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Gain-assisted superluminal optical solitons at very low light intensity

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We show the possibility of generating gain-assisted superluminal optical solitons in a room-temperature three-state active Raman gain medium. We demonstrate both analytically and numerically that under appropriate conditions a signal field of very low intensity can evolve into a stable shape-preserving wave form that propagates with superluminal velocity. Shape recovery of superluminal solitons after collision is also demonstrated numerically.

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Solitons are a class of shape-preserving wave propagation phenomena resulting from the interplay between nonlinearity and dispersion (or diffraction) that simultaneously exist during propagation. Solitons in the optical domain are of special interest because of their potential applications in information processing and transmission [1]. Up to now, most optical solitons were produced in passive optical media—e.g., glass-based optical fibers—and far-off resonance excitation schemes are generally employed in order to avoid unmanageable attenuation and distortion of optical field. Due to the lack of distinctive energy levels, however, nonlinear effects in such passive optical media are very weak, requiring very high optical power density and substantial propagation distance in order to establish their importance. Consequently, optical solitons produced in passive media generally travel with a speed very close to c, the speed of light in vacuum, and require an extended propagation distance to generate.

Recently, optical solitons with very slow propagation velocities in highly resonant absorptive media based on an electromagnetically induced transparency (EIT) effect have been studied [2]. In addition, ultraslow group-velocity-matched soliton pairs of different frequencies have also been shown [3]. However, there has been no study on the possible formation and stable propagation of optical solitons in active gain media.

Superluminal propagation (i.e., apparent group velocity exceeds c, or even becomes negative [4–6]) may occur in both absorptive and gain media. Chu and Wong [4] demonstrated experimentally a superlumianl propagation of optical wave packets through absorptive media with substantial attenuation. To obtain stable superlumianl propagations, Chiao [5] suggested to use gain media with an inverted atomic population. Steinberg and Chiao [6] showed that a stable superluminal propagation in the medium with a gain doublet is possible. The experimental works by Wang *et al.* [7], and more recently by Jiang *et al.* [8,9], as well as the theoretical works by Payne and Deng [10], further showed the intriguing aspects of gain media under near resonant excitations.

In this Rapid Communication, we investigate an active Raman gain (ARG) scheme for generating a new type of optical solitons that propagate with a superluminal group velocity. This resonant three-level ARG medium has been recently demonstrated to be able to produce both superluminal and ultraslow propagation of optical pulses [7–9] in the linear regime. We demonstrate both analytically and numerically that superluminal solitons are indeed possible in the ARG medium based on the balance of the dispersion and Kerr nonlinearity of the system. We note that all weakly driven EIT-based ultraslow solitons studied up to date require ultracold media because the Doppler broadening effect significantly degrades EIT processes under weakly driven conditions. This difficulty is removed in the ARG system because of the use of the large one-photon detuning (see below). To the best of our knowledge, our work represents the first study of superluminal optical soliton formation and propagation in a room-temperature resonant and active gain medium.

Consider a lifetime-broadened three-state atomic system (Fig. 1) that interacts with a strong, continuous-wave (cw) pump field $E_P(\omega_P)$ ($|1\rangle \leftrightarrow |2\rangle$) and a weak, pulsed (pulse length of τ_0 at the entrance of the medium) signal field $E_S(\omega_S)$ ($|2\rangle \leftrightarrow |3\rangle$). In the interaction picture, the atomic response of the system is described by

$$\left(i\frac{\partial}{\partial t}+d_2\right)A_2+\Omega_S^{*}A_3=0, \tag{1a}$$

$$\left(i\frac{\partial}{\partial t}+d_3\right)A_3+\Omega_P A_1+\Omega_S A_2=0, \tag{1b}$$

with $\Sigma_{j=1}^{3}|A_j|^2=1$, where A_j ($j=1, 2, 3$) is the probability amplitude of the bare atomic state $|j\rangle$ (with eigenenergy $\mathcal{E}_j$). $d_j=\Delta_j+i\gamma_j$ ($j=2,3$) with Δ_j and γ_j being laser frequency

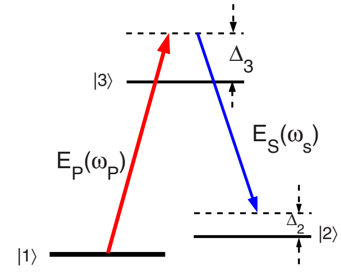

FIG. 1. (Color online) Energy-level diagram and laser excitation scheme of a three-state atomic system interacting with a strong, cw pump field of amplitude E_P and frequency ω_P and a weak, pulsed signal field of amplitude E_S and center frequency ω_S. Δ_3 and Δ_2 are one- and two-photon detunings, respectively.

detuning from the relevant transition and the decay rate of state $|j\rangle$, respectively. Here, one- and two-photon detunings are $\Delta_3=\omega_P-(\mathcal{E}_3-\mathcal{E}_1)/\hbar$ and $\Delta_2=\omega_P-\omega_S-(\mathcal{E}_2-\mathcal{E}_1)/\hbar$. In Eqs. (1), the half Rabi frequency of the pump (signal) field for the relevant atomic transition is defined by $\Omega_P=|\mathbf{p}_{31}|E_P/(2\hbar)$ $[\Omega_S=|\mathbf{p}_{32}|E_S/(2\hbar)]$ with $\mathbf{p}_{ij}$ being the electric-dipole matrix element of the $|i\rangle\leftrightarrow|j\rangle$ transition.

Under the slowly varying envelope approximation the equation for the signal field Rabi frequency is given by

$$i\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_S+\kappa A_3A_2^*=0, \tag{2}$$

where $\kappa=\mathcal{N}_a\omega_S|\mathbf{p}_{23}|^2/(2\epsilon_0 c\hbar)$ with $\mathcal{N}_a$ being the atom density. We assume that $|\Delta_3|$ is much larger than any Rabi frequencies, Doppler-broadened linewidths, atomic population and coherence decay rates, and frequency shift induced by the pump laser field.

Previous works [7–10] used a very weak signal field to avoid the nonlinear effect on the superluminal propagation. However, for a long propagating distance an increasing signal wave intensity due to the gain will bring about significant Kerr nonlinearity and thus depletion of the ground-state population, even lead to wave generation and a nonlinear phase shift [10,11]. Notice that for the large one-photon detuning Δ_3 the ground-state depletion is not large, and hence the increase of the signal field intensity is not too fast. In this situation the signal field displays only a weak nonlinear effect during propagation.

We are interested in possible soliton formation through the balance between the second-order dispersion and Kerr nonlinearity of the system. For this aim we solve Eqs. (1) and (2) by using the method of multiple scales [1]. We begin by making the asymptotic expansion $A_j=\sum_{n=0}^{\infty}\epsilon^n A_j^{(n)}$ ($j=1$, 2, 3) and $\Omega_S=\sum_{n=1}^{\infty}\epsilon^n\Omega_S^{(n)}$ with $A_1^{(0)}=1/\sqrt{1+|\Omega_P/d_3|^2}$, $A_2^{(0)}=0$, and $A_3^{(0)}=-\Omega_P/(d_3\sqrt{1+|\Omega_P/d_3|^2})$, where ϵ is a small parameter characterizing the amplitude of the signal field. It is straightforward to show that the leading-order solution gives $\Omega_S^{(1)}=F\exp\{i[k(\omega)z-\omega t]\}$ with F being a yet to be determined envelope function of slow variables $t_1=\epsilon t$, $z_1=\epsilon z$, and $z_2=\epsilon^2 z$ and $k(\omega)=\omega/c+\kappa|\Omega_P|^2/[(\omega-d_2^*)|d_3|^2]$ being the linear dispersion relation [10].

At second order, the condition of eliminating secular terms requires $\partial F/\partial z_1+(1/V_g)\partial F/\partial t_1=0$ with $V_g=K_1^{-1}$ with $K_1=1/c-\kappa|\Omega_P|^2/(d_2^{*2}|d_3|^2)$, which means that the envelope F propagates with group velocity V_g and hence $F=F(\tau_1,z_2)$ with $\tau_1=\epsilon(t-z/V_g)$.

To study soliton formation and propagation it is necessary to examine the asymptotic expansion to third order. The condition of eliminating secular terms in this order yields the nonlinear Schrödinger (NLS) equation for F after returning to original variables:

$$i\frac{\partial U}{\partial z}-\frac{K_2}{2}\frac{\partial^2 U}{\partial\tau^2}-We^{-\alpha z}U|U|^2=0, \tag{3}$$

where $U=\epsilon F$, $\tau=t-z/V_g$, $K_2=-2\kappa|\Omega_P|^2/(d_2^{*3}|d_3|^2)$ describes group velocity dispersion, $W=-\kappa|\Omega_P|^2[d_2d_3+(d_2d_3)^*]/[d_2^*|d_2|^2|d_3|^4]$ characterizes the self-phase modulation (SPM) effect of the signal field, and $-\alpha=2\kappa|\Omega_P|^2\gamma_2/(|d_2|^2|d_3|^2)$ is the gain coefficient of the signal field.

Equation (3) has complex coefficients and generally does not allow soliton solutions. However, as we show below, for the present system practical parameters can be found so that α may be made small and the imaginary parts of the complex coefficients are much smaller than the corresponding real parts. In this situation Eq. (3) can be written in the dimensionless form

$$i\frac{\partial u}{\partial s}+\frac{\partial^2 u}{\partial\sigma^2}+2u|u|^2=0, \tag{4}$$

where $s=z/(2L_D)$, $\sigma=\tau/\tau_0$, and $u=U/U_0$. $L_D=-\tau_0^2/\widetilde{K}_2$ is the characteristic dispersion length [we have set $L_D=L_{NL}$, with $L_{NL}=1/(U_0^2\widetilde{W})$ being nonlinear length], and $U_0=(1/\tau_0)[\widetilde{K}_2/\widetilde{W}]^{1/2}$ is the typical Rabi frequency of the signal field. Here the tildes over symbols denote the real part, indicating that we have neglected the small imaginary parts of the corresponding parameters. A bright soliton solution of Eq. (4) reads

$$u=2\beta\,\mathrm{sech}[2\beta(\sigma-\sigma_0+4\delta s)]e^{-2i\delta\sigma-4i(\delta^2-\beta^2)s-i\phi_0},$$

where β, δ, σ_0, and ϕ_0 are real parameters which determine the amplitude (as well as width), propagating velocity, initial position, and initial phase of the soliton, respectively. When taking $\beta=1/2$ and $\delta=\sigma_0=\phi_0=0$, we have $u=\mathrm{sech}\,\sigma\exp(is)$, or in terms of field,

$$\Omega_S=\frac{1}{\tau_0}\sqrt{\frac{\widetilde{K}_2}{\widetilde{W}}}\,\mathrm{sech}\left[\frac{1}{\tau_0}\left(t-\frac{z}{\widetilde{V}_g}\right)\right]e^{i\widetilde{K}_0z+iz/2L_D}, \tag{5}$$

which describes a bright fundamental soliton traveling with superluminal propagating velocity $\widetilde{V}_g$.

To demonstrate that the assumptions that lead to Eqs. (4) and (5) are indeed practical we consider a typical warm alkali-metal-atomatomic vapor. We choose $\gamma_2=1$ kHz, $\gamma_3=500$ MHz, $\kappa=1.0\times10^{10}$ cm^{-1} s^{-1}, $\Omega_P=4.0\times10^7$ s^{-1}, $\Delta_2=3.0\times10^6$ s^{-1}, $\Delta_3=-2.0\times10^9$ s^{-1}, and $\tau_0=1.0\times10^{-6}$ s. With these parameters we obtain $K_0=-(1.25+i0.42\times10^{-3})$ cm^{-1}, $K_1=-(41.81+i0.03)\times10^{-8}$ cm^{-1} s, $K_2=-(27.88+i0.03)\times10^{-14}$ cm^{-1} s^2,and $W=-(44.59+i0.01)\times10^{-17}$ cm^{-1} s^2. Clearly, for all complex coefficients the imaginary parts are indeed much smaller than their corresponding real parts. We obtain $\alpha=-0.8\times10^{-3}$ cm^{-1}, $L_D=L_{NL}=3.6$ cm, and $U_0=2.5\times10^7$ s^{-1}. The group velocity of the signal pulse is $\widetilde{V}_g=-8.0\times10^{-5}c$, indicating a superluminal propagation.

In Fig. 2(a), we have shown the result of numerical simulation on the soliton wave shape $|\Omega_S/U_0|^2$ versus dimensionless time t/τ_0 and distance $z/(2L_D)$ by taking Eq. (5) as an initial condition. In producing this plot we have kept the imaginary parts of all coefficients shown in Eq. (3). We see that the soliton preserves its shape after propagating a fairly long distance.

It is known that shape recovery after soliton collisions is an interesting yet important feature of solitons. This feature

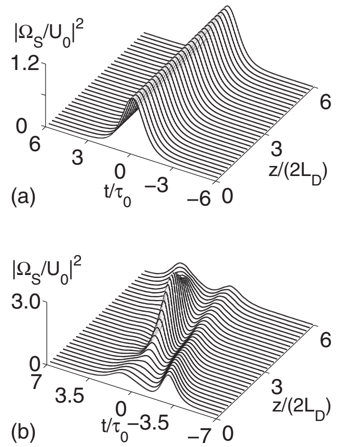

FIG. 2. (a) Soliton wave shape $|\Omega_S/U_0|^2$ versus t/τ_0 and $z/(2L_D)$ obtained by numerically solving Eq. (3) without neglecting the imaginary part of coefficients. (b) Soliton collision and shape recovery after the collision.

is demonstrated in our system in Fig. 2(b) where a collision of two bright superluminal solitons is shown. Here, one soliton is obtained by taking β=0.4, δ=0, and σ_0=1.0, whereas the other is obtained by taking β=0.6, δ=−0.3, and σ_0=−3.0. We see that both solitons have resumed their original shapes after the collision, indicating that superluminal optical solitons created in the ARG system are stable during the collision.

One of the important aspects of soliton formation is the threshold optical power density required to establish stable soliton propagation. In the present ARG system, this threshold is surprisingly low. To estimate the input power for generating the superluminal bright soliton described above we use Poynting's vector to calculate the average flux of the signal field energy over the duration of the signal pulse. We obtain

$$\bar{P} = \bar{P}_{\max}\,\mathrm{sech}^2[(t - z/\widetilde{V}_g)/\tau_0], \tag{6}$$

where $\bar{P}_{\max}=2\epsilon_0 c n_S S_0(\hbar/|\mathbf{p}_{23}|)^2\widetilde{K}_2/(\tau_0^2\widetilde{W})$ is the peak power with $n_S=1+c\widetilde{k}/\omega_S$ being the reflective index of the signal field and S_0 the cross-section area of the signal laser beam. Taking the signal beam radius $R_\perp$=0.01 cm and using the parameters given above, we obtain $\bar{P}_{\max}\approx 3$ μW. This prediction is very different from the conventional optical soliton generation scheme in optical fibers where ultrashort optical pulses of peak power of kW to MW are usually needed in order to bring about sufficient nonlinearity necessary for soliton formation.

We have also carried out numerical simulations by directly integrating Eqs. (1) and (2) with the parameters specified above to confirm the analytical prediction given above. Shown in Fig. 3(a) is the signal wave intensity $|\Omega_S/U_0|^2$ after it propagates for z=6.0 cm by taking Eq. (5) as input condition. We see that, except for small radiations (ripples) appearing on its two wings due to high-order dispersion and high-order nonlinear effects that have not been included in Eq. (3), the superluminal optical soliton is rather stable during propagation as expected. In Fig. 3(b), we have shown the simulation result of the collision between two superluminal optical solitons with the same initial condition used in Fig. 2(b). We see that although the agreement with the result shown in Fig. 2 is not so good, the full model (1) and (2) supports still nearly shape-preserving soliton propagation.

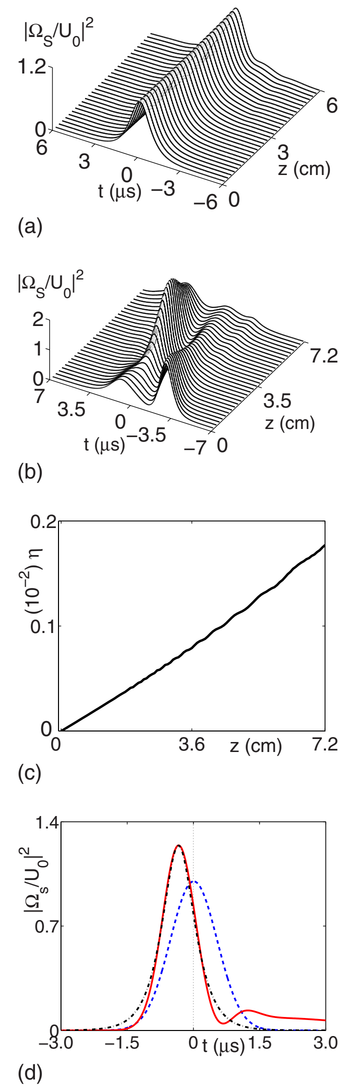

FIG. 3. (a) (Color online) Dimensionless signal wave intensity $|\Omega_S/U_0|^2$ obtained by numerically integrating Eqs. (1) and (2) using Eq. (5) as input. (b) Collision between two solitons. (c) Curve of $\eta(z)=[A(z)/W(z)]/[A(0)/W(0)]-1$, where $A(z)$ and $W(z)$ are the amplitude and width of the soliton, respectively. (d) $|\Omega_S/U_0|^2$ (solid line) at z=5.0 cm obtained by numerically integrating Eqs. (1) and (2) using a Gaussian input pulse. The central peak has a hyperbolic secant shape (dash-dotted line) and has an apparent advancement relative to that of the signal field propagating through vacuum (dashed line).

In order to estimate the rate of shape variation of the superluminal soliton during propagation, in Fig. 3(c) we have plotted the curve of $\eta(z)=[A(z)/W(z)]/[A(0)/W(0)]-1$ versus propagation length z, where $A(z)$ and $W(z)$ are the amplitude and the full width at half maximum of the soliton. We see that although $A(z)$ and $W(z)$ change with z due to the small gain of the system, the increase of $\eta(z)$ is only 0.2% after the soliton propagates to 7.2 cm. This also shows that the superluminal optical soliton in the ARG system can preserve its shape after propagating a fairly long distance.

To further demonstrate the formation and evolution of a superluminal bright soliton in an ARG system, we take a Gaussian initial signal field shape $\Omega_S(0,t) = U_0 \exp[-t^2/(2\tau_0^2)]$. This field shape has a full width at half maximum that is about 18% larger than the initial hyperbolic pulse shape used in the previous simulation. Figure 3(d) shows the evolution of this input signal field at $z=5.0$ cm using the same parameters as in Fig. 2. Clearly, the pulse shape of this initial signal field has changed significantly because the input signal field is not a soliton. Due to the Kerr effect, the signal pulse is narrowed and it arrives at a balance by the dispersion of the system. We notice that the central peak of the signal pulse can be fitted very well by a hyperbolic secant function (shown by the dash-dotted line) and has an apparent advancement relative to that of the signal field propagating through vacuum (shown by the dashed line), indicating that the main part of the signal field has evolved into a superluminal soliton. This simulation confirms the conclusion that a bright superluminal optical soliton can form by using Gaussian inputs in the ARG system.

It is worth pointing out some important differences between the solitons obtained in the present ARG scheme and in the Λ-type EIT scheme [2]. First, the EIT system is absorptive in nature, while the present system is a gain one in which the signal field operates in a stimulated Raman emission mode. Due to the gain, the signal wave grows rapidly and hence the ARG system has a stronger nonlinear effect than the EIT system. Second, although the envelope equations derived in the ARG system and the EIT system are of similar form, they have different physical origins. In fact, the dispersion near the transparency window of the EIT system has a different sign, therefore very different physical meaning, compared to the ARG system. Indeed, typically an EIT system cannot lead to a superluminal soliton. Third, the SPM coefficients for both cases are also different, reflecting the absorptive (gain) character of the EIT (ARG) system. Finally, the slow light solitons in the EIT system [2] need a ultracold condition, while the superluminal solitons in the present ARG system can be generated at room temperature.

In conclusion, we have shown that it is possible to generate superluminal optical solitons in the ARG medium at room temperature. By means of the method of multiple scales we have derived a modified NLS equation and demonstrated that under realistic physical conditions optical solitons with superluminal propagating velocity can form and propagate stably over extended distances. We stress that the superluminal optical solitons predicted here can be established with very low signal field intensity. This is in a startling contrast to the soliton in optical fibers where high peak power is required and the propagating velocity of the soliton is close to c.

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