Modern Physics Letters B, Vol. 19, Nos. 28 & 29 (2005) 1531–1534

LATTICE BOLTZMANN SIMULATION OF MOMENTUM AND ENERGY TRANSFER IN A POROUS MEDIUM

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Received 1 June 2005

The fluid flow and heat transfer in porous media has wide applications in oil industry and air conditioning system. In this study the momentum and energy transport processes in a porous medium are numerically simulated. The Brinkman-Forchheimer-extended Darcy equations and energy equation are solved using single lattice-Boltzmann method (LBM). As a first attempt, a benchmark problem is studied, i.e., momentum and energy transfer in a rectangular enclosure with differentially heated vertical walls. The effect of permeability on the critical Rayleigh numbers, the temperature distribution and flow field is discussed.

Keywords: Porous media; momentum and energy transfer; lattice Boltzmann method.

In recent years, as an efficient numerical tool lattice Boltzmann method (LBM) has been proposed to investigate the fluid flows in porous media.[1,2] The LBM allows a detailed discretization of the porous geometry and hence one can have an exact simulation of the flows without using any of semi-empirical homogenization models. In this study we present a LBM to simulate the momentum and energy transport processes

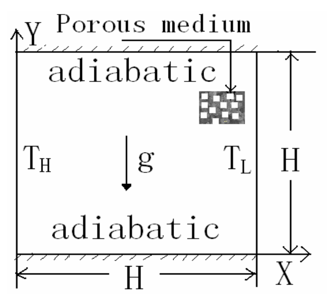

Fig. 1. Physical configuration and coordinate system.

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in porous media, in which that the Brinkman-Forchheimer-extended Darcy flow prevails, and that there exists local thermal equilibrium between the fluid and the solid. The physical configuration is shown in FIG. 1, where the geometry is essentially a two-dimensional square cavity filled with a porous material that is homogeneous and isotropic. Assuming the temperature T_H is higher than T_L, so the initial and boundary conditions can be expressed as:

$$t=0:\vec{v}=0,T=0;\quad x=0:\vec{v}=0,T=T_H;\quad x=H:\vec{v}=0,T=T_L;$$
$$y=0:\vec{v}=0,\partial T/\partial n=0;\quad y=H:\vec{v}=0,\partial T/\partial n=0.$$

For incompressible flows in a porous medium, the macroscopic governing equations can be expressed by the Brinkman-Forchheimer-extended Darcy equations (generalized momentum equation) together with the energy equation:[3]

$$\nabla\cdot\vec{v}=0, \tag{1}$$

$$\frac{\partial\vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\left(\frac{\vec{v}}{\phi}\right)=-\frac{1}{\rho}\nabla(\phi p)+\nu_e\nabla^2\vec{v}=\frac{\phi\nu}{K}\vec{v}-\frac{\phi F_\phi}{\sqrt{K}}|\vec{v}|\vec{v}-\beta g(T-T_L), \tag{2}$$

$$\sigma\frac{\partial T}{\partial t}+\vec{v}\cdot\nabla T=\nabla\cdot(\alpha_e\nabla T). \tag{3}$$

Here ρ is the fluid density, ϕ is porosity, $\vec{v}$, p, and T are volume-averaged velocity, pressure, and temperature, respectively, υ_e is an effective viscosity, σ is the ratio of heat capacities of the stagnant medium and the fluid, α_e is the thermal diffusivity, and υ is the shear viscosity of the fluid, which is generally different from υ_e, F_Φ and K are respectively the geometric function and permeability, which are estimated using Ergun's experimental investigation,[4] and is expressed by Vafai as following [5]

$$F_\phi=\frac{1.75}{\sqrt{150\phi^3}},\qquad K=\frac{\phi^3 d_p^2}{150(1-\phi)^2}, \tag{4}$$

With ϕ the porosity, and d_p the solid particle diameter. Equations (1)-(4) are well established for characterization of the fluid flows in a porous medium.

Considering the drag effects of the medium, the LBM equation can be presented as :

$$f_i(\vec{x}+\vec{e}_i\delta_t,t+\delta_t)=f_i(\vec{x},t)-[f_i(\vec{x},t)-f_i^{eq}(\vec{x},t)]/\tau+F_i\delta_t, \tag{5}$$

where f_i and f_i^{eq} are the volume-averaged distribution function and equilibrium distribution function at the representative elementary volume scale, respectively. As in Ref. [1], F_i is the force term for ith particle of fluid and can be written as :

$$F_i=\omega_i\rho(1-1/2\tau)[(\vec{e}_i\cdot\vec{F})/c^2+(\vec{e}_i\cdot\vec{v})(\vec{e}_i\cdot\vec{F})/\phi c^4-(\vec{v}\cdot\vec{F})/\phi c^2], \tag{6}$$

Where $\vec{F}=-(\phi\nu/K)\vec{v}-(\phi F_\phi/\sqrt{K})|\vec{v}|\vec{v}-\beta g(T-T_L)$. The density and velocity are defined as

$$\rho = \sum_i f_{i,} \qquad \vec{v} = \sum_i f_i \vec{e}_i / \rho + 0.5\delta_t \vec{F}, \tag{7}$$

For temperature distribution, there is an extra computational substep:

$$T_i(\vec{x} + \vec{e}_i\delta_t, t + \delta t) = T_i(\vec{x}, t) - [T_i(\vec{x}, t) - T^{eq}{}_i(\vec{x}, t)] / \tau_T, \tag{8}$$

where T_i is the temperature distribution function, τ_T is relaxation time and determined by thermal diffusivity, $T^{eq}{}_i$ is the equilibrium temperature distribution function and here we give it as

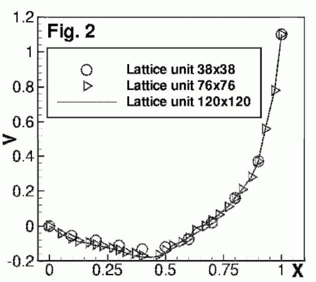
$$T_i^{eq} = \omega_i T(1 + \vec{e}_i \cdot \vec{v} / c^2), \tag{9}$$

where ω_i is the same as those used in Eq. (6). The temperature T can be defined as

$$\sigma = \sum_i T_i \tag{10}$$

D2Q9 model is selected.[6] Obviously, the governing equations (1)-(3) can be recovered from the Eq. (5) and Eq. (8) by the Chapman-Enskog procedure.

The boundary condition is treated by "Bouncing-back" scheme[7]. The following dimensionless parameters are introduced: the viscosity ratio $J=\nu_e/\nu$, the Darcy number $Da=K/H^2$, the Prandtl number $Pr=\nu/\alpha$, the Raleigh number $Ra=\beta g\ \delta TH^3/\nu\alpha$, the Reynolds number $Re=HV/\nu$, the Grsshof number $Gr=\beta g\ \nabla TH^3/\nu$, and Richardson number $Ri=Gr/Re^2$. The thermal properties of the fluid and solid are corresponding to those of water and copper which are associated with porous metal heat exchanger. The parameters of 305 K are: ρ_s=8930 kg m^{-3}, c_s=385 J kg^{-1}K$_{-1}$, k_s=400 W m^{-1} K^{-1}, while ρ_f=997kgm^{-3}, c_f=4197 J kg^{-1}K^{-1}, k_f=0.613 W m^{-1}K^{-1}, ν_f=855×10^{-6}N s m $^{-2}$, β=0.0033K^{-1}. A 120×120 lattice is used to simulate the momentum and energy transport process. This lattice mesh is determined by using different lattices from coarse to progressively finer lattices, until the numerical results are lattice-convergent to within a prescribed tolerance (~1%). Fig. 2 shows the comparison of vertical velocity at at $y=H/2$ for $Da=0.2$ and Gr=10^4 and $Ri=1.0$ with different lattice numbers. The lattice convergence is quite good. Fig. 3a and 3b show the temperature contours and streamlines in cavity at Ra=2.4×10^9. The isotherms are dense near the vertical walls, and thin thermal boundary layers are observed as the lower portion of the heating wall and upper portion of the cooling wall, indicating a strong heat flux there. However, the bulk of the enclosure is thermally stratified, due to a stagnant flow region there. From Fig. 3b, the recirculation is strong near the heating and cooling walls, however the flow in the bulk of the cavity is stagnant.



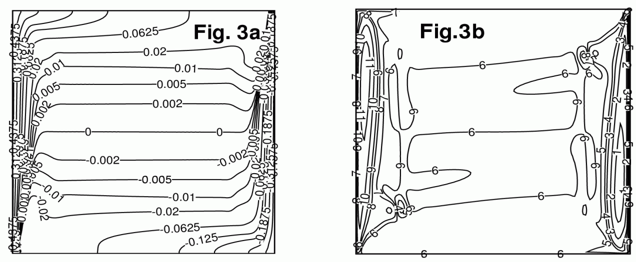


Fig. 2. Comparison of vertical velocity at y = H/2 of Da = 0.2, Gr = 10^4 and Ri = 1.0. Fig. 3. Contours of Ra = 2.4 x 10^9: (a) isotherm, (b) streamlines.

Finally, the effect of permeability on critical Rayleigh numbers are investigated and the results are tabulated in Table 1. The critical Rayleigh number increases with increasing permeability, indicating that higher permeability can stabilize the flow in a porous medium.

The momentum and energy transfer in a porous medium has been solved using LBM method. The force term in generalized momentum equation is embedded into the LBM equation. The results indicate that LBM method is capable of solving porous media problem.

Table 1. The analysis of convective instability.

Instability coefficients	Permeability $0.1\times10^{-8}m^2$	Permeability $0.1\times10^{-8}m^2$	Permeability $0.1\times10^{-8}m^2$
Critical Raleigh numbers	66.103	196.307	306.011

Acknowledgments

Supports given by the Research Grants Council of the Government of the HKSAR under Grant No. PolyU5172/02, by the National Natural Science Foundation of China under Grant No. 10572130, and by the Natural Science Foundation of Zhejiang Province under Grant No. M103082 are gratefully acknowledged.

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