Second-Harmonic Generation in Optical Fibres Induced by a Cross-Phase Modulation Effect *

CUI Wei-Na(崔维娜)**, HUANG Guo-Xiang(黄国翔)

Department of Applied Physics, Nanjing University of Science and Technology, Nanjing 210094

Department of Physics, East China Normal University, Shanghai 200062

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When two optical pulses copropagate inside a single-mode fibre, intensity-dependent refractive index couples the pulses through a cross-phase modulation (XPM). We show that a second-harmonic generation (SHG) on a continuous-wave background is possible in the optical fibre induced by the XPM effect. By means of a multi-scale method the nonlinearly coupled envelope equations for the SHG are derived and their explicit solutions are provided and discussed.

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Second-harmonic generation (SHG) has been extensively studied in nonlinear optics because of its fundamental interest as well as technological applications.[1] An SHG in glass fibres is unexpected since second-order processes are normally forbidden in materials such as silica with a centre of inversion.[2] Several early experiments showed that the SHG as well as other second-order parametric processes can be generated when an intense 1.06 μm pump pulse from a mode-locked, Q-switched, Nd:YAG laser propagates through optical fibres.[3] Up to now SHG conversion efficiency as high as 10% was achieved in a Ge-doped silica optical fibre after irradiation by a laser beam for several hours.[4] Most theoretical explanations focused on quadrupole interaction or the processes near the surface where a centro-symmetry is broken. However, up to now a satisfactory theoretical explanation on the SHG in optical fibres remains lacking.[5]

In a recent work, it was proposed that a new type of SHG in an optical fibre can be realized without need of any breaking of the centro-symmetry.[6,7] Such SHG is for optical excitations created from a cw background and, for fulfilling related phase-matching conditions, the fibre must work near zero-dispersion point and hence third-order dispersion must be taken into account. Note that without high-order dispersion the SHG is not possible because its excitation spectrum cannot satisfy the phase-matching condition for SHG.[6,7] Thus we consider the case of two optical fields co-propagating in optical fibres, whose cross-phase modulation (XPM) effect provides us a novel physical mechanism to fulfill the SHG phase-matching condition even without third-order dispersion.

The XPM effect in optical fibres and its various applications for pulse compression, optical switching, etc. have been intensively investigated.[5,8,9] When two optical pulses copropagate inside a single-mode fibre, the intensity-dependent refractive index couples the two pulses through a nonlinear phenomenon, i.e. XPM. A steady state (i.e. cw background) can be modulationally stable or unstable, depending on the parameters of the system. Most studies on the XPM concentrate on the unstable regime, where coupled optical soliton pairs appear.[5] Here we are interested in the stable regime and show that an SHG is possible for the excitations on the cw background through the XPM effect.

Consider two optical pulses copropagating in a single-mode, polarization-preserving fibre. Under a slowly varying envelope approximation, the amplitudes of electric field pulse A_1 and A_2 can be described by the following coupled nonlinear Schrödinger (NLS) equations:[8]

$$\frac{\partial A_1}{\partial z}+\frac{1}{v_{g1}}\frac{\partial A_1}{\partial t}+\frac{i}{2}\beta_1\frac{\partial^2 A_1}{\partial t^2}=i\gamma_1(|A_1|^2+2|A_2|^2)A_1, \tag{1}$$

$$\frac{\partial A_2}{\partial z}+\frac{1}{v_{g2}}\frac{\partial A_2}{\partial t}+\frac{i}{2}\beta_2\frac{\partial^2 A_2}{\partial t^2}=i\gamma_2(|A_2|^2+2|A_1|^2)A_2, \tag{2}$$

where A_j ($j=1,2$) is the pulse envelope assumed to be slowly varying with both distance z and time t; $v_{gj}=(dk_j/d\omega)^{-1}_{\omega=\omega_j}$ is the group velocity with respect to the central frequency ω_j, $\beta_j=(d^2k_j/d\omega^2)_{\omega=\omega_j}$ is the group-velocity dispersion coefficient, and $\gamma_j=n_2\omega_j/(cA_{\rm eff})$ accounts for the fibre nonlinearity with the Kerr coefficient n_2 and the effective core area $A_{\rm eff}$ of the fibre. The two terms on the right-hand side of Eqs. (1) and (2) are responsible for SPM (self-phase modulation) and XPM, respectively.

Expressing the field amplitude in terms of their modulus and phase, i.e., $A_j=a_j\exp^{i\phi_j}$, we obtain a set of coupled nonlinear equations for a_j and ϕ_j ($j=1,2$):

$$\frac{\partial a_j}{\partial z}+\frac{1}{v_{gj}}\frac{\partial a_j}{\partial t}-\frac{\beta_j}{2}\Big(2\frac{\partial a_j}{\partial t}\frac{\partial \phi_j}{\partial t}+a_j\frac{\partial^2\phi_j}{\partial t^2}\Big)=0, \tag{3}$$

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** Email: cuiweinaa@yahoo.com

$$a_j\frac{\partial\phi_j}{\partial z}+\frac{1}{v_{gj}}a_j\frac{\partial\phi_j}{\partial t}+\frac{\beta_j}{2}\Big[\frac{\partial^2a_j}{\partial t^2}-a_j\Big(\frac{\partial\phi_j}{\partial t}\Big)^2\Big]-\gamma_j(a_j^2+2a_{3-j}^2)a_j=0. \quad (4)$$

Theses equations are readily solved for obtaining a steady-state cw solution, i.e. $A_j(z)=a_{jo}\exp(i\phi_{jo})$, where the phase $\phi_{jo}=\gamma_j(a_{jo}^2+2a_{(3-j)o}^2)z$ and a_{j0} being the amplitude. Based on Eqs. (3) and (4), we investigate the linear stability of the cw solution against a small perturbation (i.e. excitation). To do so, we follow the standard procedure and look for solutions describing small variations around the exact solution with the form $A_j(z)=(a_{jo}+\varsigma_j)\exp(i\phi_{jo}+i\psi_j)$, where the functions ς_j and derivative of the phase ψ_j are assumed to be small. If the excitation varies with the form $\varsigma_j,\psi_j\sim\exp(i\omega t-ikz)$ and the group-velocity mismatch of the two optical fields can be neglected, we obtain the linear dispersion relation[8]

$$k=\frac{\omega}{v_{g1}}+\frac{1}{\sqrt{2}}[(f_1+f_2)\pm[(f_1+f_2)^2+4(c_1^2-f_1f_2)]^{1/2}]^{1/2}, \quad (5)$$

where $c_1^2=4\omega^4a_{1o}^2a_{2o}^2\gamma_1\gamma_2\beta_1\beta_2$ and $f_j=\beta_j^2\omega^4/4+\beta_j\gamma_ja_{jo}^2\omega^2$ $(j=1,2)$; k and ω are the wave vector and frequency of the excitation, respectively. From Eq. (5) we can find that when $c_1^2<f_1f_2$, one has $\omega^2>\{2[(\beta_1\gamma_2a_{20}^2+\beta_2\gamma_1a_{10}^2)^2+12\gamma_1\gamma_2\beta_1\beta_2a_{10}^2a_{20}^2]^{1/2}-(\beta_1\gamma_2a_{20}^2+\beta_2\gamma_1a_{10}^2)\}/(\beta_1\beta_2)$. The cw background is stable against the excitation (perturbation) whenever the fibre works in normal or anormal dispersion regime. Equation (5) gives two branches of linear dispersion curves for the excitation, i.e. $k_+(\omega)$ and $k_-(\omega)$, which are shown in Fig. 1.

Our interest here is a possible SHG for the excitation created on the cw background. To realize the SHG a phase-matching condition is necessary, which reads

$$\omega_2=2\omega_1,\quad k_2=2k_1, \quad (6)$$

where ω_1 and k_1 (ω_2 and k_2) are the frequency and wave vector of fundamental (second-harmonic) wave, respectively. By choosing $k_1=k_+(\omega_1)$ and $k_2=k_-(\omega_2)=k_-(2\omega_1)$, the phase-matching condition (6) can indeed be fulfilled, as shown in Fig. 1. Note that Eq. (6) is equivalent to $k_-(2\omega_1)=2k_+(\omega_1)$, which results in the solution

$$\omega_1=\{[(20g_3g_4(g_1^2-g_3^2))^2-16(9g_1^4+25g_3^4-34g_1^2g_3^2)\cdot(g_3^2g_4^2-g_1^2g_4^2-4cg_1^2)]^{1/2}+20g_3g_4(g_1^2-g_3^2)\}^{1/2}\cdot\{2(9g_1^4+25g_3^4-34g_1^2g_3^2)\}^{-1/2} \quad (7)$$

with $g_1=(\beta_1^2+\beta_2^2)/4$, $g_3=(\beta_1^2-\beta_2^2)/4$, $g_2=\beta_1\gamma_1a_{1o}^2+\beta_2\gamma_2a_{20}^2$, $g_4=\beta_1\gamma_1a_{1o}^2-\beta_2\gamma_2a_{20}^2$, $c=4a_{1o}^2a_{2o}^2\gamma_1\gamma_2\beta_1\beta_2$. As we see in Eq. (7) the frequency of fundamental wave depends on input optical power a_{jo}^2. If only optical field 1 or field 2 is incident, another vanishes, one has $g_1=g_3=\beta_1^2/4$, $g_2=g_4=\beta_1\gamma_1a_{10}^2$ or $g_1=-g_3=\beta_2^2/4$, $g_2=-g_4=\beta_2\gamma_2a_{20}^2$ and $c=0$, and hence Eq. (7) has no solution. From Eq. (7), we also see in the absence of XPM, $c_1=0$ and the solution is $k=\omega/v_{g1}+\sqrt{f_j}$ for $j=1,2$ which means condition (6) can not be satisfied and SHG does not occur for the case of a single field. It is the XPM effect that makes the phase-matching condition (6) be satisfied and hence the SHG possible. The parameters of standard single-mode optical fibre can be chosen as $n_2=3.2\times10^{-16}\,\mathrm{cm^2/W}$, $\beta_1=\beta_2=0.06\times\mathrm{ps^2/m}$, $\gamma_1=\gamma_2=0.015\,\mathrm{W^{-1}/m}$, $v_{g1}=v_{g2}=2\times10^8\,\mathrm{ms^{-1}}$ and the power of the optical field background is taken as $a_{1o}^2=2\,\mathrm{KW}$, $a_{2o}^2=4.9\,\mathrm{KW}$, we obtain $\omega_1=0.63\times10^{14}\,\mathrm{s^{-1}}$, $k_1=6.26\times10^5\,\mathrm{m^{-1}}$.

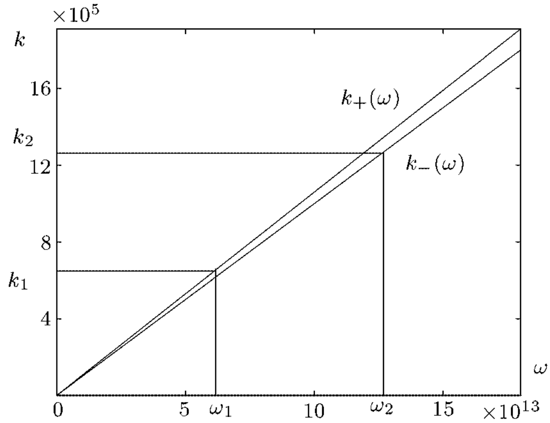

Fig. 1. The linear dispersion relation and phase matching of an SHG for the excitation on a cw background. The phase-matching condition can be satisfied if (ω_1,k_1) and (ω_2,k_2) are chosen from different dispersion branches $k_+(\omega)$ and $k_-(\omega)$, respectively. The parameters used in the figure are $\beta_1=\beta_2=0.06\times\mathrm{ps^2/m}$, $\gamma_1=\gamma_2=0.015\,\mathrm{W^{-1}/m}$, $v_{g1}=v_{g2}=2\times10^8\,\mathrm{ms^{-1}}$, $a_{1o}^2=2\,\mathrm{kW}$, $a_{2o}^2=4.9\,\mathrm{kW}$.

We now derive the nonlinear envelope (or amplitude) equations controlling the SHG. By introducing the asymptotic expansion $a_j=a_{jo}+a_{jo}(\varepsilon a_j^{(1)}+\varepsilon^2a_j^{(2)}+\cdots)$ and $\phi_j=\phi_{jo}+\phi_{jo}(\varepsilon\phi_j^{(1)}+\varepsilon^2\phi_j^{(2)}+\cdots)$ with ε being a small ordering parameter and a_j and ϕ_j being the functions of the fast variables (z,t) and the slow variables $(\varepsilon z,\varepsilon t)$, Eqs. (3) and (4) are transformed into a set of equations for $a_j^{(l)}$ and $\phi_j^{(l)}$ $(j=1,2;\ l=1,2,3.)$

$$\Big(\frac{\partial}{\partial z}+\frac{1}{v_{gj}}\frac{\partial}{\partial t}\Big)a_j^{(l)}-\frac{\beta_j}{2}\frac{\partial^2\phi_j^{(l)}}{\partial t^2}=m_j^{(l)}, \quad (8)$$

$$\Big(\frac{\partial}{\partial z}+\frac{1}{v_{gj}}\frac{\partial}{\partial t}\Big)\phi_j^{(l)}+\frac{\beta_j}{2}\frac{\partial^2a_j^{(l)}}{\partial t^2}+a_j^{(2)}\frac{\partial\phi_{jo}}{\partial z}-\gamma_j(3a_{jo}^2a_1^{(l)}+2a_{(3-j)o}^2a_j^{(2)}+4a_{(3-j)o}^2a_{(3-j)o}^{(2)})=n_j^{(l)}. \quad (9)$$

The explicit expressions of $m^{(j)}$ and $n^{(j)}$ $(j=1,2,\cdots)$ are omitted here. In the leading order $(l=1)$, Equations (8) and (10) admit the solution in a linear approximation. For the SHG we take the leading solution as a superposition of two components, i.e. $a_j^{(1)} = (\delta_{j1} + \delta_{j2} L_1) U_1 \exp(i\theta_1) + (\delta_{j1} + \delta_{j2} L_2) U_2 \exp(i\theta_2) + \text{c.c.}$, $\phi_j^{(1)} = iL_{j1} U_1 \exp(i\theta_1) + iL_{j2} U_2 \exp(i\theta_2) + \text{c.c.}$, where $L_j = [(k_j - \omega_j/v_{g1})^2 - (\beta_1^2 \omega_j^4/4 + \gamma_1 \beta_1 a_{1o}^2 \omega_j^2)]/(2\beta_1 \gamma_1 a_{2o}^2 \omega_j^2)$, $L_{1j} = -2(k_j - \omega_j/v_{g1})/(\beta_1 \omega_j^2)$, and $L_{2j} = -2(k_j - \omega_j/v_{g2})/(\beta_2 \omega_j^2)$ $(j = 1, 2)$. U_1 and U_2 are respectively the envelope functions of the fundamental wave (with the phase $\theta_1 = k_1 z - \omega_1 t$) and the second-harmonic wave (with the phase $\theta_2 = k_2 z - \omega_2 t$); k_1, k_2, ω_1 and ω_2 are chosen according to the SHG phase-matching condition (6), i.e., $k_1 = k_+(\omega_1)$ and $k_2 = k_-(\omega_2)$ with $\omega_2 = 2\omega_1$.

In the next order ($l = 2$), solvability conditions give closed equations for U_1 and U_2. After making the transformation $\varepsilon U_1 = u_1, \varepsilon U_2 = u_2$, the equations governing the envelopes of the fundamental and second harmonic waves read

$$\partial u_1/\partial z + (1/V_{g1}) \partial u_1/\partial t + i\lambda_1 u_1^* u_2 \exp(-i\Delta kz) = 0, \tag{10}$$

$$\partial u_2/\partial z + (1/V_{g2}) \partial u_2/\partial t + i\lambda_2 u_1^2 \exp(-i\Delta kz) = 0, \tag{11}$$

where V_{gj} is the group velocity of the jth excited wave, $\Delta k = 2k_1 - k_2$ is a possible phase mismatch. The nonlinear coefficients appearing in Eqs. (10) and (11) read

$$\begin{aligned}
\lambda_1 = \{ & L_1 \beta_1^2 \gamma_1 a_{2o}^2 \omega_1^4 [-k_2 L_{12} - k_1 L_{11} \\
& + (\omega_1 L_{11} + \omega_2 L_{12})/v_{g1} + \beta_1 \omega_1 \omega_2 L_{12} L_{11} \\
& - 6\gamma_1 a_{1o}^2 - 4a_{20}^2 (L_1 + L_2) - 4a_{2o}^2 L_{22} L_{21}] \\
& + \beta_1 \beta_2 \gamma_1 a_{20}^2 \omega_1^4/4 [-k_2 L_{22} L_2^2 - k_1 L_1^2 L_{21} \\
& + (\omega_1 L_1 L_2 L_{21} + \omega_2 L_1 L_2 L_{22})/v_{g2} \\
& - \beta_2 \omega_1 \omega_2 L_1 L_2 L_{21} L_{22} - 6\gamma_2 a_{2o}^2 L_{21} L_{22} \\
& - 4a_{20}^2 L_{22} + L_{21}) 4a_{1o}^2] + 2\beta_1 \gamma_1 a_{2o}^2 \omega_1^2 \\
& \cdot (k_1 - \omega_1/v_{g1}) [\beta_1 \omega_1 \omega_2 (L_{11} - L_{22}) \\
& - \beta_1 \omega_1^2 L_{11}/2 + \beta_1 \omega_2^2 L_{12}/2] \\
& + 2\beta_1 \gamma_1 a_{2o}^2 \omega_1^2 (k_1 - \omega_1/v_{g2}) [\beta_2 \omega_1 \omega \\
& - 2L_1 L_{21} L_{22} - \beta_2 \omega_1 \omega_2 L_{22} L_1 L_2 - \beta_2 L_1 L_2 \\
& \cdot (\omega_1^2 L_{21} + L_{22})/2] \}/\{2\beta_1 \gamma_1 a_{20}^2 \omega_1^2 \\
& \cdot [(k_1 - \omega_1/v_{g2}) - 4L_1 (k_1 - \omega_1/v_{g1})] \},
\end{aligned} \tag{12}$$

$$\begin{aligned}
\lambda_2 = \{ & L_2 \beta_2^2 \gamma_2 a_{1o}^2 \omega_2^4 (-k_1 L_{21} L_1^2 + \omega_1 L_{21} L_1^2/v_{g2} \\
& - \beta_2 \omega_1^2 L_{21}^2 L_1^2/2 - 3\gamma_2 a_{2o}^2 - 4a_{1o}^2 L_1 - 2a_{10}^2) \\
& - 2\beta_2 \gamma_2 a_{10}^2 \omega_2^2 [-\beta_1 \omega_2^2/2(-k_1 d_{11} + \omega_1 L_{11}/v_{g1} \\
& - \beta_1 \omega_1^2 L_{11}^2/2 - 3\gamma_1 a_{10}^2 - 4a_{20}^2 L_1 - 2a_{20}^2 L_1^2) \\
& - (k_2 - \omega_2/v_{g1})(3\beta_1 \omega_1^2 L_{11}/2)] - (k_1 - \omega_1/v_{g2}) \\
& - 3(k_1 - \omega_1/v_{g2})(\beta_2^2 \gamma_2 a_{1o}^2 \omega_1^2 \omega_2^2 L_1 L_2 L_{21})\} \\
& /\{2\beta_2 \gamma_2 a_{10}^2 \omega_2^2 [(k_2 - \omega_2/v_{g1}) \\
& - 4L_2 (k_2 - \omega_2/v_{g2})]\}.
\end{aligned} \tag{13}$$

We now consider the solution of Eqs. (10) and (11) corresponding to the SHG. For a stationary case ($\partial/\partial t = 0$), Eqs. (10) and (11) can be solved exactly.[1] Let $u_1 = f \exp(i\varphi_f)$ and $u_2 = h \exp(i\varphi_h)$ with f and h being two real functions, Eqs. (10) and (11) become $\partial f/\partial z = -\lambda_1 fh \sin\theta$, $\partial h/\partial z = \lambda_2 f^2 \sin\theta$, $(\partial \varphi_f/\partial z) f = \lambda_1 fh \cos\theta$ and $(\partial \varphi_h/\partial z) h = \lambda_2 f^2 \cos\theta$ with the relative phase angle defined by $\theta = \varphi_h - 2\varphi_f + \Delta kz$. One of the conservative quantities for these equations reads $f^2/\lambda_1 + h^2/\lambda_2 = m$, where m is the integration constant. We can see λ_1 and λ_2 determine the rate of energy transfer between fundamental wave and the second harmonic wave. Another conservative quantity is given by $\Gamma_h = \Delta kh^2/2 + \lambda_2 f^2 h \cos\theta$. With these relations we obtain

$$\begin{aligned}
\int_{z_1}^{z} dz = \frac{1}{2} \int_{H(z_1)}^{H(z)} & [\lambda_2^2 (m - \frac{\lambda_1}{\lambda_2} h^2)^2 h^2 \\
& - (\Gamma_h - \Delta kh^2/2)^2]^{-1/2} dh^2,
\end{aligned} \tag{14}$$

where $H(z) \equiv h^2(z)$. The integral equation (14) gives the general solution $H(z)$ at the distance z for arbitrary inputs $F(z_1)$ $(F(z) \equiv f^2(z))$, $H(z_1)$ at z_1. If the power of the initial second-harmonic wave is zero, i.e. $H(z_1) = 0$ and hence leading to $\Gamma_h = 0$, the integral (14) is simplified as $\int_{z_1}^{z} dz = (1/2) \int_0^{H(z)} dh^2 [\lambda_2^2 (F(z_1) - \lambda_1 h^2/\lambda_2)^2 h^2 - (\Delta kh^2/2)^2]^{-1/2}$, where $F(z_1) = m$ is the initial power of the fundamental wave. When λ_1 and λ_2 have the same sign, the general expression for the magnitude of the second harmonic wave reads

$$H(z) = \lambda_2/\lambda_1 F(z_1) B_s^2 \, sn^2 [(\lambda_1 \lambda_2 F(z_1) A_s^2 z^2)^{1/2}, \gamma_s], \tag{15}$$

where γ_s is the modulus of the elliptic function sn, given by $\gamma_s = A_{s-}^2/A_{s+}^2$ with $A_{s\pm}^2 = [(2 + \sigma) \pm [(2 + \sigma)^2 - 4]^{1/2}]/2$, where $\sigma = (\Delta k/2)^2/(F(z_1)\lambda_1 \lambda_2)$ is responsible for the properties of the fibre. The result for the energy conversion efficiency of the second-harmonic wave, $\eta_H = H(z)/F(z_1)$, and the fundamental wave, $\eta_F = F(z)/F(z_1)$, have been plotted in

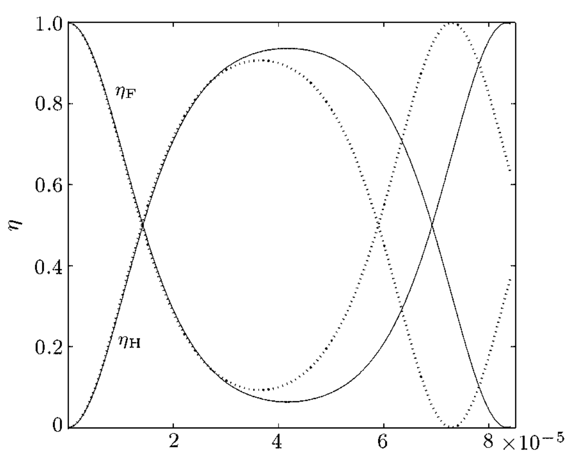

Fig. 2. The energy conversion efficiency between the fundamental and second-harmonic waves with the parameters $\beta_1 = \beta_2 = 0.06 \times \text{ps}^2/\text{m}$, $\gamma_1 = \gamma_2 = 0.015\,\text{W}^{-1}/\text{m}$, $v_{g1} = v_{g2} = 2 \times 10^8\,\text{ms}^{-1}$, $a_{1o}^2 = 2\,\text{kW}$, $a_{2o}^2 = 4.9\,\text{kW}$, and phase mismatch $\Delta k = 5 \times 10^3\,\text{m}^{-1}$ (solid lines). The dotted cures show the effect of increasing phase mismatch $\Delta k = 9 \times 10^3\,\text{m}^{-1}$.

Fig. 2, from which we can see there is a periodic energy conversion between two wave modes. The bold curves in the figure show the generation of η_H with phase mismatch $\Delta k = 5\times10^3\,\mathrm{m}^{-1}$ from initial value of $H(0) = 0$ and $F(0) = 10\,\mathrm{W}$. The dashed curves show the effect of the increment of $\Delta k = 9\times10^3\,\mathrm{m}^{-1}$. It is clear the energy conversion efficiency decreases with increasing phase mismatch Δk.

In the ideal case of a perfect phase matching $\Delta k = 0$, Eq. (15) become $H = (\lambda_2/\lambda_1)F(z_1)\tanh^2[(\lambda_1\lambda_2F(z_1)z^2)^{1/2}]$, just as seen in Fig. 3. In this case there is no back-conversion, and the maximal conversion efficiency η_H can approach 97.5%.

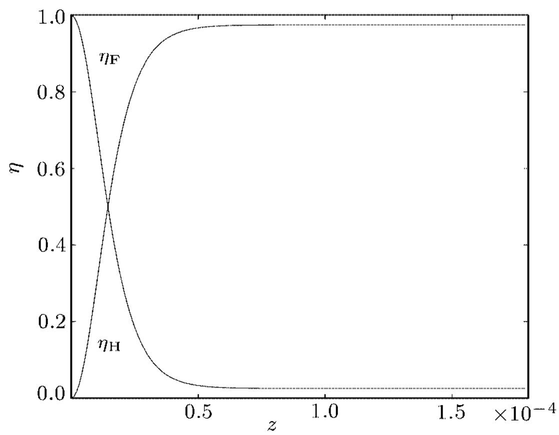

Fig. 3. In the ideal case of perfect phase matching $\Delta k = 0$, the conversion efficiency approaches the maximum with the parameters given as in Fig. 2.

For very short-pulse excitations the walk-off effect due to different group velocity velocities between the fundamental and the second-harmonic waves must be taken into account. The group velocity mismatch in the SHG will reduce the conversion efficiency. Considering a travelling-wave solution, i.e., taking u_j ($j = 1,2$) as a function of z and $\eta = t - z/V_{1g}$ and under phase-matching condition $\Delta k = 0$, the coupled amplitude Eqs. (10) and (11) are transferred as

$$\frac{\partial u_1}{\partial z} + i\lambda_1 u_1^* u_2 = 0, \tag{16}$$

$$\frac{\partial u_2}{\partial z} + \nu\frac{\partial u_2}{\partial \eta} + i\lambda_2 u_1^2 = 0, \tag{17}$$

with $\nu = 1/V_{g2} - 1/V_{g1}$. The walk-off parameter ν indicates the separation between the two pulses. If at $z = 0$, the fundamental wave and the second-harmonic wave take the form $u_1(t) = A_0/(1 + t^2/\tau_0^2)$ and $u_2(0,t) = 0$, where τ_0 is the initial pulse width and A_0 is a constant representing the initial amplitude of the fundamental wave, the solutions of Eqs. (16) and (17) have the form[1]

$$\phi_1 = \sqrt{\frac{1}{\lambda_1\lambda_2}}\frac{A}{(1+\widetilde{\eta})^{1/2}[1+(\widetilde{\eta}-\widetilde{z})^2]} \cdot\Big\{\cosh\xi + \frac{\widetilde{\eta}}{f}\sinh\xi\Big\}, \tag{18}$$

$$\phi_2 = -\frac{A\tau_{cr}}{\tau\lambda_1[1+(\widetilde{\eta}-\widetilde{z})^2]} \cdot\Big\{\frac{\widetilde{z}\cosh\xi + [f-\widetilde{\eta}(\widetilde{\eta}-\widetilde{z}/f)]\sinh\xi}{\cosh\xi + (\widetilde{\eta}/f)\sinh\xi}\Big\}, \tag{19}$$

with $\widetilde{\eta} = \eta/\tau$, $\widetilde{z} = z/l_\nu$, $\tau_{cr} = \nu/A$, $f = (\tau^2/\tau_{cr}^2 - 1)^{1/2}$, $\xi = f[\tanh^{-1}\widetilde{\eta} - \tanh^{-1}(\widetilde{\eta}-\widetilde{z})]$, where $l_\nu = \tau/\nu$ is the propagating distance over which the overlapping fundamental and second-harmonic pulses of width τ are clearly separated. This situation corresponds to a quasistationary SHG.

In conclusion, we have predicted a new type of SHG in nonlinear optical fibres based on the XPM effect of two copropagating optical pulses. We have shown that since the linear dispersion curve of the excitation created from the cw background displays two branches, the phase-matching condition of the SHG can be fulfilled if the wave vectors and frequencies of fundamental and second harmonic waves are selected suitably from different branches. By means of a multi-scale method we have derived the nonlinearly coupled envelope equations for the SHG and presented their explicit solutions. The SHG proposed here is similar to that of the elementary excitation in a two-component Bose–Einstein condensate, which the interaction between different components of the condensates provides the possibility to fulfill the SHG phase-matching condition.[11] To observe experimentally the SHG predicted here one should note that an infinite extended cw background is practically not realizable, so one realistic way is to consider the excitations created on the background with a large but finite extend, as in the case of the observation for the dark solitons in optical fibres.[10]

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