Dromion Excitations in Self-Defocusing Optical Media *

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(Received 4 November 2002)

We investigate the nonlinear dynamics of multi-dimensional optical pulses propagating in an isotropic self-defocusing medium. Using a method of multiple-scales we show that the nonlinear evolution of the pulses is governed by Davey–Stewartson equations. Dromion-like nonlinear localized structures (high-dimensional optical solitons) excited from a continuous wave background and decaying in all spatial directions are predicted through the interaction between a wavepacket superposed by short-wavelength components and a long-wavelength mean field generated by an optical rectification.

PACS: 42. 65. Tg, 42. 81. Dp, 05. 45. Yv

In recent years, optical solitons have been the subject of intensive theoretical and experimental studies. Both temporal optical soliton (localized nonlinear optical pulse in time) and spatial optical soliton (bounded nonlinear optical beam in space) are believed to be ideal information carriers for optical communications of next generation.[1] Much attention has also been paid to high-dimensional optical solitons, which are much richer and provide us with many chances (with also challenges) for revealing new high-dimensional nonlinear optical phenomena.[2] On the other hand, some high-dimensional soliton models have been found recently in soliton theory, which admit interesting multi-dimensional soliton solutions. One of them is the Davey–Stewartson (DS) model first obtained in nonlinear hydrodynamics,[3] plasma physics,[4] and then in nonlinear lattice dynamics,[5] which under some conditions is completely integrable and admit remarkable *dromion* solutions (i.e., two-dimensional (2D) solitons decaying exponentially in all spatial directions).[6]

In this Letter, we investigate the nonlinear dynamics of high-dimensional optical pulses propagating in an isotropic self-defocusing optical medium. Although studies on high-dimensional nonlinear excitations exist, including the findings of dark optical solitons and vortices,[7] it seems that nobody has been aware of the possibility of optical dromions in such a system. Here we show, for the first time to the best of our knowledge, that in self-defocusing media, an optical dromion is possible and can be created from a continuum wave (cw) background of an electric field.

Let us consider the propagation of a monochromatic electric field E in a nonlinear self-defocusing optical medium with the intensity-dependent refractive index $n = n_0 + n_2|E|^2$ [$n_2(<0)$ is the Kerr coefficient]. When looking for the solutions of Maxwell's equations in the form of a slowly varying envelope of a carrier wave with propagation constant β_0 and frequency ω, one can obtain the 2D nonlinear Schrödinger (NLS) equation[7]

$$2\mathrm{i}\beta_0\frac{\partial \mathcal{E}}{\partial T} + \frac{\partial^2 \mathcal{E}}{\partial x^2} + \frac{\partial^2 \mathcal{E}}{\partial y^2} + \beta_0^2\frac{n_2}{n_0}|\mathcal{E}|^2\mathcal{E} = 0, \qquad (1)$$

where $\mathcal{E}(x,y,T)$ is the complex slowly varying envelope of the electric field, T is the evolution time (or a longitudinal coordinate, or a propagation coordinate), x and y are the two transverse coordinates. Generally speaking, a group-velocity dispersion term can be included in Eq. (1), but here we assume that the diffraction of the system is dominant and hence the effect of the group-velocity can be neglected. Using the transformation $T \to \beta_0 t$, $\mathcal{E}(x,y,T) \to [2n_0/(|n_2|\beta_0^2|)]^{1/2}\psi(x,y,t)\exp(-\mathrm{i}t)$, one can reduce Eq. (1) to the normalized form

$$2\mathrm{i}\frac{\partial \psi}{\partial t} + \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} - 2(|\psi|^2 - 1)\psi = 0. \qquad (2)$$

It is well known that for self-defocusing media, a cw solution described by Eq. (2) is modulationally stable. We are interested in the nonlinear dynamics of localized waves propagating on the stable cw background. In this situation the slowly varying field amplitude ψ has nonzero asymptotics, i.e., $|\psi| \to 1$ for $x, y \to \pm\infty$. Taking $\psi = Q\mathrm{e}^{\mathrm{i}R}$, where Q and R are two real functions, Eq. (2) becomes

$$\frac{\partial Q}{\partial t} + \frac{Q}{2}\Big(\frac{\partial^2 R}{\partial x^2} + \frac{\partial^2 R}{\partial y^2}\Big) + \Big(\frac{\partial Q}{\partial x}\frac{\partial R}{\partial x} + \frac{\partial Q}{\partial y}\frac{\partial R}{\partial y}\Big) = 0, \qquad (3)$$

$$Q\frac{\partial R}{\partial t} - \frac{1}{2}\Big(\frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 Q}{\partial y^2}\Big) + \frac{Q}{2}\Big[\Big(\frac{\partial R}{\partial x}\Big)^2 + \Big(\frac{\partial R}{\partial y}\Big)^2\Big] + Q^3 - Q = 0. \qquad (4)$$

* Supported by the National Natural Science Foundation of China under Grant No 10274021 and the Trans-Century Training Programme Foundation for the Talents from the Ministry of Education of China.

To derive the nonlinear amplitude equations describing the dynamics of 2D optical pulses propagating on the cw background we make the asymptotic expansion $Q = 1 + \sum_{n=1}^{\infty} \varepsilon^n Q^{(n)}$ and $R = \sum_{n=1}^{\infty} \varepsilon^n R^{(n)}$, where ε is a small parameter characterizing the relative amplitude of the excitation. $Q^{(n)}$ and $R^{(n)}$ are the real functions of the multiple-scale variables $\theta = kx - \omega t$, $\xi = \varepsilon(x - C_g t)$, $\eta = \varepsilon y$ and $\tau = \varepsilon^2 t$. The constant C_g will be determined by a solvability condition. The constant term "1" in the expansion of Q represents the cw background described by the uniformly oscillating electric field $[2n_0/(|n_2|\beta_0^2)]^{1/2}\exp(-\mathrm{i}t)]$. With the above notation, Eqs. (3) and (4) are transformed into

$$\frac{\partial Q^{(n)}}{\partial t} - \frac{1}{2}\frac{\partial^2 R^{(n)}}{\partial x^2} = \alpha^{(n)}, \tag{5}$$

$$\frac{\partial R^{(n)}}{\partial t} - \frac{1}{2}\frac{\partial^2 Q^{(n)}}{\partial x^2} + 2Q^{(n)} = \beta^{(n)}. \tag{6}$$

The explicit expressions of $Q^{(n)}$ and $R^{(n)}$ ($n = 1, 2, 3\cdots$) are omitted here.

We concentrate on a weakly nonlinear excitation originated by the interaction between a long-wavelength mode and a (modulated) high-frequency mode. At the leading order ($n = 1$) one has the solution

$$Q^{(1)} = Q_{11}\exp(\mathrm{i}\theta) + \mathrm{c.c.}, \tag{7}$$

$$R^{(1)} = R_{10} + [R_{11}\exp(\mathrm{i}\theta) + \mathrm{c.c.}] \tag{8}$$

with the linear dispersion relation $\omega(k) = (k/2)\sqrt{k^2+4}$, where R_{10} is a real function representing the long-wavelength mode introduced necessarily for cancelling the secular terms appearing in higher-order approximations, k and ω are the wavenumber and frequency of the carrier wave [i.e., $\exp(\mathrm{i}\theta)$], respectively.

A solvability condition at the next order results in $C_g = \mathrm{d}\omega/\mathrm{d}k = (k^3+2k)/(2\omega)$, which is just the group velocity of the modulated carrier wave. One obtains the solution $Q^{(2)} = Q_{20} + [Q_{21}\exp(\mathrm{i}\theta) + Q_{22}\exp(2\mathrm{i}\theta) + \mathrm{c.c.}]$ and $R^{(2)} = R_{22}\exp(\mathrm{i}\theta) + \mathrm{c.c.}$ The absence of the secular terms in Eqs. (5) and (6) for $n = 2$ requires that $Q_{20} = (C_g/2)\partial R_{10}/\partial\xi - [3k^4/(4\omega^2)]|R_{11}|^2$ and $Q_{21} = -[k^2/(2\omega^2)]C_g\partial R_{10}/\partial\xi - (k/\omega)\partial R_{11}/\partial\xi$. Then we obtain $Q_{22} = [k^2(k^2+24)/(24\omega^2)]R_{11}^2$ and $R_{22} = -\mathrm{i}[(5k^2+12)/12]\omega R_{11}^2$. One can see that, through the self-interaction of the short waves, a long wave component proportional to $|R_{11}|^2$ appears in Q_{20}. Such an effect is due to an optical rectification generated from the cw background.

At the order $n = 3$, the solvability conditions of Eqs. (5) and (6) give rise to the closed equations for R_{10} and R_{11}:

$$\frac{k^4(k^2+3)}{4\omega^2}\frac{\partial^2 R_{10}}{\partial\xi^2} - \frac{\partial^2 R_{10}}{\partial\eta^2} = \frac{k^5(k^2+3)}{\omega^3}\frac{\partial|R_{11}|^2}{\partial\xi}, \tag{9}$$

$$\begin{aligned}&\mathrm{i}\frac{\partial R_{11}}{\partial\tau} + \frac{k^4(k^2+6)}{16\omega^3}\frac{\partial^2 R_{11}}{\partial\xi^2} + \frac{k^2+2}{4\omega}\frac{\partial^2 R_{11}}{\partial\eta^2}\\ &= -2\frac{k^4(-k^4+40k^2+96)}{32\omega^3}|R_{11}|^2 R_{11}\\ &\quad + \frac{k^3(k^2+3)}{2\omega^2}\frac{\partial R_{10}}{\partial\xi}R_{11}.\end{aligned} \tag{10}$$

Equations (9) and (10) are in the general form of the DS equations, which include the dispersion, diffraction and nonlinearity of the system. Obviously they can be reduced to the well-known 1D NLS equation if the excitation is independent of the variable η. From Eqs. (9) and (10) we can see that the long wave component R_{10} is nonlinearly coupled with the short wave component R_{11}. This is a type of long-wave-to-short-wave interaction, which is well known in nonlinear hydrodynamics.[8] The quadratic nonlinearity in Eqs. (9) and (10) comes from the mentioned fact that the excitation considered is created from a cw background, although the nonlinearity characterizing the self-defocusing material is cubic. The property of DS Eqs. (9) and (10) depends strongly on the value of their coefficients. Under some conditions they can be reduced to an integrable model and admit analytical 2D soliton solutions. In the following we look for this possibility by choosing suitable parameter values of the system.

Using the transformation

$$\partial R_{10}/\partial\xi = -\varepsilon^{-2}\{k\omega(k^2+6)/[2(k^2+3)^2]\}\,s,$$
$$R_{11} = \varepsilon^{-1}[k\omega/(k^2+3)]\sqrt{(k^2+6)/2}\,u$$

with

$$x' = [2\omega/\sqrt{k^2+3}](x - C_g t), \quad y' = k^2 y,$$
$$t' = \{k^4(k^2+6)/[4\omega(k^2+3)]\}t,$$

Eqs. (9) and (10) can be rewritten as

$$\frac{\partial^2 s}{\partial x'^2} - \frac{\partial^2 s}{\partial y'^2} + 4\frac{\partial|u|^2}{\partial x'^2} = 0, \tag{11}$$

$$\mathrm{i}\frac{\partial u}{\partial t'} + \frac{\partial^2 u}{\partial x'^2} + \frac{\partial^2 u}{\partial y'^2} + \tilde{\mu}|u|^2 u + \tilde{\nu} s u = 0, \tag{12}$$

with

$$\tilde{\mu} = (k^2+3)(k^2+2)/(k^2+6),$$
$$\tilde{\nu} = (-k^4+40k^2+96)/[32(k^2+3)].$$

For arbitrary wavenumber k, an exact 2D soliton solution localized in all spatial directions is not available yet. However, it is noted that for a small k, $\tilde{\mu} \approx 2$ and $\tilde{\nu} \approx 1$ and hence Eq. (12) takes the form

$$\mathrm{i}\frac{\partial u}{\partial t'} + \frac{\partial^2 u}{\partial x'^2} + \frac{\partial^2 u}{\partial y'^2} + 2|u|^2 u + su = 0. \tag{13}$$

Equations (11) and (13) are the standard DS I equations, which are completely integrable and can be solved exactly by the inverse scattering transform.[6] One of the remarkable properties of the DS I equations is that they allow 2D dromion solutions decaying exponentially in all spatial directions.

The single dromion solution of DS I Eqs. (11) and (13) reads[9]

$$u = \frac{G}{F}, \quad s = 4\frac{\partial^2}{\partial x'^2}\ln F, \tag{14}$$

$$F = 1 + \exp(\eta_1 + \eta_1^*) + \exp(\eta_2 + \eta_2^*) + \gamma\exp(\eta_1 + \eta_1^* + \eta_2 + \eta_2^*), \tag{15}$$

$$G = \rho\exp(\eta_1 + \eta_2), \tag{16}$$

with

$$\begin{aligned}
&\eta_1 = (k_r + \mathrm{i}k_i)x'' + (\Omega_r + \mathrm{i}\Omega_i)t',\\
&\eta_2 = (l_r + \mathrm{i}l_i)x'' + (\omega_r + \mathrm{i}\omega_i)t',\\
&\Omega_r = -2k_r k_i \omega_r = -2l_r l_i,\\
&\Omega_i + \omega_i = k_r^2 + l_r^2 - k_i^2 - l_i^2,\\
&\rho = |\rho|\exp(\mathrm{i}\varphi_\rho),\\
&|\rho| = 2\sqrt{2k_r k_r(\gamma - 1)},\\
&x'' = (y' + x')/2, \quad y'' = (y' - x')/2.
\end{aligned}$$

The constants $k_r, k_i, l_r, l_i, |\rho|, \varphi_\rho$ and $\gamma = \exp(2\varphi_\gamma)$ are the free real parameters with $k_r k_l > 0$, $\gamma (> 1)$ determines an amplitude, k_r represents the width of the pulse in the x'' direction and l_r that in the y''-direction; k_i and l_i are x''- and y''-components of the dromion velocity, respectively.

If taking $k_r = \sqrt{2}\mu$, $k_i = \sqrt{2}a$, $l_r = \sqrt{2}\lambda$, $l_i = \sqrt{2}p$ $(\lambda\mu \geq 0)$, $\Omega_i = 2(\mu^2 - a^2)$, $\omega_i = 2(\lambda^2 - p^2)$, $\Omega_r = -4a\mu$, and $\omega_r = -4\lambda p$, we can obtain

$$u = \frac{2\mu\exp(\mathrm{i}h)}{m\cosh f_1 + n\cosh f_2}, \tag{17}$$

$$\begin{aligned}
s = &\frac{4(m^2 + n^2)(\mu^2 + \lambda^2) - 8\mu^2}{(m\cosh f_1 + n\cosh f_2)^2}\\
&+ 8mn\big[(\mu^2 + \lambda^2)\cosh f_1\cosh f_2\\
&- (\mu^2 - \lambda^2)\sinh f_1\sinh f_2\big]\\
&\cdot (m\cosh f_1 + n\cosh f_2)^{-2},
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
m &= (\mu/[\lambda(\gamma - 1)])^{1/2},\\
n &= (\mu\gamma/[\lambda(\gamma - 1)])^{1/2},\\
h &= \sqrt{2}ax'' + 2(\mu^2 - a^2)t' + \sqrt{2}py'' + 2(\lambda^2 - p^2)t' + \varphi_\rho,
\end{aligned} \tag{19}$$

$$f_1 = \sqrt{2}\mu x'' - \sqrt{2}\lambda y'' - 4(a\mu - \lambda p)t', \tag{20}$$

$$f_2 = \sqrt{2}\mu x'' + \sqrt{2}\lambda y'' - 4(a\mu + \lambda p)t' + \varphi_\gamma. \tag{21}$$

Obviously, the expression of u in Eq. (17) denotes a localized envelope function decaying *exponentially* in all spatial directions, called the dromion.[6] The mean field component s consists of two interacting plane solitons with each plane soliton decaying in its travelling direction.

The explicit expression for the pulse propagating in the self-defocusing medium in the case of the dromion excitation has the form:

$$Q = 1 - \frac{2\mu k^4}{k^2 + 3}\sqrt{\frac{k^2 + 6}{2}}\frac{\sin(h + \theta)}{m\cosh(f_1) + n\cosh(f_2)}, \tag{22}$$

$$\begin{aligned}
R = &-\frac{k(k^2 + 6)}{4(k^2 + 3)^{3/2}}R_0 + \frac{4\mu\omega k^2}{k^2 + 3}\sqrt{\frac{k^2 + 6}{2}}\\
&\cdot\frac{\cos(h + \theta)}{m\cosh(f_1) + n\cosh(f_2)},
\end{aligned} \tag{23}$$

with

$$\begin{aligned}
R_0 = &\big[\mu\exp(\eta_1 + \eta_1^*)(1 + \gamma\exp(\eta_2 + \eta_2^*))\\
&+ \lambda\exp(\eta_2 + \eta_2^*)(1 + \gamma\exp(\eta_1 + \eta_1^*))\big]\\
&\cdot\big[1 + \exp(\eta_1 + \eta_1^*) + \exp(\eta_2 + \eta_2^*)\\
&+ \gamma\exp(\eta_1 + \eta_2 + \eta_1^* + \eta_2^*)\big]^{-1}.
\end{aligned} \tag{24}$$

The quantities h, f_1 and f_2 have been given by Eqs. (19)–(21). When returning to the original variables, one has

$$\begin{aligned}
&x'' = (1/\sqrt{2})[2\omega(x - C_g t)/\sqrt{(k^2 + 3)} + k^2 y],\\
&y'' = -(1/\sqrt{2})\{[2\omega(x - C_g t)]/\sqrt{(k^2 + 3)} - k^2 y\},\\
&t' = \{k^4(k^2 + 6)/[4\omega(k^2 + 3)]\}t.
\end{aligned}$$

From the above results we obtain the following conclusions:

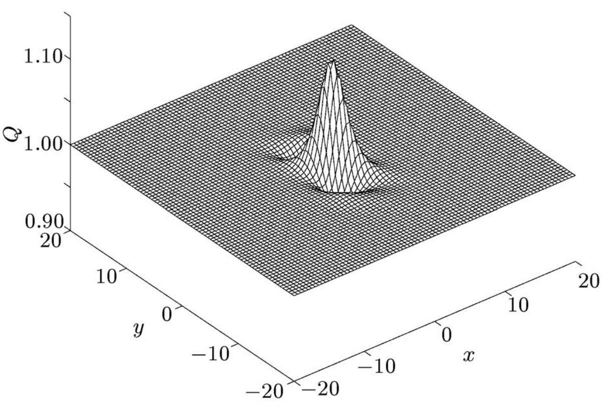

Fig. 1. Modulus Q of $\psi[= Q\exp(R)]$ in the case of a dromion excitation created from a cw background. The parameters are chosen as $\mu = 1.0$, $\lambda = 1.0$, $a = 1.0$, $p = 1.0$, $\varphi_\rho = 0$, $\varphi_\gamma = 1.0$, $k = 3/5$ at $t = 2.0$. The dromion decaying in all spatial directions is represented by the quantity $Q - 1$.

(i) In isotropic self-defocusing media a dromion-like nonlinear self-localized structure, i.e., a 2D optical soliton decaying exponentially in all spatial directions, can be created from a cw background. Figure

1 shows the configuration of the quantity $|Q|$ in the (x'', y'') coordinate system. At time t', the dromion locates at the position $(x'', y'') = (4at'/\sqrt{2} - \varphi_\gamma/[2\sqrt{2}\mu], 4pt'/\sqrt{2} - \varphi_\gamma/[2\sqrt{2}\lambda])$ and hence its velocity $\boldsymbol{V} = (4a/\sqrt{2}, 4p/\sqrt{2})$.

(ii) The phase of the envelope of the electric field, i.e., the quantity R, consists of two parts (see Eq. (23)). The first part is the term proportional to R_0, which represents two interacting kinks. It is easy to show that the kink of amplitude $4\lambda^2$ (λ-kink) travels with the velocity $\boldsymbol{V}_\lambda = (0, 4p/\sqrt{2})$. Another kink, i.e., the kink of amplitude $4\mu^2$ (μ-kink), travels with the velocity $\boldsymbol{V}_\mu = (4a/\sqrt{2}, 0)$. There is a cross-region (corresponding to an oblique collision between the kinks) where a new kink appears. If assuming that both a and p are positive with $a > p$, one can assign $x'' \to +\infty$ and $y'' \to +\infty$ as the region of "before collision," and $x'' \to -\infty$ and $y'' \to -\infty$ as the region of "after-collision." The position shifts due to the collision are given by $\Delta_\lambda = -\varphi_\gamma/(\sqrt{2}\lambda)$ (for λ-kink), $\Delta_\mu = -\varphi_\gamma/(\sqrt{2}\mu)$ (for μ-kink). Therefore, both the position shifts due to the collision are *negative.*

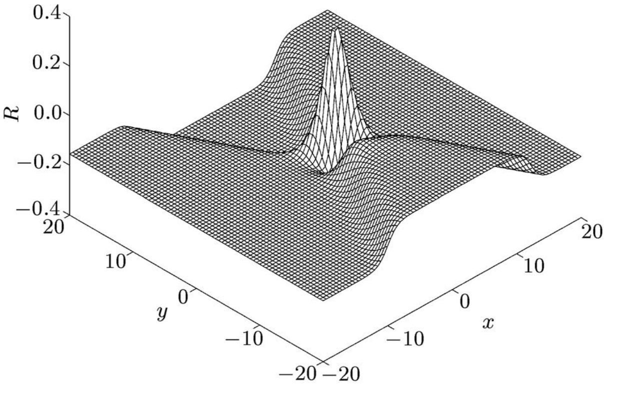

Fig. 2. Phase R of ψ in the case of a dromion excitation. The parameters are chosen as the same as Fig. 1. There are two parts for R. The first consists of two kinks travelling in different directions. The second is a dromion riding always on the cross-point of the kinks.

The second part of R is also a dromion decaying in all spatial directions. Figure 2 shows the configuration of R. We see that the dromion in R is just at the cross-point of mentioned two kinks and hence its velocity is the same as that of the cross-point. In fact, the dromion always *rides* at the cross-point of the kinks and is driven by them. This interesting feature can be seen in Fig. 2.

There is a common belief that dromion excitations exist only for optical materials with quadratic nonlinearity.[10,11] Here we have shown, for the first time, that the materials of cubic nonlinearity with negative Kerr coefficient can provide also with the possibility for producing optical dromions, but such excitations can be created only on a cw background.

In conclusion, we have investigated the nonlinear dynamics of 2D weak nonlinear optical pulses in an isotropic self-defocusing medium. Davey–Stewartson equations, which describe the interaction between a wavepacket superposed by short waves and a long-wavelength mean field generated through an optical rectification, have been obtained by means of a method of multiple scales. The results show that dromion-like nonlinear localized excitations decaying in all spatial directions are possible to generate from a cw background. Thus a self-defocusing medium is a realistic physical system for observing dromion-like high-dimensional optical solitons, which are promising for applications of all-optical switching and optical communications.[1,2]

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