Journal of Physics B: Atomic, Molecular and Optical Physics

Evolution of a coherent array of Bose–Einstein condensates in a magnetic trap

To cite this article: Hongwei Xiong *et al* 2002 *J. Phys. B: At. Mol. Opt. Phys.* **35** 4863

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS B: ATOMIC, MOLECULAR AND OPTICAL PHYSICS

J. Phys. B: At. Mol. Opt. Phys. **35** (2002) 4863–4873 PII: S0953-4075(02)37174-8

Evolution of a coherent array of Bose–Einstein condensates in a magnetic trap

Hongwei Xiong[1,2], Shujuan Liu[1], Guoxiang Huang[3,4] and Zhijun Xu[1]

[1] Department of Applied Physics, Zhejiang University of Technology, Hangzhou 310032, People's Republic of China
[2] Zhijiang College, Zhejiang University of Technology, Hangzhou 310012, People's Republic of China
[3] Department of Physics, East China Normal University, Shanghai 200062, People's Republic of China
[4] Laboratoire de Physique Théorique de la Matière Condensée, case 7020, 2 Place Jussieu, 75251, Paris Cedex 05, France

Received 17 May 2002, in final form 3 October 2002
Published 19 November 2002
Online at stacks.iop.org/JPhysB/35/4863

Abstract

We investigate the interference pattern evolution process for a coherent array of Bose–Einstein condensates in a magnetic trap after the optical lattices are switched off. It is shown that there is a decay and revival of the density oscillation for the condensates confined in the magnetic trap. We find that, due to the confinement of the magnetic trap, the interference effect is much stronger than that found in the experiment performed by Pedri *et al* (Pedri P *et al* 2001 *Phys. Rev. Lett.* **87** 220401), where the magnetic trap is also switched off. The interaction correction to the interference effect is also discussed for the density distribution of the central peak.

1. Introduction

The development of laser trapping and evaporative cooling technologies has yielded intriguing Bose–Einstein condensates (BECs) [1], a state of matter in which many atoms are in the same quantum mechanical state. The realization of BECs has enabled remarkable theoretical and experimental advances to be made on this exotic quantum system [2, 3]. Recently, optical lattices were used to investigate further the unique character of ultra-cold atoms [4]. The applications of optical lattices to ultra-cold atoms show great promise; for example, the quantum computing scheme proposed in [5, 6]. Experimentally, the macroscopic quantum interference effect [7] and thermodynamic properties [8] of the BECs in optical lattices have been thoroughly investigated. In addition, the superfluid and dissipative dynamics [9, 10] of a BEC in the optical lattices have also been investigated experimentally. In particular, through the application of optical lattices, the quantum phase transition from a superfluid to a Mott insulator in Bose–Einstein condensed gases has been observed in a recent experiment [11].

Recently, in an experiment performed by Pedri *et al* [12], the expansion of a coherent array of BECs was carried out to illustrate the interference effect when both the magnetic trap and optical lattices are switched off. In another experiment performed by Morsch *et al* [13], the expansion of the condensates is investigated when only the magnetic trap is switched off. In this paper, we present a theoretical investigation of the expansion of the condensates when only the optical lattices are switched off. Due to the confinement of the harmonic potential, we find that there are several interesting phenomena not encountered in the experiments by Pedri *et al* [12] and Morsch *et al* [13]. In the presence of the harmonic potential, the interference effect would be much stronger than the case when the magnetic trap is also switched off. In this situation, research shows that the interaction between atoms would give rise to an important correction, in contrast to the case investigated by Pedri *et al* [12].

2. The wavefunction in harmonic potential after optical lattices are switched off

In the experiment conducted by Pedri *et al* [12], the external potential of the Bose gas was given by [12]

$$V = \frac{1}{2}m(\omega_x^2 x^2 + \omega_\perp^2 (y^2 + z^2)) + sE_R \cos^2\left(\frac{2\pi x}{\lambda} + \frac{\pi}{2}\right). \tag{1}$$

The last term represents the external potential due to the presence of the optical lattices. In the above expression, ω_x and $\omega_\perp$ are the axial and radial frequencies of the harmonic potential, respectively. In addition, λ is the wavelength of the retroreflected laser beam, and sE_R denotes the depth of the optical lattices. For the optical lattices created by the retroreflected laser beam, the last term has a period $d = \lambda/2$. In other words, d can be regarded as the distance between two neighbouring wells induced by the optical lattices. In this paper, the experimental parameters in [12] are used to calculate various physical quantities, thus it would be useful to give them here. The experimental parameters in [12] are $\omega_x = 2\pi \times 9$ Hz, $\omega_\perp = 2\pi \times 92$ Hz, $\lambda = 795$ nm, $E_R = 2\pi\hbar \times 3.6$ kHz and $s = 5$.

Due to the presence of the optical lattices, there is an array of condensates formed in the combined potential, when the temperature is lower than the critical temperature. In this work, we investigate the case of the strong tunnelling between neighbouring BECs, which holds in the experiment peformed by Pedri *et al* [12]. In this situation, all the condensates are fully coherent, and can be described by a single order parameter. To emphasize the role of the optical lattices, our research is carried out mainly on the character of the coherent array of condensates in the x-direction.

In the presence of a magnetic trap, the number of atoms in each well should be different. Based on the analysis of the three-dimensional model of the condensates in the combined potential [12], the ratio between the number of condensed atoms in the kth and central wells is given by $N_k/N_0 = (1-k^2/k_M^2)^2$, where $2k_M+1$ [12] represents the total number of condensates induced by the optical lattices. In this situation, using the Gaussian approximation in the x-direction for each well, the normalized wavefunction in coordinate space takes the form

$$\varphi_0(x) = A_n \sum_{k=-k_M}^{k_M} \left(1 - \frac{k^2}{k_M^2}\right) \exp[-(x - kd)^2/2\sigma^2], \tag{2}$$

where σ denotes the width of the condensate in each well. It can be calculated by numerical minimization of the energy of the condensates [12]. In the calculations that follow in this paper, $\sigma = 0.25d$ [12] is used for $s = 5$. In the above expression, the normalized constant A_n

takes the form

$$A_n = \frac{1}{\sqrt{(16k_M^4 - 1)/15k_M^3}\pi^{1/4}\sigma^{1/2}}. \tag{3}$$

It is well known that once the wavefunction at an initial time is known, the wavefunction at a later time can be obtained through the following integral equation [14]:

$$\varphi_0(x,t) = \int_{-\infty}^{\infty} K(x,t;y,t=0)\varphi_0(y,t=0)\,\mathrm{d}y, \tag{4}$$

where $\varphi_0(y, t = 0)$ is the wavefunction at the initial time $t = 0$ which is given by equation (2), and $K(x, t; y, t = 0)$ is the well known propagator. For the atoms in the harmonic potential, the propagator can be expressed as [14]

$$K(x,t;y,t=0) = \left[\frac{m\omega_x}{2\pi \mathrm{i}\hbar \sin\omega_x t}\right]^{1/2} \exp\left\{\frac{\mathrm{i}m\omega_x}{2\hbar\sin\omega_x t}[(x^2+y^2)\cos\omega_x t - 2xy]\right\}. \tag{5}$$

From the formulae (2), (4) and (5), after a straightforward calculation, one obtains the analytical result of the wavefunction confined in the magnetic trap

$$\begin{aligned}\varphi_0(x,t) = A_n\sqrt{\frac{1}{\sin\omega_x t(\mathrm{ctg}\omega_x t + \mathrm{i}\gamma)}} \sum_{k=-k_M}^{k_M}\left(1 - \frac{k^2}{k_M^2}\right)\exp\left[-\frac{(kd\cos\omega_x t - x)^2}{2\sigma^2\sin^2\omega_x t(\mathrm{ctg}^2\omega_x t + \gamma^2)}\right] \\ \times \exp\left[-\frac{\mathrm{i}(kd\cos\omega_x t - x)^2\mathrm{ctg}\omega_x t}{2\gamma\sigma^2\sin^2\omega_x t(\mathrm{ctg}^2\omega_x t + \gamma^2)}\right] \\ \times \exp\left[\frac{\mathrm{i}(x^2\cos\omega_x t + k^2d^2\cos\omega_x t - 2xkd)}{2\gamma\sigma^2\sin\omega_x t}\right],\end{aligned} \tag{6}$$

where we have introduced a dimensionless parameter $\gamma = \hbar/m\omega_x\sigma^2$. Assuming N denotes the total number of particles in the condensates, the density distribution in the x-direction is $n(x, t) = N|\varphi_0(x, t)|^2$.

3. The periodicity of the density distribution and the motion of the $n = \pm 1$ peak

Due to the confinement of the harmonic potential, the density distribution in the x-direction should exhibit periodic character. From equation (6) it is easy to show that the period of the density distribution $n(x, t)$ is determined by $\omega_x T = \pi$. For the experiment performed in [12], $\omega_x = 2\pi \times 9$ Hz, which means that the period of the density distribution is given by 500/9 ms.

In figure 1 $n(x = 0, t)$ is displayed when only the optical lattices are switched off. The periodicity of the density is clearly shown in the figure and is in agreement with the analytical result given by $T = \pi/\omega_x$. We see that the density at $x = 0$ reaches a maximum value at time $t_m = (2m - 1)\pi/2\omega_x$, with m denoting a positive integer. At time t_m, the wavefunction takes the form

$$\varphi_0(x,t_m) = A_n\sqrt{\frac{1}{\mathrm{i}\gamma}}\sum_{k=-k_M}^{k_M}\left(1 - \frac{k^2}{k_M^2}\right)\exp\left[-\frac{x^2}{2\sigma^2\gamma^2} - \mathrm{i}\frac{xkd}{\gamma\sigma^2}\right]. \tag{7}$$

The maximum density at $x = 0$ is then

$$n(x=0,t_m) = \frac{NA_n^2}{\gamma}\left[\sum_{k=-k_M}^{k_M}\left(1 - \frac{k^2}{k_M^2}\right)\right]^2. \tag{8}$$

In the case of $k_M \gg 1$, the above expression can be approximated as follows:

$$n(x=0,t_m) \approx N\alpha_{x-ideal}^2, \tag{9}$$

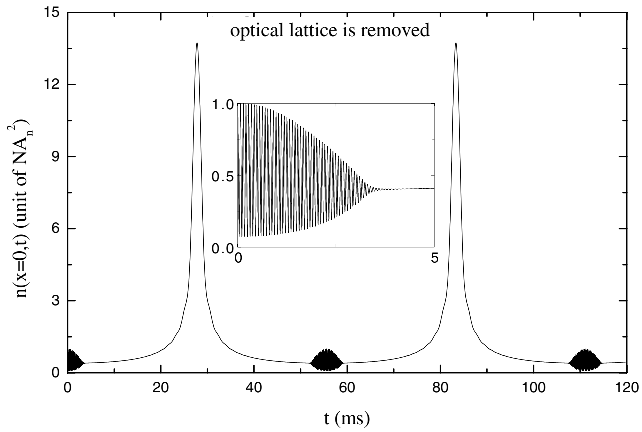

Figure 1. Displayed is the density of the condensate at $x = 0$ versus time t, after the optical lattice is switched off. Here the density $n(x = 0, t)$ is in units of NA_n^2. We can see clearly that there is a decay and revival of the density oscillation. In addition, there is a periodicity of the density due to the confinement of the magnetic trap.

where $\alpha_{x-ideal}^2$ is given by

$$\alpha_{x-ideal}^2 = \frac{5k_M m\omega_x\sigma}{3\pi^{1/2}\hbar}. \tag{10}$$

In the experiments, the depth of the optical lattices can be changed through the variation of the parameter s. From [12], we know $\sigma \propto 1/s^{1/4}$ and $k_M \propto 1/\sigma^{1/5}$. Therefore, $n(x = 0, t_m) \propto 1/s^{1/5}$. This shows that in the non-interacting model, the maximum density at $x = 0$ decreases with the increase of the depth of the optical lattices.

In figures 2(a)–(d), we show the evolution of the density distribution of the condensates confined in the magnetic trap, after the optical lattices are switched off. The density distributions are shown at $t = 0$, $0.1\pi/\omega_x$, $0.3\pi/\omega_x$ and $0.5\pi/\omega_x$ in these figures. The motion of the $n = \pm 1$ peaks is also clearly shown in these figures. The oscillating motion of the $n = \pm 1$ peaks is due to the confinement of the harmonic potential. In fact, the motion of the $n = \pm 1$ peaks can be described very well using classical harmonic motion. Using classical harmonic motion, the motion of the $n = \pm 1$ peaks is determined by the following expression:

$$x_{n=\pm 1}(t) = \pm\frac{2\pi\hbar}{m\omega_x d}\cos\left(\omega_x t - \frac{\pi}{2}\right). \tag{11}$$

When obtaining the above formula, we have used the fact that the momentum distribution is characterized by sharp peaks at the values $p_x = n2\pi\hbar/d$ [12]. The solid curve in figure 3 shows the harmonic motion of the $n = 1$ peak using the above formula, while the circles show the result given by equation (6). We see that the classical harmonic motion agrees quite well with the result given by equation (6). In a sense, equation (11) describes the motion of the centre of mass of the $n = \pm 1$ peak. Thus, we anticipate that the interaction between atoms will not change the motion of the $n = \pm 1$, although it will affect the density and width of the $n = \pm 1$ peak.

From figure 2(d) we see that at time t_m the density distribution in the x-direction exhibits a very sharp peak at the centre of the magnetic trap. The maximum density of the central peak is given by equation (9). As a comparison, assume there are N atoms confined in an identical magnetic trap, but there are no optical lattices to induce the interference effect. In

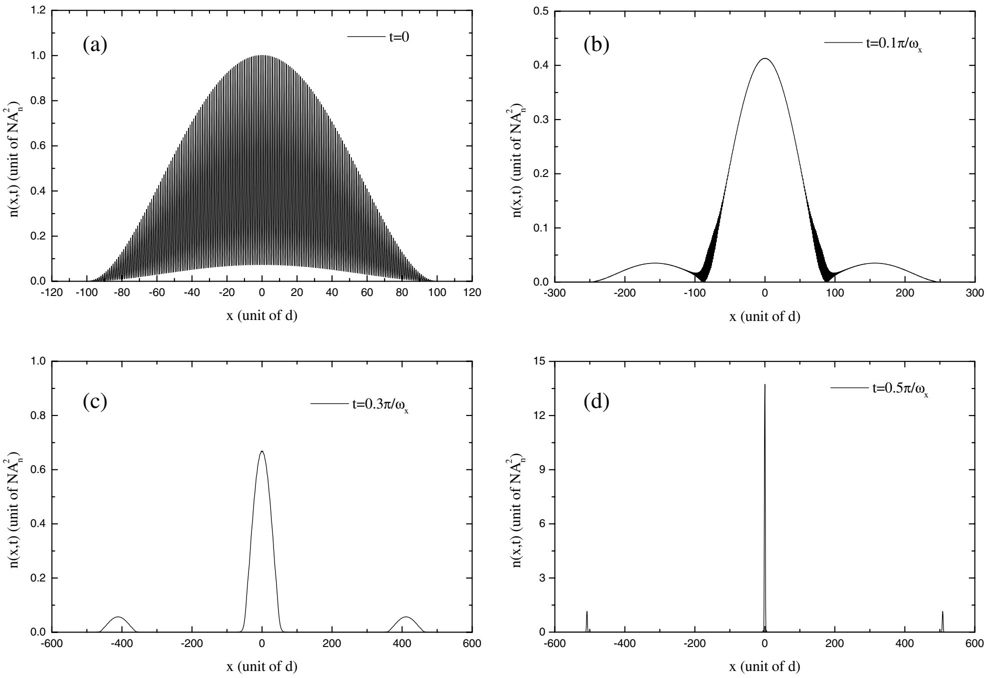

Figure 2. (a)–(d) show the evolution of the density distribution with time t, after the optical lattices are switched off. The density distributions are shown at $t = 0$, $0.1\pi/\omega_x$, $0.3\pi/\omega_x$ and $0.5\pi/\omega_x$. The emergence and motion of the $n = \pm 1$ peaks are clearly shown in these figures. Here the density distribution $n(x, t)$ is in units of NA_n^2, while the location x is in units of d, i.e. the distance between two neighbouring condensates.

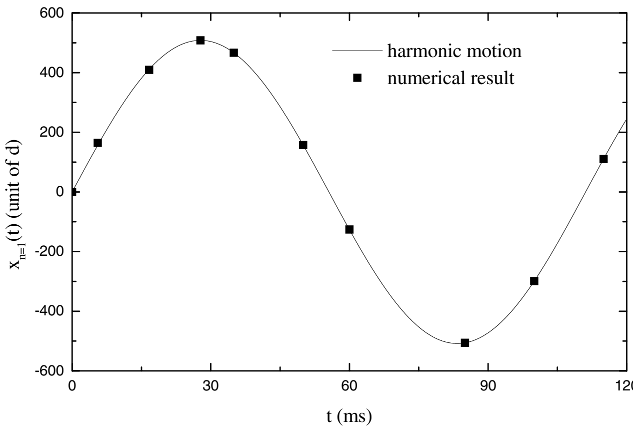

Figure 3. Displayed is the motion of the $n = 1$ peak, after the optical lattices are switched off. Here the location $x(t)_{n=1}$ of the $n = 1$ peak is in units of d. The solid curve is the result calculated from the classical harmonic motion given by equation (11). The squares show the motion of the $n = 1$ peak obtained from the numerical result given by equation (6). We see that the classical harmonic motion agrees quite well with the numerical result.

this situation, in the x-direction, the density distribution at $x = 0$ is given by

$$n_{mag}(x = 0) = \frac{\pi \mu_{mag}^2}{gm\omega_\perp^2}, \tag{12}$$

where μ_{mag} is the chemical potential of the Bose gas confined in the magnetic trap. The ratio between $n(x = 0, t_m)$ and $n_{mag}(x = 0)$ is then

$$\frac{n(x=0,t_m)}{n_{mag}(x=0)} = \frac{Ngm\omega_\perp^2\alpha_{x-ideal}^2}{\pi\mu_{mag}^2}. \tag{13}$$

For the experimental parameters in [12], $n(x = 0, t_m)/n_{mag}(x = 0) = 29.6$. This shows clearly that there is a very strong interference effect for the case considered here.

If both the magnetic trap and optical lattices are switched off, there is also a sort of interference effect. The maximum value of the density distribution in this case is given by

$$n_{bs}(x=0) = \frac{N_0}{\sqrt{\pi}\sigma}. \tag{14}$$

From equation (14) $n_{bs}(x = 0) \propto s^{1/4}$. In contrast to the case where only the optical lattices are switched off, $n_{bs}(x = 0)$ decreases with the decrease of the depth of the optical lattices.

The ratio between $n_{bs}(x = 0)$ and $n_{mag}(x = 0)$ is then

$$\frac{n_{bs}(x=0)}{n_{mag}(x=0)} = \frac{N_0gm\omega_\perp^2}{\pi^{3/2}\sigma\mu_{mag}^2}. \tag{15}$$

For the experimental parameters presented in [12], we have $n_{bs}(x = 0)/n_{mag}(x = 0) = 2.2$. From equations (9) and (14), $n(x = 0, t_m)/n_{bs}(x = 0) = 13.7$. Therefore, when only the optical lattices are switched off, the interference effect would be much stronger than the case when the magnetic trap is also switched off. When only the optical lattices are switched off, at t_m the density of the central peak is very high, and we anticipate that in this situation the interaction between atoms would give important correction.

4. The decay and revival of the density oscillation

From figure 1 we see that there is a decay and revival phenomenon of the density oscillation at $x = 0$. We now discuss this unique character. To proceed, it is useful to introduce an important timescale which determines when the interference between two neighbouring condensates begins to occur. Before the magnetic trap and optical lattices are switched off, from the Gaussian approximation of the condensates in each well, the width of the condensates in each well is given by

$$\Delta x_0^2 = \frac{\int x^2 \exp[-x^2/\sigma^2]\,\mathrm{d}x}{\int \exp[-x^2/\sigma^2]\,\mathrm{d}x}. \tag{16}$$

It is easy to obtain $\Delta x_0 = \sigma/\sqrt{2}$ from the above formula. When the optical lattices are switched off, for a time period much smaller than π/ω_x, the condensates can be approximated as a free expansion and the width of the condensates would increase in this situation. Based on the analysis of the spreading of the wavepacket, the width of each condensate is given by

$$\Delta x(t) = \Delta x_0\sqrt{1+\frac{\hbar^2t^2}{m^2\Delta x_0^4}}. \tag{17}$$

When $\Delta x(t) = d$, the condensates in neighbouring wells begin to interfere with each other. By setting $\Delta x(t) = d$ in the above formula, we obtain a timescale t_w which determines when the interference between neighbouring condensates begins to occur. From equation (17) it is easy to find that the following analytical result for t_w can give a rather good approximation:

$$t_w = \frac{\sigma dm}{\sqrt{2}\hbar}. \tag{18}$$

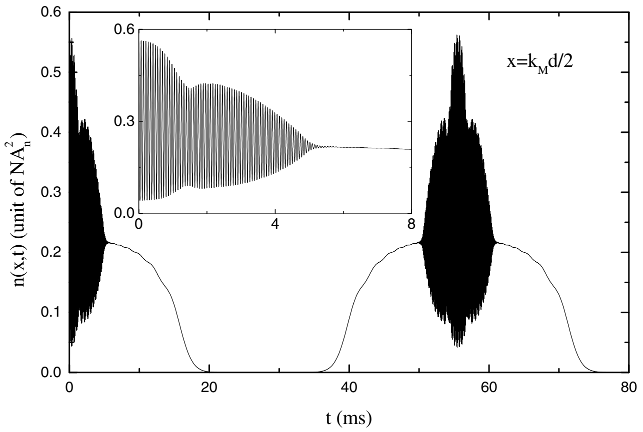

Figure 4. Displayed is the density of the condensate at $x = k_M d/2$ versus time t, after the optical lattices are switched off. Here the density $n(x, t)$ is in units of NA_n^2. There is a decay and revival phenomenon of the density oscillation.

From equations (17) and (18), for $t > t_w$, one obtains the following useful result:

$$\Delta x(t) = \frac{t}{t_w} d. \tag{19}$$

As illustrated in figure 1, the oscillation of the density at $x = 0$ will ultimately cease when $t > k_M t_w$. However, when the time approaches π/ω_x, the density oscillation will reappear. Note that the time for the revival of the density oscillation is determined solely by the axial frequency of the harmonic potential. This shows that the confinement of the harmonic potential plays a crucial role in the revival phenomenon of the density oscillation. To further verify the decay and revival of the density oscillation, figure 4 shows the density at $x = k_M d/2$. The analogous decay and revival of the density oscillation are illustrated clearly in the figure. However, the density oscillation disappears at a longer time $t = 1.47 k_M t_w$, in comparison with the case at $x = 0$.

In figure 5 we display the disappearance time of the density oscillation for different locations in the region $0 < t < \pi/\omega_x$. We can give a rather simple interpretation of this result. When the optical lattices are switched off, the width of the expanding condensates in each well will increase. For the location at $x = 0$, there are more and more expanding BECs which interfere at this point with the time development. This is the reason why the density at the point $x = 0$ will oscillate intensely. When $t > k_M t_w$, however, all expanding BECs have participated in the interference at the point $x = 0$. Therefore, the oscillation of the density at $x = 0$ will cease at a time greater than $k_M t_w$. Generalizing this result, the disappearance time of the density oscillation for different locations is given by the following simple expression:

$$t = (k_M + x/d) t_w. \tag{20}$$

The solid line in figure 5 displays the above analytical result. We see from figure 5 that this simple expression agrees well with the result given by equation (6). Perhaps the slight difference from the result given by equation (6) lies in the fact that we do not account for the effect of a non-uniform atom distribution in each well when obtaining equation (20).

It is worth pointing out that the disappearance of the density oscillation does not mean the disappearance of the interference effect between the expanding condensates. When $0 < t \ll (2k_M + 1)t_w$ there are only a few expanding condensates interfering with each other, and there are interference fringes (or density oscillation) in this situation. For

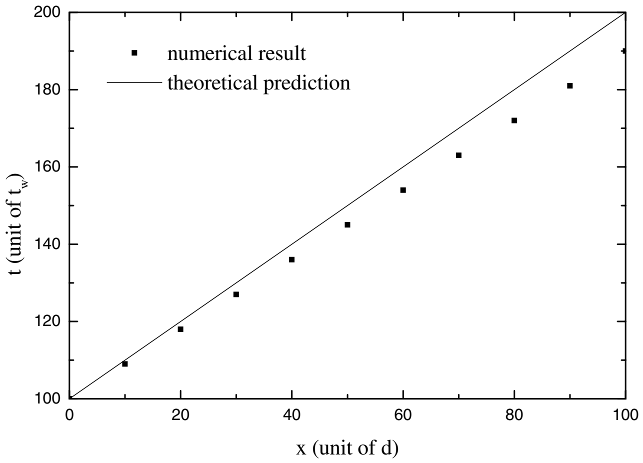

Figure 5. Displayed is the disappearance time of the density oscillation for different locations. Here the location x is in units of d, while the time t is in units of the timescale t_w. The solid line is obtained from the formula (20), while the squares show the result obtained directly from the numerical result given by equation (6). We see that the analytical formula (20) can give a good description for the disappearance of the density oscillation.

$(2k_M + 1)t_w < t < \pi/\omega_x - (2k_M + 1)t_w$, however, all expanding condensates will interfere with each other, and this means the emergence of the diffraction fringes. In fact, $n = 0$ and ± 1 peaks in figures 2(a)–(d) should be regarded as the diffraction fringes, rather than the interference fringes. Note that the phenomena of diffraction and interference are basically equivalent. Different from the interference phenomenon, however, the diffraction phenomenon should be regarded as a consequence of interference from many coherent wave sources. In a sense, equation (20) gives the emergence time of the diffraction fringes, which means the disappearance of the density oscillation.

5. Interaction correction to the central peak at t_m

In the case of the non-interacting model, we have shown that, at time t_m, there would be a sharp central peak in the magnetic trap. In this case, the interaction between atoms cannot be simply omitted, in contrast to the case when the magnetic trap is also switched off. At time t_m, the central density peak in the x-direction can be approximated as a Gaussian distribution. After the optical lattices are switched off, using the Thomas–Fermi approximation in the radial direction, the square of the modulus of the three-dimensional wavefunction at time t_m takes the form

$$|\varphi_0(x, r_\perp, t_m)|^2 = \alpha_x^2 \alpha_\perp^2 \exp\left[-\frac{2x^2}{R_x^2}\right]\left(1 - \frac{r_\perp^2}{R_\perp^2}\right), \tag{21}$$

where $R_\perp^2 = \sqrt{2\mu_0/m\omega_\perp^2}$ with $\mu_0 = m\omega_x^2 k_M^2 d^2/2$ [12]. In the above expression, $\alpha_x = (2/\pi)^{1/4}/\sqrt{R_x}$ and $\alpha_\perp = \sqrt{2/\pi R_\perp^2}$ are the normalized constants in the axial and radial directions, respectively. Obviously, $N\alpha_x^2$ represents the density $n(x = 0, t_m)$ in the x-direction. Due to the repulsive interaction between atoms, we anticipate that $\alpha_x^2 < \alpha_{x-ideal}^2$.

Assume E_{int} and E_{kin} are the interaction energy and kinetic energy of the central peak at t_m, respectively. The interaction energy of the central peak is given by

$$E_{int} = \frac{gN^2}{2}\int |\varphi_0(x, r_\perp, t_m)|^4 \,\mathrm{d}V = \frac{\sqrt{2}gN^2\alpha_x^2}{3\pi R_\perp^2}. \tag{22}$$

Assuming the total energy of the condensates is E_{all}, we have

$$E_{kin} + E_{int} + E_{ho} = E_{all}, \tag{23}$$

where E_{ho} is the potential energy of the condensates. For the central peak, E_{ho} can be safely omitted. In the case of the non-interacting model, E_{all} will transform fully to the kinetic energy and potential energy of the condensates once the optical lattices are switched off. Due to the presence of the repulsive interaction between the atoms, the maximum density of the central peak would be smaller than the result of the non-interacting model.

Note that the kinetic energy of the central peak cannot be calculated using $N\int \frac{\hbar^2}{2m}(\nabla\sqrt{|\varphi_0(x, r_\perp, t_m)|^2})^2 \,\mathrm{d}V$, because the phase factor is different for a different well (see equation (7)). From the uncertainty relation, assume $E_{kin} \propto 1/R_x^2 \propto \alpha_x^4$. In the presence of repulsive interaction, we have

$$E_{kin} = \frac{\alpha_x^4}{\alpha_{x-ideal}^4} E_{all}. \tag{24}$$

From equations (22)–(24) one obtains the following equation to determine α_x^2:

$$\beta^2 + \theta\beta - 1 = 0, \tag{25}$$

where $\beta = (\alpha_x/\alpha_{x-ideal})^2$. The value of β reflects how the repulsive interaction between atoms reduces the density of the central peak. In the above expression, the dimensionless parameter $\theta = E_{int}(\alpha_x = \alpha_{x-ideal})/E_{all}$. From equation (25) we have

$$\beta = \frac{-\theta + \sqrt{\theta^2 + 4}}{2}. \tag{26}$$

Now let us turn to discuss the total energy of the condensates which is necessary to calculate the value of β. The total energy of the condensates can be obtained through the sum of the energy of the condensates in each well before the optical lattices are switched off. Before the optical lattices are switched off, the normalized wavefunction in the kth well takes the form

$$\varphi_{0k}(x, r_\perp) = \varphi_{0k}(x)\varphi_{0k}(r_\perp) \tag{27}$$

where

$$\varphi_{0k}(x) = \left(\frac{1}{\sqrt{\pi}\sigma}\right)^{1/2} \exp\left[-\frac{(x - kd)^2}{2\sigma^2}\right], \tag{28}$$

and $\varphi_{0k}(r_\perp)$ is the normalized wavefunction in the radial direction. From equation (27), we have

$$\begin{aligned} E_{all} = \sum_{k=-k_M}^{k_M} \Bigg\{ & N_k \int \varphi_{0k}(x, r_\perp)\left[-\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega_{xe}^2 x^2 + \frac{1}{2}m\omega_\perp^2 r_\perp^2\right]\varphi_{0k}(x, r_\perp)\,\mathrm{d}V \\ & + \frac{gN_k^2}{2}\int |\varphi_{0k}(x, r_\perp)|^4 \,\mathrm{d}V \Bigg\}, \end{aligned} \tag{29}$$

where ω_{xe} is the effective harmonic frequency in the x-direction of the well induced by the optical lattices. The last term in the above expression represents the interaction energy of the condensates in each well, and it is easy to verify that it can be safely omitted. In addition,

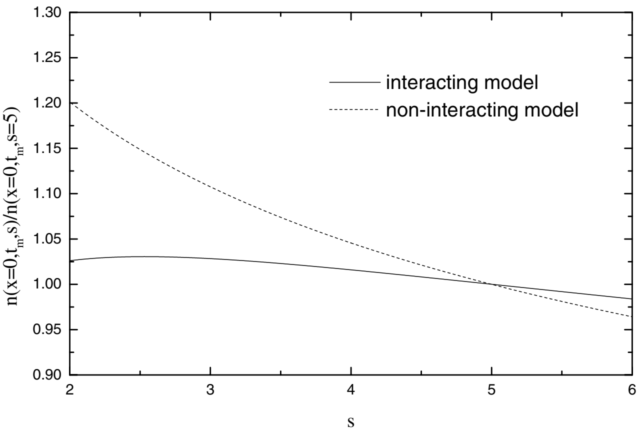

Figure 6. Displayed is the ratio between $n(x = 0, t_m, s)$ and $n(x = 0, t_m, s = 5)$ for the interacting and non-interacting models.

the kinetic energy and potential energy in the radial direction can be also omitted because $\omega_{xe} \gg \omega_\perp$ and $\hbar\omega_{xe} \gg \mu_0$. In this case, one gets

$$E_{all} \approx N\left(\frac{\hbar^2}{4m\sigma^2} + \frac{1}{4}m\omega_{xe}^2\sigma^2\right). \tag{30}$$

For the experimental parameters presented in [12], from the formulae (22), (26) and (30), calculation shows that $\beta = 0.80$. This clearly shows that the repulsive interaction between atoms would reduce the density of the central peak at t_m.

With the variation of the parameter s, the maximum density of the central peak can also be calculated based on the method given here. In the interacting model, the solid curve in figure 6 shows the ratio between $n(x = 0, t_m, s)$ and $n(x = 0, t_m, s = 5)$. The result of the non-interacting model is also shown in the figure.

6. Discussion and conclusion

In brief, in this work the evolution process of the condensates is investigated after the optical lattices are switched off. We find that the density oscillation exhibits a decay and revival phenomenon, based on the numerical result of the evolution of the density distribution. The decay of the density oscillation is interpreted as the emergence of the diffraction phenomenon, which is considered to result as a consequence of interference from a large number of coherent expanding condensates. Due to the confinement of the harmonic potential, the density distribution displays periodic character, and it is this periodic character which leads to the revival of the density oscillation. In contrast to the condensates in the magnetic trap, there is no revival of the density oscillation, when both the magnetic and optical lattices are switched off. In addition, in the case of the non-interacting model, it is shown that the maximum value of the density distribution at $x = 0$ would be approximately 30 times larger than the case when there are no optical lattices to induce the interference effect.

It is shown here that the repulsive interaction between atoms has the effect of reducing the maximum density of the central peak. In a real experiment performed in the future, it might be that the maximum density experimental result for the central peak is smaller than the theoretical prediction given here, because it is possible that there is a loss of the total energy of the condensates during the optical lattice removal process. For attractive interaction, such as for Li, the role of interaction would become very important when the atoms are confined by the combined potential. For example, when only the optical lattices are removed, based on the non-interacting model, the density at $x = 0$ would increase largely due to the interference and confinement of the magnetic trap. In addition, due to the attractive interaction between atoms, the density of the central peak would increase rapidly. In this situation, it is possible that the condensates might collapse and even explode at a subsequent time, in analogy with the dynamic process of collapsing and exploding atoms [15] produced by switching the interaction from repulsive to attractive.

Acknowledgments

This work was supported by the Natural Science Foundation of China under grant nos 10205011 and 10274021, the Science Foundation of Zhijiang College, Zhejiang University of Technology and the Natural Science Foundation of Zhejiang Province. One of us (GH) is indebted to the National Natural Science Foundation of China under grant no 19975019, and the French Ministry of Research for a visiting grant at Université Paris 7.

References

[1] Anderson M H *et al* 1995 *Science* **269** 198
Davis K B *et al* 1995 *Phys. Rev. Lett.* **75** 3969
Bradley C C *et al* 1995 *Phys. Rev. Lett.* **75** 1687
[2] Anglin J R and Ketterle W 2002 *Nature* **416** 211
[3] Dalfovo F, Giorgini S, Pitaevskii L P and Stringari S 1999 *Rev. Mod. Phys.* **71** 463
Parkin A S and Walls D F 1998 *Phys. Rep.* **303** 1
Leggett A J 2001 *Rev. Mod. Phys.* **73** 307
[4] Burger S *et al* 2001 *Preprint* cond-mat/0111235 and references therein
[5] Jaksch D *et al* 1998 *Phys. Rev. Lett.* **81** 3108
[6] Brennen G K *et al* 1999 *Phys. Rev. Lett.* **82** 1060
[7] Anderson B P and Kasevich M A 1998 *Science* **282** 1686
[8] Burger S *et al* 2002 *Europhys. Lett.* **57** 1
[9] Burger S *et al* 2001 *Phys. Rev. Lett.* **86** 4447
[10] Cataliotti F S *et al* 2001 *Science* **293** 843
[11] Greiner M *et al* 2002 *Nature* **415** 39
[12] Pedri P *et al* 2001 *Phys. Rev. Lett.* **87** 220401
[13] Morsch O *et al* 2002 *Preprint* cond-mat/0204528
[14] Feynman R P and Hibbs A R 1965 *Quantum Mechanics and Path Integrals* (New York: McGraw-Hill)
[15] Donley E A *et al* 2001 *Nature* **412** 295