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# Phase diffusion of Bose–Einstein condensation close to zero temperature

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## Abstract

The correlation function of the quantum fluctuations due to collective excitations is calculated and used to investigate the phase diffusion of a Bose–Einstein condensate close to zero temperature. It is shown that the phase diffusion time of the condensate is much longer than the result obtained by assuming that the correlation time of the quantum fluctuations is infinity. © 2002 Elsevier Science B.V. All rights reserved.

## 1. Introduction

The development of the technologies of laser trapping and evaporative cooling has yielded intriguing Bose–Einstein condensates (BECs) [1–3], a state of matter in which many atoms are in the same quantum mechanical state. The remarkable observations of gaseous BECs have opened up new avenues [4–6] of research into the physical properties and nature of Bose-condensed systems. The phase properties of a BEC are of particular interest because the phase of an order parameter, i.e., the macroscopic wave function of the condensate, reflects directly the coherent nature of the condensate.

For BEC created in experiment, one of the most important characters is that all the atoms in the condensate can be described by the wave function (i.e., the order parameter) with a single phase. Due to thermal and quantum fluctuations, however, the single phase of the condensate will become unpredictable beyond the phase diffusion time. After the realization of BECs, the phase diffusion of the condensate has been discussed intensively [7–20]. In particular, the role of quantum fluctuations on the phase diffusion process was investigated in the pioneering work by Lewenstein and You [9,10], and a far off-resonant light scattering experiment was proposed to detect the quantum diffusion. Recently, a Langevin equation

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was given by Graham [14,15] to discuss the phase diffusion due to quantum fluctuations and thermal fluctuations. The calculation of the time scale of the phase diffusion is a very important problem because the phase diffusion time determines when the phase of a BEC would be unpredictable. Recently, the phase correlation has been investigated experimentally by the JILA group [21]. It was found that there is no detectable diffusion of the phase on time scale 100 ms. The stable interference patterns shown in the experiment put forward a question [21] why the phase correlation is so robust despite the phase diffusion and complicated rearrangement dynamics of the two condensates.

In the present Letter, we address the question of the phase diffusion process of a condensate close to zero temperature. In general, the phase diffusion of the condensate can have either a thermal or a quantum origin. At extremely low temperature (in the experiment by the JILA group [21], the temperature is only $0.1T_c$, where $T_c$ denotes the critical temperature of the Bose gas), the thermal fluctuations can be omitted and hence the quantum fluctuations become dominant. We give therefore emphasis on the role of collective excitations due to quantum fluctuations in a phase diffusion process. Although the phase diffusion process due to quantum fluctuations has been investigated by several authors such as the recent researches in [14,15,20], the analysis of the time correlation of the quantum fluctuations is not given when the phase fluctuations are calculated. Obviously, the correct consideration of the time correlation of the quantum fluctuations would make more reliable prediction on the phase diffusion process. In particular, researches show that the phase diffusion time is much longer than the correlation time of the quantum fluctuations. In this case, our results show that the phase diffusion time calculated from the correlation function of the quantum fluctuations is much longer than that obtained in the previous theoretical researches [14,15,20].

The Letter is organized as follows. In Section 2, we investigate the phase fluctuations of the condensate due to quantum fluctuations. In Section 3, the phase diffusion time is calculated for the condensate close to zero temperature, where the effect of thermal fluctuations can be omitted. Section 4 contains a discussion and summary of our results.

## 2. Phase fluctuations of the condensate due to quantum fluctuations

For temperature below the critical temperature $T_c$, the condensate can be described very well by the following order parameter with a phase factor $\phi(t)$

$$\Phi(\mathbf{r}, t) = \Phi_0(\mathbf{r})e^{-i\phi(t)}, \tag{1}$$

where the phase of the condensate has the form

$$\phi(t) = \mu(N_0, T)t/\hbar, \tag{2}$$

and the time-independent real component $\Phi_0(\mathbf{r})$ is determined by the stationary Gross–Pitaevskii (GP) equation [4]:

$$\mu(N_0, T)\Phi_0(\mathbf{r})$$
$$= \left( -\frac{\hbar^2}{2m}\nabla^2 + V_{\text{ext}}(\mathbf{r}) + g\Phi_0^2(\mathbf{r}) \right)\Phi_0(\mathbf{r}), \tag{3}$$

where $V_{\text{ext}}(\mathbf{r})$ is an external harmonic potential, and $g = 4\pi\hbar^2 a_s/m$ is the coupling constant fixed by the $s$-wave scattering length $a_s$. The chemical potential $\mu(N_0, T)$ in the above equation is determined by the normalization condition for the density distribution $n_0(\mathbf{r})$ of the condensate. With a Thomas–Fermi approximation [4], one gets easily the following expression for the chemical potential:

$$\mu(N_0, T) = \frac{\hbar\omega_{\text{ho}}}{2}\left( \frac{15N_0 a_s}{a_{\text{ho}}} \right)^{2/5}, \tag{4}$$

where $\omega_{\text{ho}} = (\omega_x \omega_y \omega_z)^{1/3}$ is the geometric average of oscillator frequencies, and $a_{\text{ho}} = \sqrt{\hbar/m\omega_{\text{ho}}}$ is the harmonic oscillator length of the system. From Eqs. (2) and (4), we see that the particle number fluctuations of the condensate yield fluctuations in the chemical potential, and hence lead to the phase diffusion of the condensate.

Assuming the mean ground state occupation number is $\langle N_0 \rangle$, the average phase of the condensate is then given by

$$\phi(\langle N_0 \rangle, t) = \mu(\langle N_0 \rangle, T)t/\hbar. \tag{5}$$

The phase diffusion of the condensate can be described by considering the phase difference $\Delta\phi(t) = \phi(t) - \phi(\langle N_0 \rangle, t)$. From Eqs. (2) and (5), it is straightforward to obtain a differential equation on $\Delta\phi(t)$:

$$\frac{d\Delta\phi(t)}{dt} = X_{\text{qua}}(t) = \frac{\partial\mu(\langle N_0 \rangle, T)}{\partial\langle N_0 \rangle}\Delta N_0(t)/\hbar, \tag{6}$$

where $X_{\text{qua}}(t)$ is determined by collective excitations due to quantum fluctuations. In the above expression, $\Delta N_0(t)$ represents the fluctuations of the ground state occupation number around $\langle N_0 \rangle$. A similar equation was derived and used by Graham [14,15] to discuss the phase diffusion of the condensate. Note that $\Delta N_0(t)$ can be either negative or positive numbers. For $\Delta N_0(t) < 0$, there are collective excitations created so that the atoms would loss in the condensate. Similarly, $\Delta N_0(t) > 0$ means the annihilation of collective excitations, and the ground state occupation number would increase in this case. In addition, $\Delta N_0(t)$ should be time-dependent because it originates from quantum fluctuations. Eq. (6) is our starting point to discuss the phase diffusion of the condensate. Because it is obtained from Eqs. (2) and (5), rather than directly from a time-dependent GP equation, we anticipate that Eq. (6) is still correct for longer time where the time-dependent GP equation may be no longer valid [22].

From Eq. (6), the phase fluctuations of the condensate are given by

$$\left\langle \left( \Delta \phi(\tau) \right)^2 \right\rangle = \int_0^\tau \int_0^\tau \left\langle X_{\text{qua}}(\xi) X_{\text{qua}}(\xi') \right\rangle d\xi \, d\xi', \qquad (7)$$

where $\langle X_{\text{qua}}(\xi) X_{\text{qua}}(\xi') \rangle$ is the correlation function of the quantum fluctuations. When obtaining Eq. (7), we have assumed that $\langle (\Delta \phi)^2 \rangle = 0$ at time $t = 0$. The calculation of $\langle X_{\text{qua}}(\xi) X_{\text{qua}}(\xi') \rangle$ plays a crucial role in investigating the phase diffusion process close to zero temperature.

For the Bose gas trapped in a harmonic potential, it is convenient to use the following decomposition of the particle field operator

$$\hat{\psi}(\vec{r}) = \Phi(\vec{r}) + \sum_i \left( u_i(\vec{r}) \alpha_i + v_i^*(\vec{r}) \alpha_i^\dagger \right), \qquad (8)$$

where $\Phi(\vec{r}) = \langle \hat{\psi}(\vec{r}) \rangle$ is the well-known order parameter, and the index $i$ labels the elementary excitations of the system. For the collective excitations discussed here, the energy of the collective mode indexed by $nl$ is given by the dispersion law [23]

$$\varepsilon_{\text{nl}} = \hbar \omega_{\text{ho}} \left( 2n^2 + 2nl + 3n + l \right)^{1/2}. \qquad (9)$$

As shown in [24,25], the contributions to condensate fluctuations due to quantum fluctuations are dominated by these phonon-type collective excitations. For the collective mode $nl$, one obtains the following leading behaviour for $u_{\text{nl}}(\vec{r})$ and $v_{\text{nl}}(\vec{r})$ [26]:

$$u_{\text{nl}}(\vec{r}) \simeq -v_{\text{nl}}(\vec{r}) \simeq \sqrt{\frac{g n_0(\vec{r})}{2\varepsilon_{\text{nl}}}} \, \chi_{\text{nl}}(\vec{r}), \qquad (10)$$

where $\chi_{\text{nl}}(\vec{r})$ is the velocity potential associated with the collective mode, and satisfies the condition

$$\int d\vec{r} \, \chi_{\text{nl}}^*(\vec{r}) \chi_{ij}(\vec{r}) \, d^3\vec{r} = \delta_{\text{nl},ij}.$$

In addition, the average occupation number of the atoms corresponding to the collective mode indexed by $nl$ is given by [27,28]

$$\langle N_{\text{nl}} \rangle = \left( u_{\text{nl}}^2 + v_{\text{nl}}^2 \right) f_{\text{nl}}, \qquad (11)$$

where $f_{\text{nl}} = [\exp(\varepsilon_{\text{nl}}/k_B T) - 1]^{-1}$. When a collective excitation with index $nl$ is created from the condensate due to quantum fluctuations, its energy $\varepsilon_{\text{nl}}$ originates from the energy fluctuations $\Delta E$ of the condensate. Under this consideration, a time-energy uncertainty relation can be used to calculate the longevity $\tau_{\text{nl}}$ of the collective mode $nl$. The longevity $\tau_{\text{nl}}$ of the collective mode $nl$ is therefore approximated as $1/\omega_{\text{ho}}(2n^2 + 2nl + 3n + l)^{1/2}$. For JILA experiment [21], this means that the longevity of the collective mode is smaller than 10 ms, which is obviously much smaller than the phase diffusion time.

When all collective modes are considered, $X_{\text{qua}}(t)$ can be written as:

$$X_{\text{qua}}(t) = \frac{\partial \mu(\langle N_0 \rangle, T)}{\partial \langle N_0 \rangle} \sum_{\text{nl} \neq 0} \Delta N_{\text{nl}}(t)/\hbar, \qquad (12)$$

where $\Delta N_{\text{nl}}(t)$ reflects the changes of the ground state occupation number due to the creation and annihilation of the collective mode $nl$. Therefore, the magnitude of $\Delta N_{\text{nl}}(t)$ can be regarded as $\langle N_{\text{nl}} \rangle$. Note that $\Delta N_{\text{nl}}(t)$ itself can be either positive or negative, and varies with time due to quantum fluctuations. In the case of $\Delta N_{\text{nl}}(t) < 0$, there are $|\Delta N_{\text{nl}}(t)|$ atoms created from the condensate due to quantum fluctuations, while $\Delta N_{\text{nl}}(t) > 0$ represents the annihilation of $\Delta N_{\text{nl}}(t)$ atoms.

From Eq. (12), $\langle X_{\text{qua}}(\xi) X_{\text{qua}}(\xi')\rangle$ can be written as:

$$\langle X_{\text{qua}}(\xi) X_{\text{qua}}(\xi')\rangle$$
$$= \frac{1}{\hbar^2}\big[\partial\mu(\langle N_0\rangle, T)/\partial\langle N_0\rangle\big]^2$$
$$\times \sum_{\text{nl}\neq 0} \sum_{\text{n'l'}\neq 0} \langle \Delta N_{\text{nl}}(\xi) \Delta N_{\text{n'l'}}(\xi')\rangle. \qquad (13)$$

Assuming that there is no correlation between different collective modes, i.e., $\langle \Delta N_{\text{nl}}(\xi) \Delta N_{\text{n'l'}}(\xi')\rangle = 0$ when $nl \neq n'l'$, one gets the following expression for the correlation function:

$$\langle X_{\text{qua}}(\xi) X_{\text{qua}}(\xi')\rangle = \frac{1}{\hbar^2}\big[\partial\mu(\langle N_0\rangle, T)/\partial\langle N_0\rangle\big]^2$$
$$\times \sum_{\text{nl}\neq 0} \langle N_{\text{nl}}\rangle^2 e^{-|\xi-\xi'|/\tau_{\text{nl}}}. \qquad (14)$$

When obtaining the above result, we have used the following relation

$$\langle \Delta N_{\text{nl}}(\xi) \Delta N_{\text{nl}}(\xi')\rangle = \langle N_{\text{nl}}\rangle^2 e^{-|\xi-\xi'|/\tau_{\text{nl}}}. \qquad (15)$$

In the above expression, $\tau_{\text{nl}}$ is the longevity of the collective excitation $nl$. When $|\xi - \xi'|$ is much larger than $\tau_{\text{nl}}$, the correlation between the collective excitations at times $\xi$ and $\xi'$ can be omitted. Therefore, $\tau_{\text{nl}}$ can be approximated as the correlation time of the correlation function $\langle \Delta N_{\text{nl}}(\xi) \Delta N_{\text{nl}}(\xi')\rangle$.

Because the contributions to the quantum fluctuations come mainly from the low-lying collective excitations, as a reasonable approximation, the correlation function $\langle X_{\text{qua}}(\xi) X_{\text{qua}}(\xi')\rangle$ can be approximated as an exponential form:

$$\langle X_{\text{qua}}(\xi) X_{\text{qua}}(\xi')\rangle = Q_{\text{qua}} e^{-|\xi-\xi'|/\tau_{\text{qua}}}. \qquad (16)$$

When the above exponential form is used, $\tau_{\text{qua}}$ should be regarded as the average correlation time of the collective excitations, and is determined by the following expression:

$$\tau_{\text{qua}}^2 = \frac{\int_{-\infty}^{\infty} d\tau\, \tau^2 \langle X_{\text{qua}}(t) X_{\text{qua}}(t+\tau)\rangle}{\int_{-\infty}^{\infty} d\tau\, \langle X_{\text{qua}}(t) X_{\text{qua}}(t+\tau)\rangle}. \qquad (17)$$

In terms of Eqs. (14) and (17), the correlation time of the quantum fluctuations is given by $\tau_{\text{qua}} = \sqrt{2}/\omega_{\text{ho}}$. In addition, in Eq. (16), the magnitude $Q_{\text{qua}}$ of the correlation function is given by

$$Q_{\text{qua}} = \frac{1}{\hbar^2}\big[\partial\mu(\langle N_0\rangle, T)/\partial\langle N_0\rangle\big]^2 \langle \delta^2 N_{\text{qua}}\rangle. \qquad (18)$$

In the above expression, $\langle \delta^2 N_{\text{qua}}\rangle = \sum_{\text{nl}\neq 0}\langle N_{\text{nl}}\rangle^2$ can be taken as the particle number fluctuations [25] of the condensate due to the collective excitations. Using the formulas (9)–(11), after a straightforward (although rather complex) calculation, we obtain the result of $\langle \delta^2 N_{\text{qua}}\rangle$:

$$\langle \delta^2 N_{\text{qua}}\rangle = 0.958\left(\frac{a_s}{a_{\text{ho}}}\right)^{4/5}\left(\frac{T}{T_c}\right)^2 N^{22/15}$$
$$+ 14.174\left(\frac{a_s}{a_{\text{ho}}}\right)^{4/5} N^{12/15}, \qquad (19)$$

where $N$ is the total number of atoms in the trap. The second term on the right-hand side of the above equation represents the fluctuations due to the effect of the quantum depletion which is given in [24]. This term has a finite contribution to the condensate fluctuations when the temperature approaches zero. Thus, we anticipate that there is still phase diffusion in the case of zero temperature.

We now turn to discussing the phase diffusion of the condensate due to quantum fluctuations. From Eqs. (7) and (16), the phase fluctuations of the condensate, which play a crucial role in discussing the phase diffusion, read

$$\big\langle (\Delta\phi(\tau))^2\big\rangle = 2Q_{\text{qua}}\tau_{\text{qua}}\big(\tau - \tau_{\text{qua}} + \tau_{\text{qua}} e^{-\tau/\tau_{\text{qua}}}\big). \qquad (20)$$

The phase diffusion time $\tau_{\text{phase}}$ can be obtained by setting $\langle(\Delta\phi(\tau))^2\rangle = \pi^2$ in the above expression.

## 3. Phase diffusion time of the condensate close to zero temperature

We now turn to discussing the phase diffusion time using the phase fluctuations given by Eq. (20). It is useful to discuss the phase fluctuations given by Eq. (20) for two special cases. When the time $\tau$ is much larger than the time scale of the correlation time $\tau_{\text{qua}}$, the phase fluctuations of the condensate can be approximated as:

$$\big\langle (\Delta\phi(\tau))^2\big\rangle \approx 2Q_{\text{qua}}\tau_{\text{qua}}\tau. \qquad (21)$$

Therefore, if the phase diffusion time $\tau_{\text{phase}}$ calculated from Eq. (20) is much larger than $\tau_{\text{qua}}$, the phase diffusion time in this situation takes the following

analytical form:

$$\tau_{\text{phase}} = \frac{\pi^2}{2Q_{\text{qua}}\tau_{\text{qua}}}. \tag{22}$$

In the case of $\tau \ll \tau_{\text{qua}}$, however, the phase fluctuations of the condensate can be approximated as:

$$\left\langle \left(\Delta\phi(\tau)\right)^2 \right\rangle \approx Q_{\text{qua}}\tau^2. \tag{23}$$

Different from the result given by Eq. (21), the phase fluctuations are proportional to $\tau^2$ when $\tau$ is much smaller than the correlation time $\tau_{\text{qua}}$. Therefore, if $\tau_{\text{phase}}$ calculated from Eq. (20) is much smaller than $\tau_{\text{qua}}$, the analytical result of the phase diffusion time $\tau'_{\text{phase}}$ is then

$$\tau'_{\text{phase}} = \frac{\pi\hbar}{\delta N_{\text{qua}}\partial\mu(\langle N_0\rangle, T)/\partial\langle N_0\rangle}, \tag{24}$$

where $\delta N_{\text{qua}} = \sqrt{\langle \delta^2 N_{\text{qua}}\rangle}$.

We now turn to discuss the phase correlation experiment by the JILA group [21]. The experimental values in the experiment are: $N_0 = 5 \times 10^5$, $T \approx 50$ nk, $T_c \approx 500$ nk, and $a_s \approx 5 \times 10^{-7}$ cm. It may be helpful to make a comparison between the particle number fluctuations due to quantum fluctuations and thermal fluctuations. For temperature much lower than the critical temperature, the analytical result $\langle \delta^2 N_{\text{th}}\rangle = \pi^2 N(T/T_c)^3/6\zeta(3)$ [25] can give a rather well description for the particle number fluctuations due to thermal fluctuations. For the values typical for the experiment by the JILA group [21], a simple calculation shows that $\langle \delta^2 N_{\text{qua}}\rangle/\langle \delta^2 N_{\text{th}}\rangle = 62.6$. Therefore, the thermal fluctuations can be safely omitted when the phase diffusion process is investigated for the experiment by JILA group [21]. Using the formula (20) (or Eq. (22)), the numerical result of the phase diffusion time $\tau_{\text{phase}}$ is 119 s, which is much larger than the correlation time $\tau_{\text{qua}}$. When obtaining this result, we have used the exponential form of the correlation function given by Eq. (16). In fact, we can obtain $\tau_{\text{phase}}$ directly from Eqs. (7) and (14), and it is worth pointing out that there is no important correction to the phase diffusion time, in comparison with the result obtained by using the exponential form (16). The merit of the exponential form (16) is that it clearly shows the role of particle number fluctuations on the phase diffusion process, and the analytical result of the phase fluctuations is rather concise using this exponential form.

If the correlation time of the collective excitations is assumed to be infinity, however, using Eqs. (19) and (24), the numerical result of $\tau'_{\text{phase}}$ is 0.62 s, which is much smaller than the result given by Eq. (22). Although the phase fluctuations due to quantum fluctuations are investigated in deep in Ref. [20], the finiteness of the correlation time of the quantum fluctuations was not considered, and the dephasing time was approximated as 1 s. In addition, it is worth pointing out that although a Langevin equation was proposed by Graham [14,15] to investigate the phase fluctuations due to thermal fluctuations and quantum fluctuations, the phase fluctuations due to collective excitations were proportional to $\tau^2$, because the finiteness of the longevity of the collective excitations was not considered too.

For temperature close to zero, our result of the phase diffusion time given by Eq. (22) is reasonable because of two reasons:

(i) In the experiment by the JILA group [21], the phase of the condensate was found to be very robust. In fact, the rigidity of the phase was also shown in other experiments, such as the observation of the interference between two BECs [32], and the recent experiments where the optical lattice [33,34] is used to investigate the coherent properties of the BECs. For example, recently a BEC [34] is created with up to $2 \times 10^5$ atoms and no discernible thermal component. The radial trapping frequencies are relaxed over a period of 500 ms to 24 Hz such that the harmonic potential becomes spherically symmetric. Then three optical standing waves are aligned orthogonal to each other, in order to form a three-dimensional lattice potential. In this situation, the condensate is distributed over more than 150 000 lattice sites. When the magnetic trap and lattice potential are both switched off, it is interested to find that there is a high-contrast interference pattern, which means that phase is still robust after the BEC has been formed for nearly 1 s, and even after the interference between a large number of BECs.

(ii) In the present Letter, the correlation time of the quantum fluctuations is calculated and found to be much smaller than the time scale of the phase diffusion time. In this situation, we should regard the quantum fluctuations as a white noise to investigate the phase diffusion process. Recall that the particular collective excitations are rather stable when it is created in the experiment [29–31] by applying a small

time-dependent perturbation, it seems the longevity of the collective excitations is very long. However, we should note that in the problem discussed here for the mechanism of the phase diffusion, the collective excitations are created and annihilated through the quantum fluctuations. As pointed out in this Letter, the longevity of these collective excitations is found to be much smaller than the phase diffusion time.

We should note that when obtaining the phase diffusion time, the time-energy uncertainty relation is used to calculate approximately the longevity of the collective modes created due to quantum fluctuations. A more accurate average longevity of these collective modes can be obtained when interparticle interaction effect is included. Nevertheless, a reasonable order of magnitude on the phase diffusion time can be obtained, using Eq. (20) and the time-energy uncertainty relation. Additionally, the collective time $\tau_{qua}$ can be obtained from Eq. (20) when $\tau_{phase}$ is measured in experiment. This gives us a chance to check whether standard many-body theory can be used successfully to investigate the interested quantity $\tau_{qua}$.

## 4. Discussion and conclusion

In summary, the phase diffusion time of the condensate due to quantum fluctuations is discussed at extremely low temperature. Because the correlation time of the quantum fluctuations is much smaller than the time scale of the phase diffusion, the quantum fluctuations can be regarded as white noise when the phase diffusion of the condensate is investigated for extremely low temperature. In this situation, the phase diffusion time calculated here is much longer than the result obtained in the previous theoretical researches [14,15,20]. It is obvious that the present work cannot be applied directly to the experiment conducted by the JILA group [21], where the complicated rearrangement of the two condensates would be very important to the phase diffusion process. We shall extend our idea in a future work to the case of the specific situation realized by the JILA group [21] and discuss the phase diffusion at finite temperature. Recently, BECs have been realized in quasi-one and quasi-two dimensions [35], where new phenomena such as quasicondensates with a fluctuating phase [36–38] may be observed. A simple method is developed recently [25,

39–41] to discuss the particle number fluctuations of the low-dimensional condensate. This makes it possible to discuss the phase diffusion process in low-dimensional condensates.

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