, and hence an initially highly opaque optical medium becomes transparent. Moreover, significant reduction of group velocity and the tremendous enhancement of Kerr nonlinearity of the probe field can be realized [2–5]. Based on these striking features, the possibility of generating ultraslow optical solitons in EIT media has been predicted recently [6–12]. In nearly all theoretical studies on such solitons, the control field has been simply assumed to be a constant, with its intensity much stronger than the probe field. However, in realistic EIT experiments the control field cannot be taken to be too strong, and hence its significant depletion is expected during the formation and propagation of ultraslow optical solitons.

On the other hand, pulse propagation in resonant optical media has become an active research field since the pioneering work by McCall and Hahn on self-induced transparency (SIT) in resonant two-level atomic systems [13,14]. Like other famous models (e.g., Korteweg–de Vries model, nonlinear Schrödinger model, and sine-Gordon model), two-level SIT equations, which can be solved by the inverse scattering transform and hence are completely integrable, have now become a standard mathematical model in soliton theory [15,16]. In the past four decades, considerable progress in this direction has been achieved, which includes the extension of the SIT theory to different physical systems such as multilevel atomic systems and semiconductors [17–41], stimulated Raman scattering, and four-wave mixing, etc. [42–46]. Especially, optical simultons (i.e., simultaneously propagating multicomponent optical solitons in resonant atomic systems with more than two levels) [18,30] and adiabatons (i.e., multicomponent optical pulses in resonant atomic systems obtained under nearly adiabatic conditions) [21,24] have received a lot of interest [18–41]. It is natural to ask what is the interrelation between ultraslow optical solitons, predicted recently [6–12], and the adiabatons and simultons, which have been widely investigated in literature [18–41].

In this work, we address the above problem and show that the ultraslow optical solitons, adiabatons, and simultons are possible in multilevel systems but they are valid in different physical regimes. To demonstrate this, we make a detailed study on the nonlinear pulse propagation in a Λ-type three-level atomic system by considering the evolution of both the probe and control fields for various pulse intensities and time lengths. Our result shows that ultraslow optical solitons are valid under the conditions of weak dispersion and weak nonlinearity and the adiabatons are valid in the case of stronger dispersion and stronger nonlinearity, whereas the simultons are only possible under conditions of strong dispersion and strong nonlinearity. Between these different regimes the system also yields solitonlike nonlinear excitations with different extents of depletion of the control field. The transition from ultraslow solitons to adiabatons and simultons can be realized through adjusting the system parameters, includ-

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ing especially changing the pulse lengths, light field intensities, and atomic detunings. The results provided here are helpful for a deep understanding of the interrelation between ultraslow optical solitons, adiabatons, and simultons, and also useful for potential applications in optical information processing and transmission.

The paper is arranged as follows. Section 2 gives a simple description of our theoretical model. In Section 3, numerical simulations are carried out, the formation and propagation of stable ultraslow optical solitons for the probe field are studied, and the depletion effect of the control field is shown. In Section 4, a simple theory for explaining the numerical results presented in Section 3 is provided. Finally, the last section contains a discussion and summary of our main results.

2. THE MODEL

We consider a resonant, lifetime-broadened Λ-type three-level atomic system with energy levels $|1\rangle$, $|2\rangle$, and $|3\rangle$, as shown in Fig. 1. A probe field of the center frequency $\omega_p/(2\pi)$ is coupled to the $|1\rangle \to |3\rangle$ transition and a control field of the frequency $\omega_c/(2\pi)$ is coupled to the $|2\rangle \to |3\rangle$ transition. The electric field vector of the system is $\mathbf{E} = \Sigma_{l=p,c} \mathbf{e}_l \mathcal{E}_l \exp[i(k_l z - \omega_l t)] + \text{c.c.}$, where $\mathbf{e}_l$ is the unit vector of the polarization component of the electric field with envelope $\mathcal{E}_l$ $(l=p,c)$. The half-Rabi frequencies are defined by $\Omega_p = (\mathbf{e}_p \cdot \mathbf{p}_{13})\mathcal{E}_p/\hbar$, and $\Omega_c = (\mathbf{e}_c \cdot \mathbf{p}_{23})\mathcal{E}_c/\hbar$, respectively, where $\mathbf{p}_{ij}$ is the electric dipole matrix element associated with the transition from $|i\rangle$ to $|j\rangle$. In the interaction picture, the equations of motion for the atomic system and electric field are

$$i\frac{\partial}{\partial t}A_1 + \Omega_p^* A_3 = 0, \tag{1a}$$

$$\left(i\frac{\partial}{\partial t} + d_2\right)A_2 + \Omega_c^* A_3 = 0, \tag{1b}$$

$$\left(i\frac{\partial}{\partial t} + d_3\right)A_3 + \Omega_p A_1 + \Omega_c A_2 = 0, \tag{1c}$$

$$i\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_p + \kappa_1 A_3 A_1^* = 0, \tag{1d}$$

$$i\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_c + \kappa_2 A_3 A_2^* = 0, \tag{1e}$$

where A_j $(j=1,2,3)$ is the probability amplitude of the bare atomic state $|j\rangle$ (with eigenenergy $\varepsilon_j = \hbar\omega_j$). $d_j = \Delta_j + i\gamma_j$, with $\Delta_3 = \omega_p - (\omega_3 - \omega_1)$ and $\Delta_2 = \omega_p - \omega_c - (\omega_2 - \omega_1)$ being the one- and two-photon detunings and γ_j being the decay of the state $|j\rangle$. The coupling constant is $\kappa_{1(2)} = \mathcal{N}_a \omega_{p(c)} |\mathbf{p}_{13(23)}|^2 / (2\varepsilon_0 c\hbar)$ with $\mathcal{N}_a$ being the atomic density. When obtaining Eqs. (1), a rotating-wave approximation and a slowly-varying envelope approximation have been used.

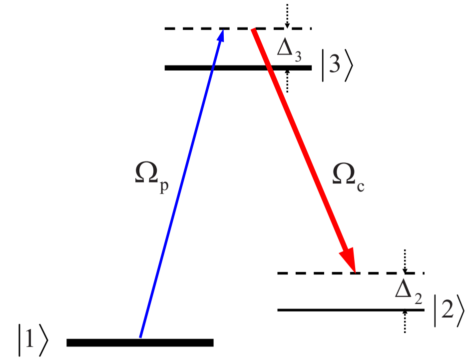

Fig. 1. (Color online) Energy-level configuration and excitation scheme of a lifetime-broadened three-state atomic system interacting with a control field of half-Rabi frequency Ω_c and a probe field of half-Rabi frequency Ω_p. Δ_3 and Δ_2 are one-photon and two-photon detunings, respectively.

Generally, for the motion of resonant atomic systems, density matrix equations should be adopted. Nevertheless, as shown by many previous studies [47,48], for EIT-like coherent atomic systems the density matrix equations can be replaced by probability amplitude equations without a significant difference. For a detailed discussion on the comparison between the result from the probability amplitude equations and that from the density matrix equations, see Section 5 and Appendix A given below.

3. NUMERICAL RESULT ON THE EVOLUTION OF PROBE AND CONTROL FIELDS

From the study of ultraslow solitons [6–12] we know that for weak pulsed Ω_p and strong continuous Ω_c, the Maxwell–Schrödinger Eqs. (1a)–(1e) can be reduced to a nonlinear Schrödinger equation, and hence an analytical soliton solution can be obtained. On the other hand, from the study on adiabatons and simultons [18,21,24,30], for pulsed strong Ω_c and Ω_p with short pulse length, Maxwell–Schrödinger Eqs. (1a)–(1e) can be solved exactly. However, for an intermediate pulse length and field intensity, an analytical solution of the Maxwell–Schrödinger Eqs. (1a)–(1e) is not available. Hence we turn to consider the evolution of the probe and control fields by using a numerical simulation in this intermediate regime. In our calculation, system parameters are chosen as (typical for transitions in hyperfine-split ^{87}Rb atoms) $\gamma_3\tau = \pi \times 5.6$, $\gamma_2\tau = 1.0\times 10^{-3}$, $\Delta_2\tau = 2.0$, $\Delta_3\tau = 1.15\times 10^3$, $\Omega_{c0}\tau = 100.0$, $\Omega_{p0}\tau = 34.0$, and $\kappa_1\tau = \kappa_2\tau = 1.0\times 10^4$ cm^{-1}, with the pulse length $\tau = 1.0\times 10^{-6}$ s. The evolution of the probe and the control fields with different initial probe amplitudes and after propagating to $z=2$ cm are shown in Figs. 2(a)–2(d), respectively. In all panels, solid curves result when the evolution of the control field is not taken into account, while dashed curves result when the evolution of both the probe and control fields are considered simultaneously. From these plots we see that for a large probe field intensity, the depletion of the control field is significant and thus cannot be neglected. In this case though, the probe field may have a solitonlike structure but it is not stable [see Fig. 2(a)]. However, for a small probe field intensity the depletion of the control field becomes less

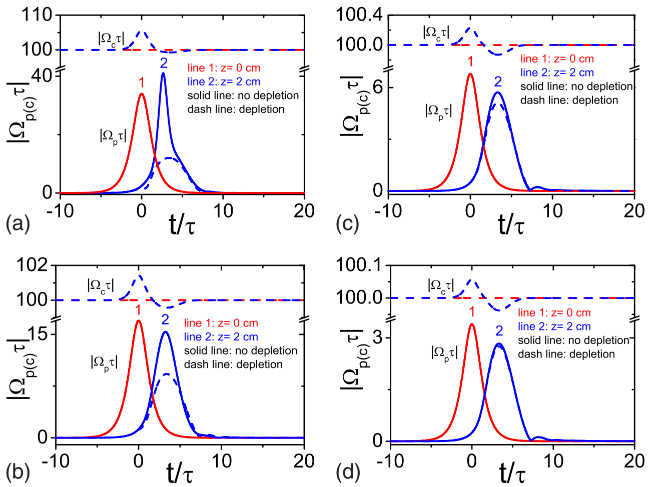

Fig. 2. (Color online) Evolution of the probe field and the control field via propagation distance z and time t with different initial amplitudes. (a) $\Omega_p(0,t)=\Omega_{p0}\,\mathrm{sech}(t/\tau)$; (b) $\Omega_p(0,t)=0.5\Omega_{p0}\,\mathrm{sech}(t/\tau)$; (c) $\Omega_p(0,t)=0.2\Omega_{p0}\,\mathrm{sech}(t/\tau)$; (d) $\Omega_p(0,t)=0.1\Omega_{p0}\,\mathrm{sech}(t/\tau)$. In all panels, solid curves are results when the evolution of control field is disregarded; dashed curves are results of probe and control fields after propagating 2 cm. Parameters are given in the text. The probe pulse is unstable in the case of panel (a). The result of panel (d) is relevant to the ultraslow optical soliton predicted in [6–12] but with a small depletion of the control field.

important, and the optical soliton for the probe field can propagate stably [see Fig. 2(d)], which is relevant to the situation of the ultraslow optical soliton in EIT systems predicted in [6–12] but with a slightly depleted control field. The depletion of the control field is due to an energy exchange between the probe and control fields.

In Fig. 3 we show the evolution of probe and control

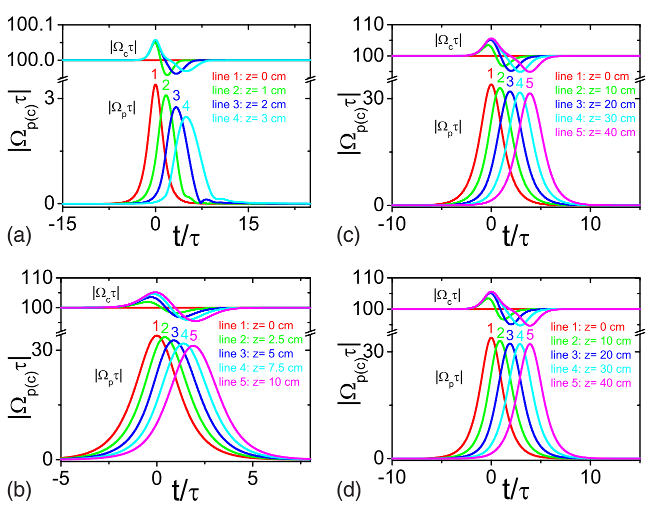

Fig. 3. (Color online) Evolution of probe and control fields via propagation distance z and time t with different detunings, optical intensities, and pulse lengths. (a) $\Omega_p(0,t)=0.1\Omega_{p0}\,\mathrm{sech}(t/\tau)$ with other parameters the same as those used in Fig. 2; (b) $\tau=2.0\times10^{-7}$ s, $\Omega_p(0,t)\tau=34.0\,\mathrm{sech}(t/\tau)$, $\Omega_{c0}\tau=100.0$ with other parameters the same as those used in Fig. 2; (c) $\tau=1.0\times10^{-7}$ s, $\Omega_p(0,t)\tau=34.0\,\mathrm{sech}(t/\tau)$, $\Omega_{c0}\tau=100.0$ with other parameters the same as those used in Fig. 2; (d) $\tau=1.0\times10^{-7}$ s, $\Omega_p(0,t)\tau=34.0$, $\Omega_{c0}\tau=100.0$, $\Delta_2\tau=\Delta_3\tau=0.0$ with other parameters the same as those used in Fig. 2. The result of panel (d) is close to the adiabaton obtained in [21,24,25].

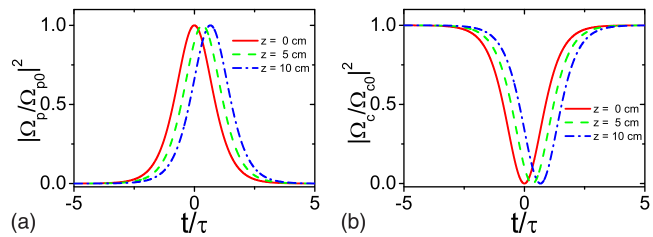

Fig. 4. (Color online) Evolution of simultons via propagation distance z and time t. (a) Evolution of probe field $|\Omega_p/\Omega_{p0}|^2$; (b) evolution of control field $|\Omega_c/\Omega_{c0}|^2$. In both panels, solid curves denote the initial conditions, dashed curves are the results after propagating 5 cm, and dashed-dotted curves are the results after propagating 10 cm. Parameters are shown in the text.

fields via propagation distance z and time t with different energy level detunings, optical intensities, and pulse lengths. In panel (a) we take $\Omega_p(0,t)=0.1\Omega_{p0}\,\mathrm{sech}(t/\tau)$ with other parameters the same as those used in Fig. 2. In this case we obtain an ultraslow optical soliton with a slightly depleted control field; in panel (d) we choose $\tau=1.0\times10^{-7}$ s, $\Omega_p(0,t)\tau=34.0$, $\Omega_{c0}\tau=100.0$, and $\Delta_2\tau=\Delta_3\tau=0.0$ with other parameters the same as the ones used in Fig. 2. In this situation we obtain a structure like an adiabaton with a significant control depletion that is very close to the phenomena observed by the authors of [21,24,25]. Panels (b) and (c) are the intermediate cases between panels (a) and (d). From panels (a) to (d) we see that one can easily obtain a continuous transition from the ultraslow soliton to the adiabaton by manipulating the energy level detunings, probe field intensities, and pulse lengths. We see that the depletion of the control field increases from panel (a) to (d) as the probe and control field intensities increase.

We stress that the conditions for generating the ultraslow optical soliton and the adiabaton are quite different. The formation of the ultraslow soliton requires weak nonlinearity (i.e., small probe intensity) and weak dispersion (i.e., nonzero detunings and longer probe pulse length—see the theoretical explanation in the next section). However, for the formation of adiabatons the system must have stronger nonlinearity and stronger dispersion, and hence both probe and control fields must have large and comparable intensity and shorter time duration. Notice that, different from the adiabatons obtained in [21,24], the decay rates of the atomic levels in Fig. 2(d) are included in our numerical calculation.

We have also made an additional numerical simulation by choosing $\tau=1.0\times10^{-9}$ s, $\Omega_p(0,t)=\Omega_{p0}\,\mathrm{sech}(t/\tau)$, $\Omega_c(0,t)=\Omega_{c0}\tanh(t/\tau)$, $\Omega_{p0}\tau=90.0$, and $\Omega_{c0}\tau=100.0$. Other parameters are the same as the ones in Fig. 2. The result is shown in Fig. 4. From the figure we see that in this case the probe field and control field have comparable amplitude variations, and they are quite stable and completely matched during propagation. Such coupled soliton behavior is nothing but the optical simultons that were predicted by Eberly and his collaborators [18,30]. Obviously, the optical simultons can only be obtained under the conditions of strong dispersion and strong nonlinearity, which require large light intensity of the probe and control fields with a very short pulse length.

4. THEORY

Now we give a simple explanation on the above numerical results analytically. The formation and propagation of ultraslow solitons in an EIT medium can be studied by a standard perturbation theory developed in [7]. If Ω_p is much less than Ω_c, in a leading-order solution, one can treat the control field as a constant and thus Eq. (1e) can be disregarded. It is easy to show that the evolution equation describing the dynamics of the probe field reads

$$i\left(\frac{\partial}{\partial z}+\frac{1}{V_g}\frac{\partial}{\partial t}\right)\Omega_p-\frac{K_2}{2}\frac{\partial^2\Omega_p}{\partial t^2}-W|\Omega_p|^2\Omega_p=0, \quad (2)$$

where $V_g=1/\mathrm{Re}(K_1)$ determines the probe field group velocity, K_2 represents group velocity dispersion, and W originates from the self-phase modulation effect of the system. The expressions of K_1 and K_2 can be obtained by the expansion of linear dispersion relation $K(\omega)$ around $\omega=0$, i.e., $K(\omega)=K_0+K_1\omega+K_2\omega^2/2+\mathcal{O}(\omega^3)$. Here,

$$K(\omega)=\frac{\omega}{c}+\kappa_1\frac{\omega+d_2}{D(\omega)}, \quad (3)$$

and $D(\omega)=|\Omega_c|^2-(\omega+d_2)(\omega+d_3)$; W is given by

$$W=\kappa_1\frac{(\omega+d_2)(|\Omega_c|^2+|\omega+d_2|^2)}{D|D|^2}. \quad (4)$$

The nonlinear Schrödinger equation has complex coefficients and hence generally it does not allow soliton solutions. However, if a practical set of system parameters can be found so that the imaginary part of these coefficients can be made much smaller than their corresponding real parts, it is possible to obtain a shape-preserving localized solution that can propagate for a rather long distance without significant distortion. Actually, we can take $\Delta_j \gg \gamma_j$ ($d_j \approx \Delta_j$), and thus the complex coefficients can be approximated as real ones:

$$\frac{V_g}{c}=\left[1+\kappa_1 c\frac{|\Omega_c|^2+\Delta_2^2}{\tilde{D}^2}\right]^{-1}, \quad (5a)$$

$$\tilde{K}_2=\frac{2\kappa_1}{\tilde{D}^2}\left[\Delta_2+\frac{(|\Omega_c|^2+\Delta_2^2)(\Delta_2+\Delta_3)}{\tilde{D}}\right], \quad (5b)$$

$$\tilde{W}=\kappa_1\frac{\Delta_2(|\Omega_c|^2+\Delta_2^2)}{\tilde{D}^3}, \quad (5c)$$

with $\tilde{D}=|\Omega_c|^2-\Delta_2\Delta_3$. The tilde denotes the real part of the coefficients.

In order to solve Eq. (2), we have used the condition $\mathrm{Im}(K_0)\approx 0$, which can be satisfied by a weak EIT condition. Then the equation supports the localized solution

$$\Omega_p=\frac{1}{\tau}\sqrt{\frac{\tilde{K}_2}{\tilde{W}}}\,\mathrm{sech}\left[\frac{1}{\tau}\left(t-\frac{z}{V_g}\right)\right]e^{i[\tilde{K}_0-\tilde{K}_2/(2\tau^2)]z}. \quad (6)$$

The solution in Eq. (6) describes a fundamental bright soliton traveling with propagating velocity V_g. With the parameters used in Fig. 2, we obtain $V_g/c=1.97\times 10^{-5}$, i.e., the soliton travels with an ultraslow propagating velocity much less than the light speed in vacuum. The dispersion length $L_d=\tau^2/|K_2|=2.0$ cm. The propagation of such an ultraslow optical soliton corresponds to the situation shown in Fig. 2(d).

From Eqs. (5b), (5c), and (6) we see that under strict EIT conditions, i.e., $\Delta_2=0$, the system displays no nonlinear effects. Hence, to generate the soliton with the form of Eq. (6) in the system, a nonzero two-photon detuning (i.e., $\Delta_2\neq 0$) is necessary. However, Δ_2 cannot be too large unless the validity condition of the nonlinear Schrödinger Eq. (2) will be violated.

With the solution in Eq. (6) one readily obtain the solution for A_2 and A_3:

$$A_2=-\frac{\Omega_c^*}{D}\Omega_p, \quad (7a)$$

$$A_3=\frac{\omega+d_2}{D}\Omega_p. \quad (7b)$$

In previous theoretical approaches [6–12], the depletion of the control field is not taken into account. Here, we consider this problem by solving Eq. (1e) through the use of the solution given above. Substituting the above result into Eq. (1e) we obtain a linear and inhomogeneous equation for Ω_c. It is easy to obtain the solution,

$$\Omega_c=\Omega_c^{(0)}+\Omega_c^{(1)}+\Omega_c^{(2)}, \quad (8)$$

where $\Omega_c^{(0)}$ is a constant describing a continuous background, and $\Omega_c^{(1)}(t,z)=\Omega_c^{(1)}(t-z/c)$ describes a hump that propagates with speed c. The concrete waveform of $\Omega_c^{(1)}$ is determined by the initial condition. The third term of Eq. (8) is given by

$$\Omega_c^{(2)}=i\kappa_2\frac{\Omega_c^{(0)}\Delta_2}{\tilde{D}^2}\frac{V_g}{\tau}\frac{\tilde{K}_2}{\tilde{W}}\tanh\left[\frac{1}{\tau}\left(t-\frac{z}{V_g}\right)\right], \quad (9)$$

which contributes a hole (or dark soliton) to the light intensity of the control field. The motion of the hole matches that of the probe field, i.e., it moves with the same propagating velocity of the probe field soliton (6). The appearance of the control field hole is obviously due to the energy exchange between the control field and the probe field via the atomic system as an intermediary.

The solution given above can be used to explain the result of the numerical simulation presented in the last section. For example, the horizontal line in the upper part of Fig. 3 is continuous background $\Omega_c^{(0)}$; the hump above the horizontal line is the contribution by $\Omega_c^{(1)}(t-z/c)$; the hole below the horizontal line is the contribution of $\Omega_c^{(2)}$.

5. DISCUSSION AND SUMMARY

In the above calculations, probability amplitude equations [i.e., Eqs. (1a)–(1c)] have been used for the description of atomic motion. Strictly speaking, for a lifetime-broadened system, density matrix equations should be adopted in order to get a complete description that includes the effects of spontaneous emission and dephasing.

However, for an EIT-based partially open system, it can be shown that the probability amplitude approach and the density matrix approach are roughly equivalent. The main reason for such equivalence is due to the fact that the controlling field Ω_c induces a quantum coherence in the system and greatly suppresses the spontaneous emission. The dominant processes in the system are hence coherent, reversible transitions between the hyperfine ground states. The quantity determining the importance of the incoherent processes is given by the fraction of the population undergoing spontaneous emission integrated over time, i.e., $P_{\rm loss}=\gamma_3\int_0^\infty dt|A_3(t)|^2$, which is indeed small because A_3 is nearly vanishing in EIT-like systems. For a detailed discussion and comparison between the two approaches in EIT systems, we refer to [47,48]. In Appendix A, we have presented the equations of motion of the density matrix for our system and have given the relations between the decay rate γ_j in the probability amplitude equations [Eqs. (1a)–(1c)] and spontaneous emission decay rates and dephasing rates in the density matrix equations. Another reason for the choice of the probability amplitude approach is due to its simplicity in the mathematical treatment and transparency for the physical explanation on the results obtained in the numerical simulation.

To check the above argument we have made an additional numerical calculation for the time evolution of the probe and controlling fields based on the density matrix equations [Eqs. (A1a)–(A1h)]. The result is shown in Fig. 5, where the corresponding result based on the probability amplitude equations [Eqs. (1a)–(1e)] is also presented. We see that the difference between two approaches is indeed very small.

In conclusion, we have investigated nonlinear optical pulse propagation in a lifetime-broadened three-state atomic system. We have shown that a depletion of the control field may occur and even be significant during the formation and propagation of ultraslow optical solitons of the probe field. We have also shown that the ultraslow optical solitons are relevant to the limit of weak dispersion and nonlinearity, the adiabatons correspond to the limit of stronger dispersion and stronger nonlinearity, and the simultons are corresponding to the limit of strong dispersion and strong nonlinearity. Between these limits the system also allows solitonlike nonlinear excitations with different depletions of the control field. We believe that the results provided in this work are useful not only for a deep understanding of the interrelation between ultraslow optical solitons and adiabatons but also for potential applications in optical information processing and transmission.

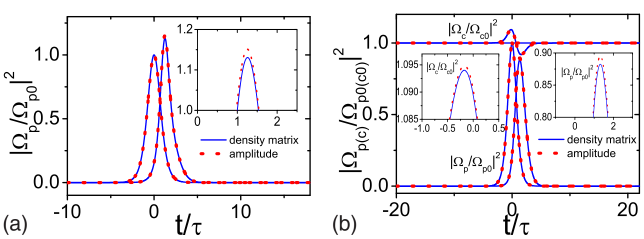

Fig. 5. (Color online) Evolution of probe and control fields as functions of distance z and time t. The results shown are for $z=0$ cm (the pulse on the left side) and 1 cm (the pulse on the right side). (a) The case of control field Ω_c is a constant; (b) the evolution of both the probe and control fields is considered. The insets in the figures show the very small difference between the density matrix description and the probability amplitude description. In both panels, solid curves are the results based on the density matrix Eq. (A1) and dotted curves are the results based on the probability amplitude Eq. (1). Parameters are chosen as $\Gamma_{31}\tau=\Gamma_{32}\tau=\gamma_{31}\tau=\gamma_{32}\tau=5.6\pi$, $\gamma_{21}\tau=1.0\times10^{-3}$ [the ionization rate $\gamma_i\tau=0$ ($i=1$ to 3) in density matrix equation], with other ones the same as those in Fig. 2. The initial condition is taken as $A_1=\rho_{11}=1$, $A_2=A_3=\rho_{22}=\rho_{33}=\rho_{21}=\rho_{31}=\rho_{32}=0$, $\Omega_p(0,t)=\Omega_{p0}\,\mathrm{sech}(t/\tau)$, $\Omega_c(0,t)=\Omega_{c0}$.

APPENDIX A: DENSITY MATRIX EQUATIONS

Density matrix equations that describe the interaction between three-level atoms and probe and controlling fields are:

$$i\left(\frac{\partial}{\partial t}+\gamma_1\right)\rho_{11}-i\Gamma_{31}\rho_{33}+\Omega_p^*\rho_{31}-\Omega_p\rho_{31}^*=0, \quad \text{(A1a)}$$

$$i\left(\frac{\partial}{\partial t}+\gamma_2\right)\rho_{22}-i\Gamma_{32}\rho_{33}+\Omega_c^*\rho_{32}-\Omega_c\rho_{32}^*=0, \quad \text{(A1b)}$$

$$i\left(\frac{\partial}{\partial t}+\gamma_3+\Gamma_3\right)\rho_{33}+\Omega_p\rho_{31}^*-\Omega_p^*\rho_{31}+\Omega_c\rho_{32}^*-\Omega_c^*\rho_{32}=0, \quad \text{(A1c)}$$

$$\left(i\frac{\partial}{\partial t}+d_{21}\right)\rho_{21}-\Omega_p\rho_{32}^*+\Omega_c^*\rho_{31}=0, \quad \text{(A1d)}$$

$$\left(i\frac{\partial}{\partial t}+d_{31}\right)\rho_{31}-\Omega_p(\rho_{33}-\rho_{11})+\Omega_c\rho_{21}=0, \quad \text{(A1e)}$$

$$\left(i\frac{\partial}{\partial t}+d_{32}\right)\rho_{32}-\Omega_c(\rho_{33}-\rho_{22})+\Omega_p\rho_{21}^*=0, \quad \text{(A1f)}$$

$$i\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_p+\kappa_1\rho_{31}=0, \quad \text{(A1g)}$$

$$i\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_c+\kappa_2\rho_{32}=0, \quad \text{(A1h)}$$

with $\rho_{ij}\leftrightarrow A_iA_j^*$, $d_{ij}=\Delta_i-\Delta_j+i\gamma_{ij}$, and $\gamma_{ij}=(\Gamma_i+\Gamma_j)/2+(\gamma_i+\gamma_j)/2+\gamma_{ij}^{\rm col}$ with $\Gamma_j=\Sigma_{i<j}\Gamma_{ij}$. Here Γ_{ij} denotes the spontaneous emission decay rate from state $|i\rangle$ to state $|j\rangle$, and γ_j is the ionization rate of the state $|j\rangle$. $\gamma_{jl}^{\rm col}$ represents the dipole dephasing rate that reflects the loss of phase coherence without change of population, as might occur with elastic collisions. From Eqs. (A1a)–(A1c), one has $\Sigma_{j=1}^3\dot{\rho}_{jj}=-\Sigma_{j=1}^3\gamma_j\rho_{jj}$, and hence the system is partially open.

If the system is an open system, i.e., $\Gamma_{31}=\Gamma_{32}=0$, Eq. (1) and Eq. (A1) are mathematically equivalent, so one has $\rho_{ij}=A_iA_j^*$ [49]. Since in our system (a partially open one) states $|1\rangle$ and $|2\rangle$ are two hyperfine ground states, one has a vanishing γ_1 and a very small γ_2. The quantum interference effect induced by the controlling field suppresses the spontaneous emission greatly. The dominant processes in the system are thus coherent, reversible transitions between the hyperfine ground states. In this case, the difference between the result given by the probability amplitude approach and the one obtained by the density matrix approach is not significant.

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