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Three-Wave Resonant Interactions in Self-Defocusing Optical Media *

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A three-wave resonant interaction for nonlinear excitations created from a continuous-wave background is shown to be possible in an isotropic optical medium with a self-defocusing cubic nonlinearity. Under suitable phase-matching conditions the nonlinear envelope equations for the resonant interaction are derived by using a method of multiple-scales. Some explicit three-wave solitary wave and lump solutions are discussed.

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It is well known that bright solitons do not exist in a self-defocusing medium, while dark solitons can be excited from a modulational stable cw background. Because of fundamental interest and potential applications, in recent years dark solitons in self-defocusing optical media have attracted much attention theoretically and experimentally.[1] Recently, possible (2+1)-dimensional envelope solitons, called the dromions, have been shown to be possible in a self-defocusing optical medium.[2] However, it seems that up to now nobody has been aware of the possibility of resonances between the excited waves created from a cw background.

Wave resonant interaction is a classical chapter in nonlinear Optics.[3] For a passive optical medium there exists a common belief for the wave resonant interaction, i.e., under suitable phase-matching conditions a three-wave resonance (TWR) (including second-harmonic generation as a special case) occurs if the medium has a quadratic ($\chi^{(2)}$) nonlinearity, while four-wave mixing processes (including spatial and temporal solitons as special cases) appear if the medium is of a cubic (i.e. $\chi^{(3)}$) nonlinearity. It seems that a TWR cannot be realized if a medium is centre-symmetric (and hence $\chi^{(2)}=0$). We should note that such a conclusion is only valid for the excitations created from a vanishing electric field background.

In this Letter, we show that a TWR can occur in a self-defocusing optical medium with only a cubic nonlinearity. The excited waves considered here are generated from a cw background and the interaction between them is shown to have a quadratic character. Under suitable phase-matching conditions, the envelope equations for the TWR are derived by using a method of multiple-scales. Some explicit three-wave soliton solutions are provided and discussed.

We consider the propagation of a monochromatic electric field $\mathcal{E}$ in a centre-symmetric self-defocusing optical medium (i.e. $\chi^{(2)}=0$ and $\chi^{(3)}<0$). The intensity-dependent refractive index reads $n=n_0+n_2|\mathcal{E}|^2$, where $n_2(<0)$ is the Kerr coefficient. When looking for a solution of Maxwell's equations in the form of a slowly varying envelope of a carrier wave with propagation constant β_0, one can obtain the (2+1)-dimensional nonlinear Schrödinger equation[4]

$$2\mathrm{i}\beta_0\Big(\frac{\partial\mathcal{E}}{\partial t}+v_g\frac{\partial\mathcal{E}}{\partial z}\Big)+\frac{\partial^2\mathcal{E}}{\partial x^2}+\frac{\partial^2\mathcal{E}}{\partial y^2}+\beta_0^2\frac{n_2}{n_0}|\mathcal{E}|^2\mathcal{E}=0, \quad (1)$$

where $\mathcal{E}(x,y,z,t)$ is a complex slowly varying envelope of the electric field. The above equation can be rewritten as

$$2\mathrm{i}\beta_0\frac{\partial\mathcal{E}}{\partial T}+\frac{\partial^2\mathcal{E}}{\partial x^2}+\frac{\partial^2\mathcal{E}}{\partial y^2}+\beta_0^2\frac{n_2}{n_0}|\mathcal{E}|^2\mathcal{E}=0, \quad (2)$$

where T denotes time t (if $\mathcal{E}$ does not depends on z), or z/v_g (if $\mathcal{E}$ does not depends on t), or $(z+v_gt)/(2v_g)$. Using the transformation $T\to\beta_0t'$ and $\mathcal{E}\to[2n_0/(|n_2|\beta_0^2)]^{1/2}\psi\exp(-\mathrm{i}t')$, we can reduce Eq. (2) to the normalized form

$$2\mathrm{i}\frac{\partial\psi}{\partial t}+\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}-2(|\psi|^2-1)\psi=0, \quad (3)$$

where the prime has been dropped. Letting $\psi=Q\exp(\mathrm{i}R)$ with Q and R being two real functions, Eq. (2) is recast into the hydrodynamic form

$$\frac{\partial Q}{\partial t}+\frac{Q}{2}\Big(\frac{\partial^2R}{\partial x^2}+\frac{\partial^2R}{\partial y^2}\Big)+\frac{\partial Q}{\partial x}\frac{\partial R}{\partial x}+\frac{\partial Q}{\partial y}\frac{\partial R}{\partial y}=0, \quad (4)$$

$$Q\frac{\partial R}{\partial t}-\frac{1}{2}\Big(\frac{\partial^2Q}{\partial x^2}+\frac{\partial^2Q}{\partial y^2}\Big)+\frac{Q}{2}\Big[\Big(\frac{\partial R}{\partial x}\Big)^2+\Big(\frac{\partial R}{\partial y}\Big)^2\Big]+Q^3-Q=0. \quad (5)$$

It is obvious that a cw solution, denoted by $(Q,R)=(1,0)$ [or equivalently $\mathcal{E}=[2n_0/(|n_2|\beta_0^2)]^{1/2}\exp(-\mathrm{i}t)$], exists. It is well known that this cw solution is modulationally stable.[4] It is noted that the dynamics of the excitations generated from the cw background [represented by $(\tilde{Q},\tilde{R})==(Q-1,R)$] is controlled by coupled nonlinear equations with both *quadratic* and cubic nonlinearities. It is easy to obtain the linear

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dispersion relation of the excitations

$$\omega^2 = \frac{1}{4}k^2(k^2+4) \tag{6}$$

with $k^2 = k_x^2 + k_y^2$, where k and ω are the wavevector and the frequency of the excitations, respectively.

We are interested in a possible TWR of exciting waves. For an efficient TWR, the phase-matching conditions

$$\omega_1 + \omega_2 = \omega_3, \tag{7}$$

$$\boldsymbol{k}_1 + \boldsymbol{k}_2 = \boldsymbol{k}_3, \tag{8}$$

should be required. From (6) it is easy to show that these conditions can be fulfilled if we choose $\boldsymbol{k}_1 = (k_1\cos\varphi, k_1\sin\varphi)$, $\boldsymbol{k}_3 = (k_3, 0)$ and $\boldsymbol{k}_2 = \boldsymbol{k}_3 - \boldsymbol{k}_1 = (k_3 - k_1\cos\varphi, -k_1\sin\varphi)$, where k_1 and k_3 are positive, and φ satisfies

$$\cos\varphi = \frac{1}{2k_1k_3}\bigg\{k_1^2 + k_3^2 + 2 - 2\bigg[1 + \bigg[k_3\Big(1+\frac{1}{4}k_3^2\Big)^{1/2} - k_1\Big(1+\frac{1}{4}k_1^2\Big)^{1/2}\bigg]^2\bigg]^{1/2}\bigg\}. \tag{9}$$

It is easy to show that for any non-vanishing k_1 and k_3 we have $0 < \cos\varphi < 1$ and hence $-\pi/2 < \varphi < \pi/2$. Consequently, in the self-defocusing optical medium a TWR is possible for the excitations created from the cw background.

We now derive the envelope equations controlling the TWR. It is assumed that $Q = 1 + \epsilon Q^{(1)} + \epsilon^2 Q^{(2)} + \cdots$, $R = \epsilon R^{(1)} + \epsilon^2 R^{(2)} + \cdots$, where ϵ is a small parameter denoting the amplitude of an excitation, $Q^{(j)}$ and $R^{(j)}$ $(j = 1, 2, 3, \cdots)$ are the functions of the fast variables x, y, and t and the slow variables $x_1 = \epsilon x$, $y_1 = \epsilon y$ and $t_1 = \epsilon t$. Then Eqs. (4) and (5) read

$$\frac{\partial Q^{(j)}}{\partial t} + \frac{1}{2}\Big(\frac{\partial^2 R^{(j)}}{\partial x^2} + \frac{\partial^2 R^{(j)}}{\partial y^2}\Big) = \alpha^{(j)}, \tag{10}$$

$$\frac{\partial R^{(j)}}{\partial t} - \frac{1}{2}\Big(\frac{\partial^2 Q^{(j)}}{\partial x^2} + \frac{\partial^2 Q^{(j)}}{\partial y^2}\Big) + 2Q^{(j)} = \beta^{(j)}. \tag{11}$$

The explicit expressions of $\alpha^{(j)}$ and $\beta^{(j)}$ are omitted here.

In the leading order $(j = 1)$, Eqs. (10) and (11) yield the solution $Q^{(1)} = Q_{11}\exp(\mathrm{i}\theta)$+c.c. and $R^{(1)} = R_0 + [R_{11}\exp(\mathrm{i}\theta)\text{+c.c.}]$ with $\theta = \boldsymbol{k}\cdot\boldsymbol{r} - \omega t$, $\boldsymbol{k} = (k_x, k_y)$ and $\boldsymbol{r} = (x, y)$, where $R_{11} = (k^2+4)/(2\mathrm{i}\omega)Q_{11}\exp(\mathrm{i}\theta)$, $\omega = \omega(k_x, k_y)$ has been given by Eq. (6). Obviously, any linear superposition of such modes is also a solution. Because we are interested in a TWR and hence we take $Q^{(1)} = \sum_{l=1}^{3}[Q_{1l}\exp(\mathrm{i}\theta_l) + \text{c.c.}]$ and $R^{(1)} = R_0 + \sum_{l=1}^{3}[R_{1l}\exp(\mathrm{i}\theta_l) + \text{c.c.}]$ with $\theta_l = \boldsymbol{k}_l\cdot\boldsymbol{r} - \omega_l t$ and $R_{1l} = (k_l^2+4)/(2\mathrm{i}\omega_l)Q_{1l}\exp(\mathrm{i}\theta_l)$.

In the next order $(j = 2)$, using the TWR conditions (7) and (8) we obtain the closed equations controlling the evolution of the envelopes Q_{1l} $(l = 1, 2, 3)$. Then by taking $Q_l = \epsilon Q_{1l}$ and returning to the original variables we obtain

$$\frac{\partial Q_1}{\partial t} + \boldsymbol{v}_1\cdot\nabla Q_1 = \lambda_1 Q_2^* Q_3^*, \tag{12}$$

$$\frac{\partial Q_2}{\partial t} + \boldsymbol{v}_2\cdot\nabla Q_2 = \lambda_2 Q_1^* Q_3^*, \tag{13}$$

$$\frac{\partial Q_3}{\partial t} + \boldsymbol{v}_3\cdot\nabla Q_3 = \lambda_3 Q_1^* Q_2^*, \tag{14}$$

where $\nabla = (\partial/\partial x, \partial/\partial y)$, $\boldsymbol{v}_l = (\mathrm{d}\omega_l/\mathrm{d}k_{lx}, \mathrm{d}\omega_l/\mathrm{d}k_{ly})$ $(l = 1, 2, 3)$ is the group velocity of the lth waves; λ_1, λ_2, and λ_3 are the complex coupling coefficients, which carry the signs of wave energy. Their explicit expressions are omitted here. Equations (12)–(14) are the envelope ones describing the TWR, as is well known in wave resonance theory.[5]

Next we discuss the soliton solutions of Eqs. (12)–(14). Using the transformation $q_j = (\lambda_i\lambda_k/\gamma_i\gamma_k)^{1/2}Q_j$ Eqs. (12)–(14) can be cast into the form

$$\frac{\partial q_i}{\partial t} + \boldsymbol{v}_i\cdot\nabla q_i = \gamma_i q_j^* q_k^*, \tag{15}$$

where i, j, and k are cyclic and equal to 1, 2, 3; γ in Eq. (15) has been scaled to unity magnitude $\gamma_i^2 = 1$. It is interesting that the above equations are completely integrable and can be solved by the inverse scattering transform.[6] Three types of three-wave soliton solutions can be obtained, which are presented in the following.

Let $\xi = x - ut$, $\eta = y - vt$ (u and v are constants), the (2+1)-dimensional Eq. (15) becomes the (1+1)-dimensional ones:

$$\frac{\partial q_i}{\partial \xi} + C_i\frac{\partial q_i}{\partial \eta} = \tilde{\gamma}_i q_j^* q_k^*, \tag{16}$$

where $C_i = (v_{iy} - v)/(v_{ix} - u)$ and $\tilde{\gamma}_i = \gamma_i/(v_{ix} - u)$. Then using the results from the inverse scattering method we obtain[6]

$$q_1 = \frac{4\sigma_1(\beta_{12}\beta_{13})^{1/2}}{D(\widetilde{\gamma_2}\widetilde{\gamma_3})^{1/2}}\exp[-\mathrm{i}(\phi_1 - 2\rho_1 Z_1)] \cdot [\exp(2\sigma_2 Z_2) + \frac{\zeta_1^* - \zeta_2^*}{\zeta_1^* - \zeta_2}\exp(-2\sigma_2 Z_2)], \tag{17}$$

$$q_2 = \frac{4\sigma_1(\beta_{13}\beta_{32})^{1/2}}{D(\widetilde{\gamma_1}\widetilde{\gamma_3})^{1/2}}\exp[-\mathrm{i}(\phi_2 - 2\rho_2 Z_2)] \cdot [\exp(-2\sigma_1 Z_1)) + \frac{\zeta_1 - \zeta_2}{\zeta_1^* - \zeta_2}\exp(2\sigma_1 Z_1)], \tag{18}$$

$$q_3 = \frac{-16\mathrm{i}\sigma_1\sigma_2\beta_{12}}{D(\zeta_1 - \zeta_2^*)(\beta_{12}\beta_{13}\widetilde{\gamma_1}\widetilde{\gamma_2})^{1/2}} \cdot \exp[\mathrm{i}(\phi_1 + \phi_2 - 2\rho_1 Z_1 - 2\rho_2 Z_2)], \tag{19}$$

with

$$D = \exp(2\sigma_1 Z_1 + 2\sigma_2 Z_2) + \exp(-2\sigma_1 Z_1 + 2\sigma_2 Z_2) + \exp(-2\sigma_1 Z_1 - 2\sigma_2 Z_2) + \Big|\frac{\zeta_1^* - \zeta_2^*}{\zeta_1 - \zeta_2^*}\Big|\exp(2\sigma_1 Z_1 - 2\sigma_2 Z_2), \tag{20}$$

where $Z_1 = \xi - C_1\eta - \xi_{10}$, $Z_2 = \xi - C_2\eta - \xi_{20}$, $\zeta_1 =$

$2(\rho_1+\mathrm{i}\sigma_1)/\beta_{23}$, $\zeta_2=2(\rho_2+\mathrm{i}\sigma_2)/\beta_{13}$, $\beta_{ij}=C_j-C_i$ $(C_2>C_3>C_1)$, ρ_j, σ_j, ϕ_j and ξ_{j0} $(j=1,2)$ are the constants.

We can easily obtain the asymptotic form of the solution. As $t\to-\infty$ one has

$$q_1\simeq\frac{2\sigma_1(\beta_{12}\beta_{13})^{1/2}}{(\widetilde{\gamma}_2\widetilde{\gamma}_3)^{1/2}}\,\mathrm{sech}(2\sigma_1Z_1)\exp(\mathrm{i}\phi_1),$$
$$q_2\simeq\frac{2\sigma_1(\beta_{13}\beta_{32})^{1/2}}{(\widetilde{\gamma}_1\widetilde{\gamma}_3)^{1/2}}\,\mathrm{sech}(2\sigma_2Z_2)\exp(\mathrm{i}\phi_2),\ q_3\simeq0. \tag{21}$$

As $t\to\infty$, one can obtain

$$\begin{aligned}q_1\simeq&\frac{2\sigma_1(\beta_{12}\beta_{13})^{1/2}}{(\widetilde{\gamma}_2\widetilde{\gamma}_3)^{1/2}}\,\mathrm{sech}(2\sigma_1Z_1-\delta)\\&\cdot\exp(\mathrm{i}\phi_1+\delta'),\\q_2\simeq&\frac{2\sigma_1(\beta_{13}\beta_{32})^{1/2}}{(\widetilde{\gamma}_1\widetilde{\gamma}_3)^{1/2}}\,\mathrm{sech}(2\sigma_2Z_2-\delta)\\&\cdot\exp(\mathrm{i}\phi_2-\delta'),\ q_3\simeq0,\end{aligned} \tag{22}$$

where δ and δ' are defined by $(\zeta_1-\zeta_2)/(\zeta_1^*-\zeta_2)=\mathrm{e}^{-\delta}\mathrm{e}^{\mathrm{i}\delta'}$.

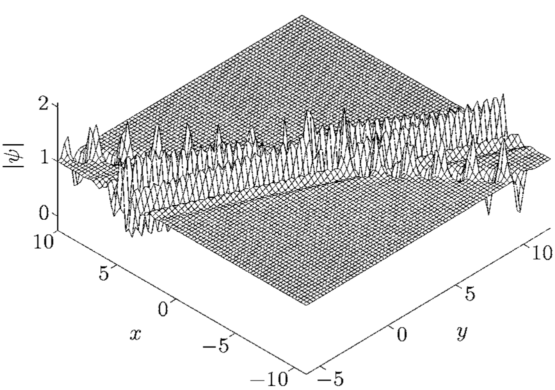

Fig. 1. Modula $|\psi|$ in the case of the three-wave soliton interaction (without energy transfer) created from the cw background. The parameters are chosen to be $k_1=2$, $k_3=6$, $u=-1$, $v=5$, $\xi_{10}=1$, $\xi_{20}=2$, $\phi_1=2$, $\phi_2=1$, $\rho_1=1$, $\rho_2=1$, $\sigma_1=0.3$, and $\sigma_3=3$ at time $t=1$.

From the above result we can see that the solution (17)–(19) describe a process of collision between the solitons q_1 and q_2. In the colliding region, a new soliton q_3, called the Mach sterm, is produced. During the collision, there is no energy exchange between q_1 and q_2. Figure 1 shows the modula of ψ (exact to the leading-order),

$$\begin{aligned}|\psi|=&Q\approx1+Q^{(1)}\\=&1+\left(Q_1\mathrm{e}^{\mathrm{i}\theta_1}+Q_2\mathrm{e}^{\mathrm{i}\theta_2}+Q_3\mathrm{e}^{-\mathrm{i}\theta_3}+\mathrm{c.c.}\right).\end{aligned} \tag{23}$$

The phase shift due to the collision can be clearly seen in the figure.

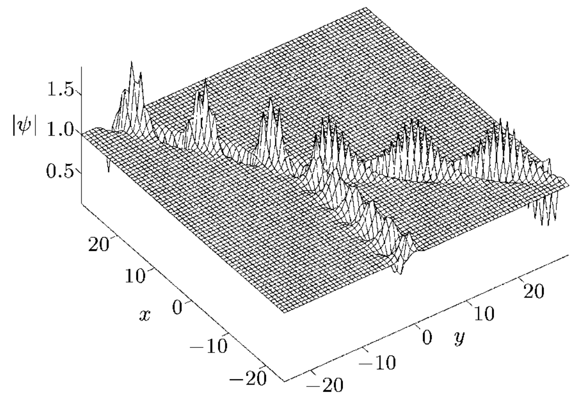

Fig. 2. Modula $|\psi|$ in the case of three-wave soliton interaction (with energy transfer) created from the cw background. The parameters are chosen as $k_1=2$, $k_3=1$, $d_{13}=1$, $\kappa_1=0$, $\kappa_2=0$, $\zeta=0$, $\nu=10$, $m=0$, $n=-1.5$, $B_1=1$, and $B_2=5$ at time $t=1$.

Another interesting case is a three-soliton resonance, i.e., two solitons are resonant with the third. To show this, in Eqs. (15) we choose $v_{2x}>v_{3x}>v_{1x}$ and define $b_j=(v_{iy}-v_{ky})/(v_{kx}-v_{ix})$ and $r=(b_2-b_1)(v_{2x}-v_{3x})=(b_2-b_3)(v_{2x}-v_{1x})$, where s, d_{13}, and d_{23} are the constants related by $s=d_{13}(v_{3x}-v_{2x})=d_{23}(v_{2x}-v_{1x})$. Then we have the following solution[7]

$$q_1=\sqrt{\frac{\lambda_2\lambda_3}{\gamma_2\gamma_3}}K_1\mathit{\Gamma}_2\mathit{\Delta}^{-1},\quad q_2=\sqrt{\frac{\lambda_1\lambda_3}{\gamma_1\gamma_2}}K_2\mathit{\Gamma}_1\mathit{\Gamma}_3^*\mathit{\Delta}^{-1},$$
$$q_3=\sqrt{\frac{\lambda_1\lambda_2}{\gamma_1\gamma_2}}K_3\mathit{\Gamma}_2\mathit{\Delta}^{-1}, \tag{24}$$

where

$$\begin{aligned}K_1=&-\mathrm{i}[(v_{2x}-v_{1x})(v_{3x}-v_{1x})/\lambda_2\lambda_3]^{1/2}\vartheta/(v_{2x}-v_{1x}),\\K_2=&\,\mathrm{i}[(v_{2x}-v_{1x})(v_{2x}-v_{3x})/\lambda_1\lambda_3]^{1/2}(b_1-b_3)\vartheta/r,\\K_3=&-\mathrm{i}[(v_{2x}-v_{1x})(v_{3x}-v_{1x})/\lambda_1\lambda_2]^{1/2}\vartheta/(v_{2x}-v_{3x}),\\\mathit{\Delta}=&\,1+(1+rp)(|\mathit{\Gamma}_1|^2+|\mathit{\Gamma}_2|^2),\vartheta=2\nu s(1+rp),\\\mathit{\Gamma}_1=&\,B_1\exp\{\mathrm{i}[(b_1\kappa_1+\zeta d_{13}+mb_1-mb_2)\\&\cdot(x-v_{3x}t)+\kappa_1(y-v_{3y}t)]\}\\&\cdot\exp\{-(\nu d_{13}-nb_3)(x-v_{3x}t)+n(y-v_{3y}t)\},\\\mathit{\Gamma}_2=&\,B_2\exp\{\mathrm{i}[(b_2\kappa_2+\zeta d_{23}+mb_2-mb_3)\\&\cdot(x-v_{1x}t)+\kappa_2(y-v_{1y}t)]\}\\&\cdot\exp\{-(\nu d_{23}-nb_3)(x-v_{1x}t)+n(y-v_{1y}t)]\},\end{aligned} \tag{25}$$

with $p=-n(\nu s)^{-1}$, κ_1, κ_2, ζ, ν, m, and n being the real constants. We note that for this solution to hold, the condition $1+rp>0$ is required. Figure 2 shows the modula $|\psi|$ in the case of the three-wave soliton solution (24). We can see that during a collision, the energies of q_1 and q_2 solitons are transferred into the q_3 soliton which completes a resonant triad.

The three-wave interaction Eq. (15) also admits the so-called three-wave lump solutions. To show this we transform Eq. (15) from space–time coordinates to the characteristic coordinates

$$\frac{\partial q_i}{\partial X_i} = \gamma_i q_j^* q_k^*, \tag{26}$$

where X_i is the ith characteristic, defined by $\partial/\partial X_i = -\partial/\partial t - \boldsymbol{v}_i \cdot \nabla$. The relations between these coordinate and x, y, t are given by $X_3 = [x(v_{1y} - v_{2y}) + y(v_{2x} - v_{1x}) + t(v_{1x}v_{2y} - v_{2x}v_{1y})]/[v_{1x}(v_{3y} - v_{2y}) + v_{2x}(v_{1y} - v_{3y}) + v_{3x}(v_{2y} - v_{1y})]$, $X_2 = (y - v_{1y}t)/(v_{1y} - v_{2y}) - [(v_{1y} - v_{3y})/(v_{1y} - v_{2y})]X_3$, and $X_1 = -t - X_2 - X_3$. The one-lump solution of Eq. (26) reads[8,9]

$$q_j = \frac{g_i^* g_k}{D}, \quad D = 1 + \sum_{i=1}^{3} \gamma_i G_i, \tag{27}$$

where $G_i(X_i) = \int_{X_i}^{\infty} g_i^*(u) g_i(u) \mathrm{d}u$, and g_i is arbitrary functions of the single variable X_i. In Fig. 3 we have plotted the modula $|\psi|$ when choosing $g_i = c_i \mathrm{e}^{-p_i X_i^2}$. We can see that the solution shows a collision among three lumps, which are localized in all spatial directions.

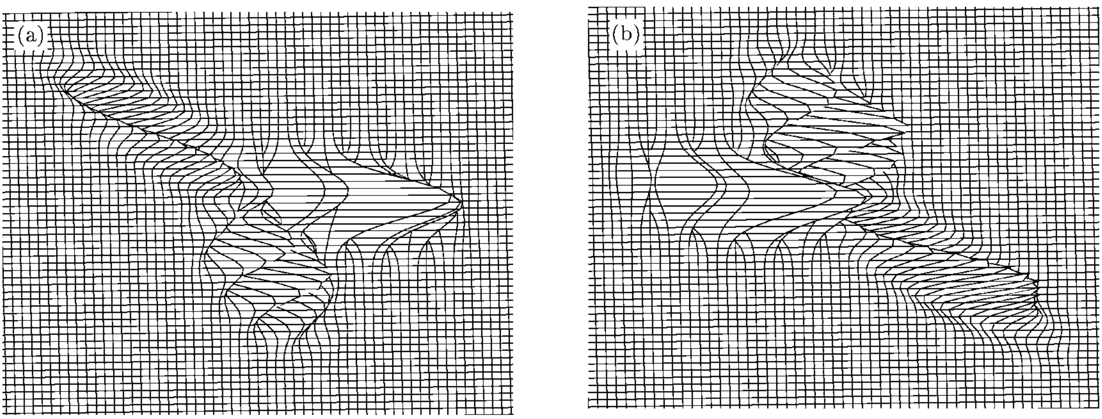

Fig. 3. Modula $|\psi|$ in the case of three-wave lump excitation created from the cw background. The parameters are chosen as $k_1 = 2$, $k_3 = 4$, $c_1 = \sqrt{1/14}$, $c_2 = \sqrt{1/2}$, $c_1 = \sqrt{7/12}$, $p_1 = \pi/2$, $p_2 = \pi/2$, and $p_3 = \pi/2$ at times $t = -2.5$ before collision (a) and 2.5 after collision (b).

In conclusion, based on a self-defocusing nonlinear Schrödinger equation we have investigated a wave resonant interaction in an isotropic optical medium with a cubic nonlinearity. We have shown that a three-wave resonance is indeed possible for the exciting waves created from a cw background. By adequately choosing the wavevectors and frequencies of the three exciting waves the phase-matching conditions for the three-wave resonant interaction can be fulfilled. We have also derived the three-wave resonant interaction equations by using a method of multiple-scales. Some explicit three-wave soliton solutions (including those with and without energy transfer) and localized three-wave lump solutions are presented. We note that conventional three-wave resonant interactions are realized only in optical materials with a quadratic nonlinearity.[3,5] Here we have shown for the first time that the materials of a cubic nonlinearity with negative Kerr coefficient can also provide with the possibility of three-wave resonant interactions. The idea presented in this work can be applied to investigate the three-wave resonance of the excitations created in Bose–Einstein condensates.[10]

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