# Comment on "Superluminal solitons in a Λ-type atomic system with two-folded levels"

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We show that the analysis of the optical solitons in a resonant, lifetime-broadened Λ-type atomic system with two-folded lower levels, given recently by D. Han *et al.* [J. Opt. Soc. Am. B **24**, 2244 (2007)], is incorrect. Although in the anomalous dispersion regime near resonance one may obtain a superluminal optical soliton, such a soliton suffers serious absorption and hence cannot propagate a significant distance. However, by choosing appropriate system parameters to work in the normal dispersion regime and hence inside the transparency windows of double dark resonance, an ultraslow optical soliton with very low light intensity can form and propagate stably for a fairly long distance. 

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In a recent work, Han *et al.* [1] considered nonlinear wave propagation in a Λ-type atomic system with two-folded lower levels driven by a coupling field. The authors claimed that a superluminal optical soliton (i.e., the propagating velocity of the soliton exceeds $c$ or even becomes negative) can propagate stably in the system. However, we find that their conclusion is incorrect. Because of the mistake of a sign in their Hamiltonian, the Maxwell–Schrödinger equations presented in their paper that describe the dynamics of probe field and atomic motion are not correct, which results in an incorrect linear dispersion relation and hence an incorrect group velocity of the probe field. In addition, by using their physical parameters, the soliton works in a far-off resonant regime and its propagating velocity is not superluminal but subluminal and very close to $c$. Actually, a superluminal optical soliton, even obtained, works outside the transparency windows of a double dark resonance and hence suffers a serious absorption. However, we shall show that the system may support an ultraslow optical soliton with very low light intensity if the parameters are chosen in normal dispersion regimes near resonance. In this case the system works inside the transparency windows and hence the optical soliton may propagate stably for a fairly long distance.

The system given in [1] consists of a resonant, lifetime-broadened Λ-type atomic system with energy levels $|1\rangle$, $|2\rangle$, and $|3\rangle$ and an additional lower level $|4\rangle$. A weak, pulsed probe field (with pulse duration $\tau_0$) of center frequency $\omega_p/(2\pi)$ is coupled to the $|1\rangle\to|2\rangle$ transition; a strong and continuous-wave pumping field of frequency $\omega_c/(2\pi)$ is coupled to the $|2\rangle\to|3\rangle$ transition; and a strong cw coupling field of frequency $\omega_d/(2\pi)$ is coupled to the two-folded lower levels $|3\rangle$ and $|4\rangle$, i.e., the $|3\rangle\to|4\rangle$ transition, respectively (see Fig. 1 of [1]).

The electric-field vector of the system can be written as $\mathbf{E}=\Sigma_{l=p,c,d}\mathbf{e}_l\mathcal{E}_l\exp[i(\mathbf{k}_l\cdot\mathbf{r}-\omega_l t)]+\text{c.c.}$, where $\mathbf{e}_l$ is the unit vector in the $l$th polarization direction and $k_l=\omega_l/c$ $(l=p,c,d)$. The Hamiltonian of the system has the form $\hat{H}=\hat{H}_0+\hat{H}'$, where $\hat{H}_0$ describes a free atom and $\hat{H}'$ describes the interaction between the atom and the optical field. In the Schrödinger picture, the state vector of the systems is $|\psi(t)\rangle_s=\Sigma_{j=1}^4 C_j(z,t)|j\rangle$, where $|j\rangle$ is the eigenstate of $\hat{H}_0$. Under electric-dipole and rotating-wave approximations, the Hamiltonian takes the form

$$\hat{H}=\sum_{j=1}^{4}\epsilon_j|j\rangle\langle j|=\hbar\{\Omega_1\exp[i(k_pz-w_pt)]|2\rangle\langle 1| + \Omega_2\exp[i(\mathbf{k}_c\cdot\mathbf{r}-\omega_ct)]|2\rangle\langle 3| + \Omega_3\exp[i(\mathbf{k}_d\cdot\mathbf{r}-\omega_dt)]|3\rangle\langle 4|+\text{H.c.}\}, \tag{1}$$

where $\epsilon_j$ is the energy of state $|j\rangle$, H.c. represents the Hermitian conjugate and $\Omega_1=(\mathbf{e}_p\cdot\mathbf{p}_{21})\mathcal{E}_p/\hbar$, $\Omega_2=(\mathbf{e}_c\cdot\mathbf{p}_{23})\mathcal{E}_c/\hbar$, and $\Omega_3=(\mathbf{e}_d\cdot\mathbf{p}_{34})\mathcal{E}_d/\hbar$ are the half Rabi frequencies for corresponding transitions, respectively. $\mathbf{p}_{ij}$ is the electric dipole matrix element associated with the transitions $|i\rangle\leftrightarrow|j\rangle$. The detunings are given by $\Delta_1=(\epsilon_2-\epsilon_1)/\hbar-\omega_p$, $\Delta_2=(\epsilon_2-\epsilon_3)/\hbar-w_c$, and $\Delta_3=(\epsilon_3-\epsilon_4)/\hbar-\omega_d$. For simplicity, we choose the wave vector direction of the probe field along the $z$ axis, i.e., $\mathbf{k}_p=k_p\mathbf{e}_z$.

To investigate the time evolution of the system, it is more convenient to employ an interaction picture, which is obtained by applying the operator $\hat{U}=\exp(i\hat{H}_0t/\hbar)$ to act on both $\hat{H}'$ and $|\Psi(t)\rangle_s$. Thus we can obtain the Hamiltonian and the state vector in the interaction picture, i.e., $\hat{H}_{\text{int}}=\hat{U}\hat{H}'\hat{U}^{-1}$ and $|\Psi(t)\rangle_{\text{int}}=\hat{U}|\Psi(t)\rangle_s$. A further simplification can be obtained by making the transformation $C_j=A_j\exp\{i[\mathbf{k}_j\cdot\mathbf{r}-(\epsilon_j/\hbar+\lambda_j)t]\}$, with $\mathbf{k}_1=0$, $\mathbf{k}_2=k_p\mathbf{e}_z$, $\mathbf{k}_3=k_p\mathbf{e}_z-\mathbf{k}_c$, $\mathbf{k}_4=k_p\mathbf{e}_z-\mathbf{k}_c-\mathbf{k}_d$, $\lambda_1=0$, $\lambda_2=-\Delta_1$, $\lambda_3=-\Delta_1+\Delta_2$, and $\lambda_4=-\Delta_1+\Delta_2+\Delta_3$. Then we obtain

 

$$\hat{H}_{\rm int} = \hbar[\Delta_1|2\rangle\langle 2| + (\Delta_1 - \Delta_2)|3\rangle\langle 3| + (\Delta_1 - \Delta_2 - \Delta_3)|4\rangle\langle 4|] - \hbar[\Omega_1|2\rangle\langle 1| + \Omega_2|2\rangle\langle 3| + \Omega_3|3\rangle\langle 4| + \text{H.c.}] \quad (2)$$

Using the Schrödinger equation, $i\hbar\,\partial|\Psi(t)\rangle_{\rm int}/\partial t = H_{\rm int}|\Psi(t)\rangle_{\rm int}$, one can readily obtain the equations of motion for atomic-state amplitudes $A_j$ ($j=1$ to 4). Under a slowly varying envelope approximation, Maxwell equation $\nabla^2\mathbf{E} - (1/c^2)\partial^2\mathbf{E}/\partial t^2 = [1/(\epsilon_0 c^2)]\partial^2\mathbf{P}/\partial t^2$ with

$$\mathbf{P} = \mathcal{N}_a\{\mathbf{p}_{21}A_2A_1^* \exp[i(k_p z - \omega_p t)] + \mathbf{p}_{23}A_2A_3^* \times\exp[i(\mathbf{k}_c\cdot\mathbf{r} - \omega_c t)] + \mathbf{p}_{34}A_3A_4^* \times\exp[i(\mathbf{k}_d\cdot\mathbf{r} - w_d t)] + \text{c.c.}\}$$

can be reduced to an equation for the probe field half Rabi frequency $\Omega_1$. Thus we have the following Maxwell–Schrödinger (MS) equations:

$$\left(i\frac{\partial}{\partial t} + d_2\right)A_2 + \Omega_1 A_1 + \Omega_2 A_3 = 0, \quad (3a)$$

$$\left(i\frac{\partial}{\partial t} + d_3\right)A_3 + \Omega_2^* A_2 + \Omega_3 A_4 = 0, \quad (3b)$$

$$\left(i\frac{\partial}{\partial t} + d_4\right)A_4 + \Omega_3^* A_3 = 0, \quad (3c)$$

$$i\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_1 + \kappa_{12}A_2A_1^* = 0, \quad (3d)$$

where $\Sigma_{j=1}^4|A_j|^2=1$, where $d_2=-\Delta_1+i\gamma_2$, $d_3=-\Delta_1+\Delta_2+i\gamma_3$, and $d_4=-\Delta_1+\Delta_2+\Delta_3+i\gamma_4$ with $\gamma_j$ ($j=2,3,4$) describing the decay rate of the energy level $|j\rangle$. $\kappa_{12}=\mathcal{N}_a\omega_p|\mathbf{p}_{12}|^2/(2\epsilon_0 c\hbar)$ is the coupling constant describing the interaction between the electric field and the atoms, with $\mathcal{N}_a$ being the atomic density and $\epsilon_0$ being the vacuum dielectric constant.

Before solving the MS Eqs. (3a)–(3d), we first examine the linear property of the system. We assume that atoms are initially populated in the state $|1\rangle$. Since in the linear regime the probe field is very weak, the ground state is not depleted during time evolution and hence $A_1\approx 1$. Taking $\Omega_1$ and $A_j$ ($j=2$ to 4) as being proportional to $\exp[i(K(\omega)z-\omega t)]$, one obtains the linear dispersion relation

$$K(\omega) = \frac{\omega}{c} - \kappa_{12}\frac{|\Omega_3|^2 - (\omega+d_3)(\omega+d_4)}{D(\omega)}, \quad (4)$$

where

$$D(\omega) = |\Omega_3|^2(\omega+d_2) + |\Omega_2|^2(\omega+d_4) - (\omega+d_2)(\omega+d_3)(\omega+d_4).$$

In most operation conditions $K(\omega)$ can be Taylor expanded around the center frequency of the probe field, i.e., $\omega=0$. We obtain $K(\omega)=K_0+K_1\omega+\frac{1}{2}K_2\omega^2+\cdots$, where the coefficients $K_j=[\partial^j K(\omega)/\partial\omega^j]|_{\omega=0}$ ($j=0,1,2,\ldots$), which can be obtained from Eq. (4) explicitly. $K_0=\phi+i\alpha/2$ gives the phase shift $\phi$ per unit length and absorption coefficient $\alpha$, $K_1=1/V_g$ determines the group velocity $V_g$ of the probe pulse, and $K_2$ represents the group-velocity dispersion that contributes to the probe pulse's shape change and an additional loss.

Shown in Fig. 1 are the dispersion (solid curve) and absorption (dashed curve) curves of the probe field $\Omega_1$, which are characterized by Re($K$) and Im($K$), respectively. The parameters are taken as $\gamma_2\simeq 1.0\times 10^7\ \text{s}^{-1}$, $\gamma_3\simeq 1.0\times 10^4\ \text{s}^{-1}$, $\gamma_4\simeq 1.0\times 10^2\ \text{s}^{-1}$, $\kappa_{12}=1.0\times 10^9\ \text{cm}^{-1}\,\text{s}^{-1}$, $\Delta_1=\Delta_2=\Delta_3=0$, $\Omega_2=3.6\times 10^7\ \text{s}^{-1}$, and $\Omega_3=1.8\times 10^7\ \text{s}^{-1}$. Two transparency windows (called double dark resonance) open due to the quantum interference effect induced by the pumping field $\Omega_2$ and the coupling field $\Omega_3$.

From the figure we see that the dispersion of the system can be divided into normal and anomalous dispersion regimes. In the normal dispersion regimes we have $V_g < c$ (subluminal propagation); in the anomalous dispersion regimes one has $V_g > c$ and even $V_g$ becomes negative (superluminal propagation). It is obvious that in the anomalous dispersion regimes the system always has a very large absorption, whereas in the normal dispersion regimes the absorption is reduced considerably due to the electromagnetically induced transparency (EIT) effect, i.e., the appearance of the EIT transparency windows. Consequently, the system is nearly transparent (opaque) if working in the normal (anomalous) dispersion regimes. This conclusion remains valid even when an envelope soliton forms when the weak nonlinear effect of the system is taken into account.

In [1], the authors mistook a sign in their Hamiltonian, and hence the MS equations derived by them are not correct, which results in an incorrect linear dispersion relation and an incorrect group-velocity expression of the probe field. Actually, the sign of the second term of their linear dispersion relation [i.e., Eq. (6) in [1]] should be minus.

Using the parameters given in [1], we obtain $\phi\simeq -9.9996\times 10^{-3}\ \text{rad cm}^{-1}$, $\beta\simeq 5.9998\times 10^{-5}\ \text{cm}^{-1}$, $K_1=(34.54+i0.01)\times 10^{-12}\ \text{cm}^{-1}\,\text{s}$, $K_2=(0.12+i12.47)$

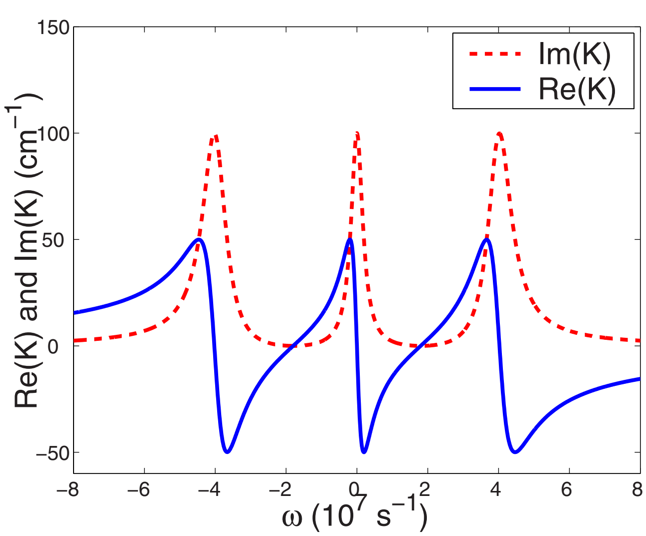


Fig. 1. (Color online) The dispersion curve Re($K$) (solid curve) and the absorption curve Im($K$) (dashed curve) of the probe field as functions of $\omega$. The parameters are given in the text.

$\times 10^{-20}\,\mathrm{cm^{-1}\,s^2}$, and $W=(11.11+i0.07)\times 10^{-24}\,\mathrm{cm^{-1}\,s^2}$. With these parameters, we obtain the following conclusions: (i) The group velocity of the probe field is $\tilde{V}_g \simeq 0.96c$, i.e., the propagation of the probe pulse is subluminal and it travels with a velocity close to but slower than $c$. (ii) It is impossible to generate a stable optical soliton in this case because the real part of the coefficient $K_2$ is much larger than its corresponding imaginary part. (iii) The center frequency of the probe field locates at $\omega \simeq -1.0\times 10^{11}\,\mathrm{s^{-1}}$, which is very far away from the resonant region of the system, thus not interesting for the active resonant system under study [2].

Although the system cannot support stable superluminal solitons as claimed in [1], it can, however, support ultraslow optical solitons in normal dispersion regimes near resonance. To demonstrate this, we derive a nonlinear envelope equation that describes the time evolution of the probe field envelope by employing the method of multiple scales [3]. For this aim we make the asymptotic expansion $A_j=\delta_{1j}+\sum_{n=1}^{\infty}\epsilon^n A_j^{(n)}(j=1,2,3,4)$ and $\Omega_1=\sum_{n=1}^{\infty}\epsilon^n\Omega_1^{(n)}$, where $\epsilon$ is a small parameter characterizing the small population depletion in the state $|1\rangle$. To obtain a divergence-free expansion, all quantities on the right-hand side of the asymptotic expansion are considered functions of the multiscale variables $z_l=\epsilon^l z$ ($l=0$ to 2) and $t_l=\epsilon^l t$ ($l=0$, 1). Substituting the expansion and the multiscale variables into Eqs. (3a)–(3d), we obtain a chain of linear but inhomogeneous equations for $A_j^{(n)}$ and $\Omega_1^{(n)}$, which can be solved order by order.

The leading order ($j=1$) solution is just that obtained in the linear regime. The expression of $\Omega_1$ has the form $F\exp(i\theta)$, here $\theta=K(\omega)z_0-\omega t_0$ with $F$ being a yet to be determined envelope function depending on the slow variables $t_1$ and $Z_j$ ($j=1,2$). At the second order ($j=2$), a divergence-free condition requires $i[\partial F/\partial z_1+(1/V_g)\partial F/\partial t_1]=0$, i.e., the envelope $F$ travels with the group velocity $V_g$. At the third order ($j=3$) a divergence-free condition yields the equation

$$i\frac{\partial F}{\partial z_2}-\frac{K_2}{2}\frac{\partial^2 F}{\partial t_1^2}-W\exp(-\alpha_1 z_2)F|F|^2=0, \tag{5}$$

with $\alpha_1=\epsilon^{-2}\alpha$ and

$$W=-\kappa_{12}(|\Omega_3|^2-d_3d_4)[||\Omega_3|^2-d_3d_4|^2 + |\Omega_2|^2(|d_4|^2+|\Omega_3|^2)]/(D|D|^2),$$

which characterizes the self-phase modulation effect of the system.

Combining the envelope equations obtained in the second and the third orders and returning to the original variables, we arrive at the following dimensionless equation:

$$i\frac{\partial u}{\partial s}+\frac{\partial^2 u}{\partial \sigma^2}+2u|u|^2=id_0u, \tag{6}$$

where $s=-z/(2L_D)$, $\sigma=(t-z/\tilde{V}_g)/\tau_0$, $u=\epsilon F/U_0e^{-i\alpha z/2}$, and $d_0=L_D/L_A$. Here $L_D=\tau_0^2/\tilde{K}_2$ is the characteristic dispersion length, $L_A=1/\alpha$ is the characteristic absorption length, and $U_0=(1/\tau_0)\sqrt{\tilde{K}_2/\tilde{W}}$ is the characteristic Rabi frequency of the probe field. The tildes above $V_g$, $K_2$, and $W$ denote their real part. To obtain Eq. (6), we have assumed that the real part of these coefficients is much larger than their corresponding imaginary part, which is realistic for the present system, as shown below.

If $d_0 \ll 1$ (i.e., $L_A \gg L_D$), the terms on the right-hand side of Eq. (6) are high-order ones and thus can be neglected within the propagation distance around $L_D$. In this case Eq. (6) reduces to the standard nonlinear Schrödinger (NLS) equation $i\partial u/\partial s+\partial^2 u/\partial\sigma^2+2u|u|^2=0$, which is completely integrable and allows multisoliton solutions. A single bright soliton solution reads $u=\mathrm{sech}\,\sigma\exp(is)$, or in terms of field

$$\Omega_1=Ue^{i\phi z}=\frac{1}{\tau_0}\sqrt{\frac{\tilde{K}_2}{\tilde{W}}}\mathrm{sech}\left[\frac{1}{\tau_0}\left(t-\frac{z}{\tilde{V}_g}\right)\right]\exp\left[i\phi z-i\frac{z}{2L_D}\right], \tag{7}$$

which describes a fundamental bright soliton traveling with velocity $\tilde{V}_g$.

Now we consider a practical atomic system that can be realized by a typical alkali atomic (such as $^{87}$Rb) vapor at ultracold low temperature. The parameters can be chosen as $\gamma_2\simeq 1.0\times 10^7\,\mathrm{s^{-1}}$, $\gamma_3\simeq 1.0\times 10^4\,\mathrm{s^{-1}}$, and $\gamma_4\simeq 1.0\times 10^2\,\mathrm{s^{-1}}$. We take $\kappa_{12}=1.0\times 10^9\,\mathrm{cm^{-1}\,s^{-1}}$ ($\mathcal{N}_a=5.4\times 10^{10}\,\mathrm{cm^{-3}}$), $\Omega_2=3.6\times 10^7\,\mathrm{s^{-1}}$, $\Omega_3=1.8\times 10^7\,\mathrm{s^{-1}}$, $\Delta_1=\Delta_2=1.0\times 10^9\,\mathrm{s^{-1}}$, $\Delta_3=1.0\times 10^8\,\mathrm{s^{-1}}$, and $\lambda_p=c/\nu_p=0.8\times 10^4\,\mathrm{cm}$. With these parameters we get $K_0=(1.67-i0.03)\,\mathrm{cm^{-1}}$, $K_1=(3.56+i0.15)\times 10^{-7}\,\mathrm{cm^{-1}\,s}$, $K_2=(3.77+i0.21)\times 10^{-13}\,\mathrm{cm^{-1}\,s^2}$, and $W=(5.90+i0.11)\times 10^{-16}\,\mathrm{cm^{-1}\,s^2}$. We see that the imaginary parts of these coefficients are indeed much smaller than their corresponding real part. If $\tau_0=1.0\times 10^{-6}\,\mathrm{s}$, we obtain $L_D=2.65\,\mathrm{cm}$, $L_A=32\,\mathrm{cm}$, and $U_0=2.5\times 10^7\,\mathrm{s^{-1}}$.

Using the above parameters we see that at the distance of the forming soliton (i.e., $L_A=2.65\,\mathrm{cm}$) the absorption of the probe field can be safely neglected because $L_A \gg L_D$. In this case the system can be described reasonably by the standard NLS equation, and hence the probe field Rabi frequency takes the soliton form (7). With the above parameters we obtain $\tilde{V}_g=9.3\times 10^{-5}c$, which means that the soliton presented above is subluminal and propagates with an ultraslow velocity.

The input power of the ultraslow optical soliton in the normal dispersion regimes described by Eq. (7) can be easily calculated by Poynting's vector [4]. By a simple calculation we obtain the average flux of energy over carrier-wave period $\bar{P}=\bar{P}_{\max}\,\mathrm{sech}^2[(t-z/\tilde{V}_g)/\tau_0]$, with the peak power $\bar{P}_{\max}=2\mathcal{E}_0cn_pS_0|\mathcal{E}_p|^2_{\max}=2\mathcal{E}_0cn_pS_0(\hbar/|\mathbf{p}_{13}|)^2\tilde{K}_2/(\tau_0^2\tilde{W})$. Here $n_p$ is the refractive index and $S_0$ is the cross-sectional area of the probe field. Using the above parameters and $S_0\approx 0.01\,\mathrm{cm^2}$, we obtain $\bar{P}_{\max}=8.3\times 10^{-2}\,\mathrm{mW}$. Thus to produce such an ultraslow soliton only very low input light intensity is needed. This sharply contrasts with nonresonant media, such as optical fibers, where picosecond or femtosecond laser pulses are required to reach a high peak power for the formation of a soliton.

We now discuss the stability of the ultraslow optical soliton by using numerical simulations. In Fig. 2(a), we have plotted the wave shape of $|\Omega_1/U_0|^2$ as a function of

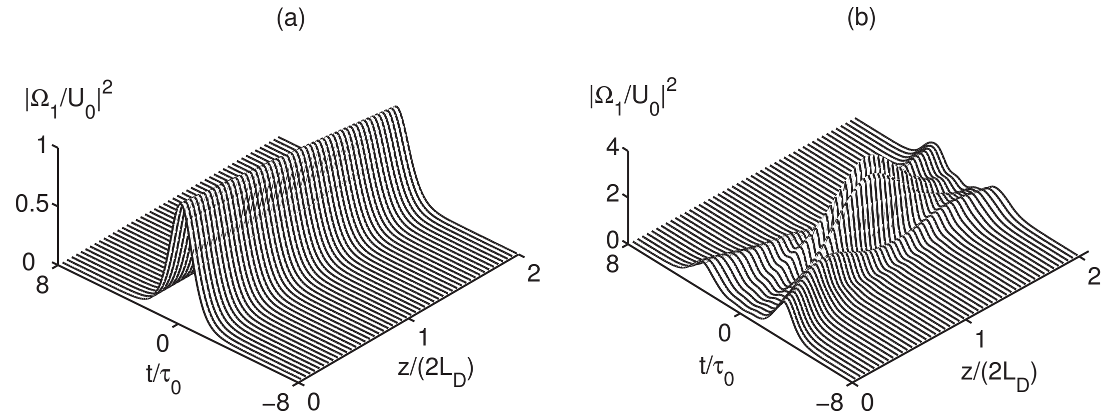


Fig. 2. (a) The waveshape of $|\Omega_1/U_0|^2$ as a function of $t/\tau_0$ and $z/(2L_D)$ with the parameters given just below Eq. (7). The solution is numerically obtained from Eq. (6) with full complex coefficients taken into account. The initial condition is given by $\Omega_1(0,t)/U_0 = \mathrm{sech}(t/\tau_0)$. (b) The wave shape during a collision between two ultraslow optical solitons. The initial condition is given by $\Omega_1(0,t)/U_0 = \mathrm{sech}[(t-3.0)/\tau_0] + 1.2\ \mathrm{sech}[1.2(t+3.0)/\tau_0]$.

$t/\tau_0$ and $z/(2L_D)$ with the parameters given below Eq. (7). The solution is obtained by numerically solving Eq. (6) with the full complex coefficients included. The space and time derivatives are performed by using a split-step Fourier method for superior conservation of energy and other invariants. The initial condition is given by $\Omega_1(0,t)U_0 = \mathrm{sech}(t/\tau_o)$. We see that the amplitude of the soliton undergoes only a slight decrease and its width undergoes a slight increase due to the influence of the imaginary part of the coefficients. We have also made a simulation of the interaction between two ultraslow optical solitons. Assume initially that we have two solitons created in the system. As time goes on they collide and then depart each other. Shown in Fig. 2(b) is the wave shape of the two solitons during their collision. The initial condition in the simulation is given by $\Omega_1(0,t)/U_0 = \mathrm{sech}[(t-3.0)/\tau_0] + 1.2\ \mathrm{sech}[1.2(t+3.0)/\tau_0]$. We see that after the collision the two solitons recover their initial waveforms.

In conclusion, we have made a detailed study of the possibility of the formation and propagation of optical solitons in a resonant, lifetime-broadened $\Lambda$-type atomic system, in which the two-folded lower levels are coupled by a coherent laser field. We have shown that the analysis given in [1] is incorrect. The reason is that the Hamiltonian presented by them has a mistake in sign, the Maxwell–Schrödinger equations describing the dynamics of probe field and atomic motion derived by them are thus also incorrect, which results in an incorrect linear dispersion relation and hence an incorrect group velocity of the probe field. We have also shown that if a superluminal optical soliton is generated, it suffers a serious absorption because it works outside the transparency windows of the double dark resonance. However, the system can support an ultraslow optical soliton if working in the normal dispersion regimes. The ultraslow optical soliton can propagate stably for a fairly long distance because in this case the system works inside the transparency windows. In addition, we have demonstrated that such an ultraslow optical soliton can be generated by using a very low light intensity. Because of their robust propagating property, the ultraslow optical solitons in such a system may have potential applications in optical information processing and transmission.


## ACKNOWLEDGMENTS

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2. The spatial width of the soliton is given by $w_d = \tilde{V}_g \tau_0$. For our ultraslow optical soliton, Eq. (7) $w_d = 2.79$ cm. However, according to the results of Han *et al.* [1], where $V_g = 1.0377\ c$ and $\tau_0 = 1.0 \times 10^{-3}$ s, the soliton they obtained has $w_d = 3.12 \times 10^5$ m, which is obviously impractical for atomic physics experiments.
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