(2+1)-Dimensional Envelope Solitons in a Disk-Shaped Bose–Einstein Condensate*

HUANG Guo-Xiang(黄国翔)

Key Laboratory for Optical and Magnetic Resonance Spectroscopy and Department of Physics, East China Normal University, Shanghai 200062

(Received 4 March 2003)

We show that, due to the nonlinear coupling between a wavepacket superposed by short-wavelength collective modes and a long-wavelength mean field generated by the self-interaction of the wavepacket, the (2+1)-dimensional envelope solitons decaying in all spatial directions, i.e., dromions, are possible nonlinear excitations in a disk-shaped Bose–Einstein condensate with a repulsive interatomic interaction.

PACS: 03. 75. Fi, 03. 65. Ge, 42. 65. Tg

Recently, the study on nonlinear property of matter wave has received much attention due to the experimental realization of Bose–Einstein condensation in weakly interacting atomic gases.[1] The most spectacular experimental progress achieved is the demonstration of atomic four-wave mixing,[2] the observation of solitons[3] and vortices,[4] the discovery of superradiance[5] and the matter-wave amplification[6] in Bose–Einstein condensates (BECs). This research into nonlinear matter waves based on BEC has enabled the extension of linear atom optics to a nonlinear regime, i.e., nonlinear atom optics.[7]

For the study of soliton dynamics in BECs, works up to now are mainly concentrated on the one-dimensional (1D) solitons moving in a cigar-shaped trap.[8] Recently, a Boussinesq–Korteweg–de Vries (BKdV) description for 1D dark soliton excitations with a repulsive atom–atom interaction has been developed.[9] This approach has been extended in a systematic way to the case of a quasi-1D condensate.[10] In a recent work, the 2D weak nonlinear matter wave pulses created in a disk-shaped BEC have been considered.[11] By using a method of multiple-scales a Kadomtsev–Pitvashvili (KP) equation is derived from an order parameter equation. It is predicted that lump-like 2D solitons are possible and their decay into vortices have been investigated in details.[11]

However, all theoretical approaches developed in Refs. [9-11] are valid only for weak nonlinear excitations with a long-wavelength and hence the dispersion of the excitations must be weak. We note that in addition to long-wavelength excitations, BECs also support the nonlinear excitations with a short wavelength, which displays a strong dispersion. One expects that a nonlinear excitation in this case will show an interesting new character. It is just this problem that will be addressed in this Letter.

The dynamics of a weakly interacting Bose gas at low temperature is described by the time-dependent Gross–Pitaevskii (GP) equation[1]

$$\mathrm{i}\hbar\frac{\partial\Psi}{\partial t}=\Big[-\frac{\hbar^2}{2m}\nabla^2+V_{\rm ext}(\boldsymbol{r})+g|\Psi|^2\Big]\Psi, \quad (1)$$

where Ψ is called the order parameter, $\int{\rm d}\boldsymbol{r}|\Psi|^2=N$ is the atomic number in the condensate, $g=4\pi\hbar^2a_s/m$ is the interaction constant with a_s the s-wave scattering length ($a_s>0$ for a repulsive interaction). We consider a disk-shaped harmonic trap of the form $V_{\rm ext}(\boldsymbol{r})=\dfrac{m}{2}[\omega_\perp^2(x^2+y^2)+\omega_z^2z^2]$ with $\omega_\perp\ll\omega_z$, where $\omega_\perp$ and ω_z are the frequencies of the trap in the transverse (x and y) directions and the z-direction, respectively. Expressing the order parameter in terms of its modulus and phase, $\Psi=\sqrt{n}\exp(\mathrm{i}\phi)$, we obtain a set of coupled equations for n and ϕ. By introducing $(x,y,z)=a_z(x',y',z')$, $t=\omega_z^{-1}t'$, $n=n_0n'$ with $a_z=[\hbar/(m\omega_z)]^{1/2}$ and $n_0=N/a_z^3$, we arrive the following dimensionless equations of motion after dropping the primes:

$$\frac{\partial n}{\partial t}+\nabla\cdot(n\nabla\phi)=0, \quad (2)$$

$$\frac{\partial\phi}{\partial t}+\frac{1}{2}z^2+V_\parallel(x,y)+Qn+\frac{1}{2}\Big[(\nabla\phi)^2-\frac{1}{\sqrt{n}}\nabla^2\sqrt{n}\Big]=0 \quad (3)$$

with $Q=4\pi Na_s/a_z$ and $\int{\rm d}\boldsymbol{r}n=1$. $V_\parallel(x,y)=(\omega_\perp/\omega_z)^2(x^2+y^2)/2$ is the dimensionless trapping potential in the x and y directions.

We are interested in an excitation created in the condensate with a thin disk-shaped trap. The thin disk-shaped trap here implies that the conditions $a_z\ll l_0$ and $\hbar\omega_\perp\ll n_0g\ll\hbar\omega_z$ can be fulfilled, where $l_0=(4\pi n_0a_s)^{-1/2}$ is healing length. In this situation we can make the quasi-2D approximation,[11]

* Supported by the National Natural Science Foundation of China under Grant No 10274021 and the Trans-Century Programme Foundation for the Talents of the Ministry of Education of China.

$\sqrt{n} = P(x,y,t)G_0(z)$, $\phi = -\mu t + \varphi(x,y,t)$, where $G_0(z) = \exp(-z^2/2)$ is the ground-state wavefunction of the 1D harmonic oscillator with the potential $z^2/2$ in the z-direction, μ is the chemical potential of the condensate and φ is a phase function contributed from the excitation, which is assumed to be a function of x and y because the created excitation can only propagate in the x and y directions as mentioned above. Then Eqs. (2) and (3) are reduced to

$$\frac{\partial P}{\partial t} + \frac{\partial P}{\partial x}\frac{\partial \varphi}{\partial x} + \frac{\partial P}{\partial y}\frac{\partial \varphi}{\partial y} + \frac{P}{2}\Big(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}\Big) = 0, \tag{4}$$

$$-\frac{1}{2}\Big(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}\Big) - \Big(\mu - \frac{1}{2}\Big)P + \Big[\frac{\partial\varphi}{\partial t} + V_{\|}(x,y) + \frac{1}{2}\Big(\frac{\partial\varphi}{\partial x}\Big)^2 + \frac{1}{2}\Big(\frac{\partial\varphi}{\partial y}\Big)^2\Big]P + Q'P^3 = 0, \tag{5}$$

where $Q' = I_0 Q$ is an effective interaction constant with $I_0 = \int_{-\infty}^{\infty}\mathrm{d}z G_0^4(z)/\int_{-\infty}^{\infty}\mathrm{d}z G_0^2(z) = 1/\sqrt{2}$. In principle, one can take into account the contribution of the higher-order eigenmodes of the harmonic oscillator in the z-direction, as carried out in Ref. [10] for a cigar-shaped trap. However, as here we have assumed $n_0 g \ll \hbar\omega_z$, the contribution from these higher-order eigenmodes is small and can be safely neglected. Furthermore, on the other hand, for the thin disk-shaped trap ($\omega_\perp/\omega_z \ll 1$) the trapping potential in the (x,y) plane is a slowly varying function of x and y and hence the size of the condensate in the radial direction is much larger than the size the soliton excitations (with the order of the healing length) considered below. In the propagation of the soliton for short times, the boundary of the condensate does not come into play and we can therefore simulate the experimental situation by considering the condensate being uniform in the (x,y) plane.

We first analyse the linear dispersion relation of the system, which can be obtained by assuming in Eqs. (4) and (5) that $P = u_0 + a(x,y,t)$ ($u_0 > 0$) with $(a,\varphi) = (a_0,\varphi_0)\exp[\mathrm{i}(k_1 x + k_2 y - \omega t)] + \text{c.c.}$ and u_0, a_0 and φ_0 being constants. The result is

$$\omega(k_1,k_2) = (4Q'u_0^2 + k^2)^{1/2}k/2, \tag{6}$$

where $k^2 = k_1^2 + k_2^2$. Equation (6) is a Bogoliubov-type linear excitation spectrum in 2D. Obviously, in addition to a long-wavelength excitation ($k = 0$), the system may support a short-wavelength excitation ($k \neq 0$). The sound speed of the system reads $c = \lim_{k\to 0}\Big[\Big(\frac{\partial\omega}{\partial k_1}\Big)^2 + \Big(\frac{\partial\omega}{\partial k_2}\Big)^2\Big]^{1/2} = \sqrt{Q'}u_0$.

In order to investigate the weak nonlinear excitations with a short wavelength, following the line of Davey and Stewartson[12] we make the asymptotic expansion $P = u_0 + \epsilon a^{(1)} + \epsilon^2 a^{(2)} + \epsilon^3 a^{(3)} + \cdots$, $\varphi = \epsilon\varphi^{(1)} + \epsilon^2\varphi^{(2)} + \epsilon^3\varphi^{(3)} + \cdots$, and assume that $a^{(j)}$ and $\varphi^{(j)}$ ($j = 1,2,3,\cdots$) are the functions of the *fast* variable $\theta = kx - \omega t$ and the multiple-scale (slow) variables $\xi = \epsilon(c_g^{-1}x - t)$, $\eta = \epsilon y$, $\tau = \epsilon^2 t$, where ϵ is a smallness parameter characterizing the relative amplitude of the excitation; c_g is a constant yet to be determined. Then substituting above expansion to Eqs. (4) and (5), we obtain $-\omega\frac{\partial a^{(j)}}{\partial\theta} + \frac{1}{2}u_0 k^2\frac{\partial^2\varphi^{(j)}}{\partial\theta^2} = \alpha^{(j)}$, $\Big[-\frac{1}{2}k^2\frac{\partial^2}{\partial\theta^2} + 2Q'u_0^2\Big]a^{(j)} - \omega u_0\frac{\partial\varphi^{(j)}}{\partial\theta} = \beta^{(j)}$, for $j = 1,2,3,\cdots$. The explicit expressions of $\alpha^{(j)}$ and $\beta^{(j)}$ are omitted here.

At leading ($j = 1$) order the solution reads

$$\varphi^{(1)} = A_0 + [A\exp(\mathrm{i}\theta) + \text{c.c.}], \tag{7}$$

$$a^{(1)} = \frac{\mathrm{i}}{2}\frac{u_0 k^2}{\omega}A\exp(\mathrm{i}\theta) + \text{c.c.}, \tag{8}$$

where A_0 is a mean flow (i.e., zero-mode) necessarily to be introduced for cancelling a secular term appearing in high-order approximations. A is an envelope function of the carrier wave $\exp(\mathrm{i}\theta)$. Both A_0 and A are yet to be determined functions of the slow variables ξ, η, and τ; $\omega(k)$ is just the linear dispersion relation of the excitation, given in Eq. (6) with $k_1 = k$ and $k_2 = 0$; c.c. represents a corresponding complex conjugate term.

At the next order ($j = 2$), a solvability condition requires that $c_g = \frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{1}{2\omega}[2Q'u_0^2 k + k^3]$, i.e., the group velocity of the carrier wave. The singularity-free second-order solution reads $\varphi^{(2)} = A_2\exp(2\mathrm{i}\theta) + \text{c.c.}$, and $a^{(2)} = B_{20} + [B_{21}\exp(\mathrm{i}\theta) + B_{22}\exp(2\mathrm{i}\theta) + \text{c.c.}]$, where A_2 and B_{2j} ($j = 0,1,2$) are functions of A_0, A, $\partial A_0/\partial\xi$ and $\partial A/\partial\xi$.

At the order $j = 3$ the solvability conditions give rise to the equations for A_0 and A:

$$\alpha_1\frac{\partial^2 A_0}{\partial\xi^2} - \frac{\partial^2 A_0}{\partial\eta^2} = \alpha_2\frac{\partial}{\partial\xi}(|A|^2), \tag{9}$$

$$\mathrm{i}\frac{\partial A}{\partial\tau} + \beta_1\frac{\partial^2 A}{\partial\xi^2} + \beta_2\frac{\partial^2 A}{\partial\eta^2} + \beta_3|A|^2 A - \beta_4 A\frac{\partial A_0}{\partial\xi} = 0, \tag{10}$$

with the coefficients explicitly given by $\alpha_1 = \frac{1}{c^2} - \frac{1}{c_g^2}$, $\alpha_2 = \frac{k^2}{2c_p^3 c_g}(2c_p^2 + 3c_p c_g + c^2)$, $\beta_1 = \frac{1}{2\omega c_g^2}\Big(c_p^2 - \frac{c^4}{c_p^2} + \frac{k^2}{4}\Big)$, $\beta_2 = \frac{1}{2\omega}\Big(c_p^2 + \frac{k^2}{4}\Big)$, $\beta_3 = \frac{1}{2\omega}\Big[c^2k^2 + \Big(\frac{15}{8}\frac{c^2}{c_p^2} - \frac{1}{4}\Big)k^4 + k^2\Big(1 + \frac{3}{2}\frac{c^2}{c_p^2}\Big)\frac{c_p^2(2c^2 + k^2) + 3c^2k^2/4}{c^2 + k^2}\Big]$, and $\beta_4 = \frac{1}{2\omega c_g}(c_g + 2c_p)k^2$, where c_p is phase speed defined by $w(k)/k$. We see that due to nonlinear effect a coupling occurs between the envelope of the short-wavelength excitations and the long-wavelength mean flow of the system. Such nonlinear coupling is one type of interactions between long waves and short waves.

Equations (9) and (10) are general Davey–Stewartson (DS) equations, which appear also in fluid physics, nonlinear optics, lattice dynamics, and plasma physics,[12–15] and have generated much interest in recent years.[16]

We now investigate the 2D soliton solutions of the DS Eqs. (9) and (10). Using the transformation $\frac{\partial A_0}{\partial \xi} = -\epsilon^{-2}\frac{\beta_1 k^4}{\alpha_1\beta_4}s$, and $A = \epsilon^{-1}\Big(\frac{4\beta_1}{\alpha_2\beta_4}\Big)^{1/2}k^4 u$, Eqs. (9) and (10) can be rewritten in the following form

$$\frac{\partial^2 s}{\partial x'^2} - \frac{\partial^2 s}{\partial y'^2} + 4\frac{\partial^2}{\partial x'^2}(|u|^2) = 0, \tag{11}$$

$$\mathrm{i}\frac{\partial u}{\partial t'} + \frac{\partial^2 u}{\partial x'^2} + \frac{\partial^2 u}{\partial y'^2} + 2|u|^2 u + su = R[u], \tag{12}$$

where $x' = (k^2/\sqrt{\alpha_1})(c_g^{-1}x - t)$, $y' = k^2 y$, $t' = (\beta_1 k^4/\alpha_1)t$ and $R[u] = (1-\kappa_1)\frac{\partial^2 u}{\partial y'^2} + 2(1-\kappa_2)|u|^2 u$ with $\kappa_1 = \frac{\alpha_1\beta_2}{\beta_1}$ and $\kappa_2 = \frac{2\alpha_1\beta_3}{\alpha_2\beta_4}$.

For an arbitrary value of the wavenumber k, an exact 2D soliton solution decaying in all spatial directions is not available yet. However, we note that for small k one has $1-\kappa_1 = -\frac{1}{6c^2}k^2 + \frac{1}{18c^4}k^4 - \frac{1}{54c^6}k^6 + O(k^8)$ and $1-\kappa_2 = -\frac{2}{3c^2}k^2 - \frac{1}{18c^4}k^4 + \frac{1}{108c^6}k^6 + O(k^8)$, thus $R[u]$ is a small quantity proportional to k^2. In this case, one can take $R[u]$ as a perturbation. As the first step we neglect $R[u]$ here. The effect of nonvanishing $R[u]$ will be considered in the future work.[17] Then Eq. (12) is simplified as

$$\mathrm{i}\frac{\partial u}{\partial t'} + \frac{\partial^2 u}{\partial x'^2} + \frac{\partial^2 u}{\partial y'^2} + 2|u|^2 u + s\,u = 0. \tag{13}$$

Eqs. (11) and (13) are standard type-I Davey–Stewartson (DSI) equations. They are completely integrable and can be solved exactly by the inverse scattering transform. One of remarkable properties of the DSI equations is that they allow dromion solutions decaying in all spatial directions.[16]

The dromion solution of the DSI Eqs. (11) and (13) reads[16] $u = \frac{G}{F}$, $s = 4\frac{\partial^2}{\partial x'^2}\ln F$, where $F = 1 + \exp(\eta_1+\eta_1^*) + \exp(\eta_2+\eta_2^*) + \gamma\exp(\eta_1+\eta_1^*+\eta_2+\eta_2^*)$, and $G = \rho\exp(\eta_1+\eta_2)$ with $\eta_1 = (k_r+\mathrm{i}k_i)x'' + (\Omega_r+\mathrm{i}\Omega_i)t'$, $\eta_2 = (l_r+il_i)y'' + (\omega_r+i\omega_i)t'$, $\Omega_r = -2k_r k_i$, $\omega_r = -2l_r l_i$, $\Omega_i + \omega_i = k_r^2 + l_r^2 - k_i^2 - l_i^2$ and $\rho = |\rho|\exp(i\varphi_\rho)$, $|\rho| = 2[2k_r l_r(\gamma-1)]^{1/2}$, where $k_r, k_i, l_r, l_i, |\rho|$, φ_ρ and γ are real integrable constants. If choosing $k_r l_r > 0$, we have $\gamma = \exp(2\varphi_\gamma)$, $\varphi_\gamma > 0$. Here x'' and y'' are the orthogonal transformation of x' and y', i.e., $x'' = \frac{1}{\sqrt{2}}(x'+y')$ and $y'' = \frac{1}{\sqrt{2}}(y'-x')$. If taking $k_r = \sqrt{2}\sigma$, $l_r = \sqrt{2}\lambda$ $(\lambda\sigma \geq 0)$, $k_i = \sqrt{2}a$, $l_i = \sqrt{2}p$, $\Omega_i = 2(\sigma^2 - a^2)$, one has $\Omega_r = -4a\sigma$, $\omega_r = -4\lambda p$, and $\omega_i = 2(\lambda^2 - p^2)$. Then we obtain

$$u = \frac{2\sigma\exp(\mathrm{i}h)}{n_1\cosh f_1 + n_2\cosh f_2}, \tag{14}$$

$$\begin{aligned} s = {} & \frac{4(n_1^2+n_2^2)(\sigma^2+\lambda^2) - 8\sigma^2}{(n_1\cosh f_1 + n_2\cosh f_2)^2} \\ & + \big\{8n_1 n_2[(\sigma^2+\lambda^2)\cosh f_1\cosh f_2 \\ & - (\sigma^2-\lambda^2)\sinh f_1\sinh f_2]\big\} \\ & \cdot\big(n_1\cosh f_1 + n_2\cosh f_2\big)^{-2}, \end{aligned} \tag{15}$$

where $n_1 = \Big(\frac{\sigma}{\lambda(\gamma-1)}\Big)^{1/2}$, $n_2 = \Big(\frac{\sigma\gamma}{\lambda(\gamma-1)}\Big)^{1/2}$ with $h = \sqrt{2}ax'' + \sqrt{2}py'' + 2(\sigma^2+\lambda^2-a^2-p^2)t' + \varphi_\rho$, $f_1 = \sqrt{2}\sigma x'' - \sqrt{2}\lambda y'' - 4(a\sigma - \lambda p)t'$ and $f_2 = \sqrt{2}\sigma x'' + \sqrt{2}\lambda y'' - 4(a\sigma+\lambda p)t' + \varphi_\gamma$. Obviously, the expression of u in Eq. (14) denotes a localized envelope function decaying in all spatial directions, called dromion.[16] From (14) we know that the dromion has an amplitude $\frac{2\sigma}{(n_1^2+n_2^2)^{1/2}}$ and, in the $Ox''y''$ coordinate system, at time t' it locates at the position $(x'', y'') = \Big(\frac{4a}{\sqrt{2}}t' - \frac{\varphi_\gamma}{2\sqrt{2}\sigma}, \frac{4p}{\sqrt{2}}t' - \frac{\varphi_\gamma}{2\sqrt{2}\lambda}\Big)$. Hence the dromion has a constant velocity $\boldsymbol{V}_d = \Big(\frac{4}{\sqrt{2}}a, \frac{4}{\sqrt{2}}p\Big)$.

The mean field component s consists of two interacting plane solitons with each plane soliton decaying in its travelling direction. It is easy to show that s has the following asymptotic form $s|_{x''\to-\infty} = 4\lambda^2\mathrm{sech}^2\,\lambda(\sqrt{2}y'' - 4pt')$, $s|_{x''\to+\infty} = 4\lambda^2\mathrm{sech}^2[\lambda(\sqrt{2}y'' - 4pt') + \varphi_\gamma]$, $s|_{y''\to-\infty} = 4\sigma^2\mathrm{sech}^2\,\sigma(\sqrt{2}x'' - 4at')$, and $s|_{y''\to+\infty} = 4\sigma^2\mathrm{sech}^2[\sigma(\sqrt{2}x'' - 4at') + \varphi_\gamma]$. Thus in the $Ox''y''$ coordinate system the plane soliton with the amplitude $4\lambda^2$ (λ-soliton) travels with the velocity $\boldsymbol{V}_\lambda = \Big(0, \frac{4p}{\sqrt{2}}\Big)$, while the plane soliton with the amplitude $4\sigma^2$ (σ-soliton) travels with the velocity $\boldsymbol{V}_\sigma = \Big(\frac{4a}{\sqrt{2}}, 0\Big)$. There is a cross region (corresponding to an oblique collision between the plane solitons) where a new plane soliton, i.e., a Mach stem, appears. If assuming that both a and p are positive with $a > p$, one can assign $x'' = +\infty$, $y'' = +\infty$ as the region of "before collision" and $x'' = -\infty$, $y'' = -\infty$ as the region of "after collision." The phase shift (i.e. position shift) for each plane soliton due to the collision is given by $\Delta_\lambda = -\frac{\varphi_\gamma}{\sqrt{2}\lambda} = -\frac{1}{2\sqrt{2}\lambda}\ln\gamma$ (for λ-soliton) and $\Delta_\sigma = -\frac{\varphi_\gamma}{\sqrt{2}\sigma} = -\frac{1}{2\sqrt{2}\sigma}\ln\gamma$ (for σ-soliton). Therefore, both phase-shifts are negative.

We note that the centre-point of the cross region of two plane solitons is just the position of the dromion and its motional velocity is also $\boldsymbol{V}_d$. Thus the dromion rides exactly on the cross-point of the two plane solitons and travels with the common velocity $\boldsymbol{V}_d$. From this result we see that the dromion, which represents the high-frequency component of the excitation, can be taken as being driven by the "truck," i.e., the long-wavelength (low-frequency) component denoted by two plane solitons.

Now we give the explicit expression for the order parameter when the dromion excitation presented above is created. At the leading order approximation we obtain

$$\Psi = P(x,y,t)\exp\Big(-\mathrm{i}\mu t - \frac{1}{2}z^2 + \mathrm{i}\varphi(x,y,t)\Big), \quad (16)$$

where

$$P(x,y,t) = u_0\Big(1 - B_0\frac{\sin\Phi}{n_1\cosh f_1 + n_2\cosh f_2}\Big), \quad (17)$$

$$\varphi = -\frac{\beta_1}{\sqrt{\alpha_1}\beta_4}k^2 D_0(x,y,t) + \Big(\frac{4\beta_1}{\alpha_2\beta_4}\Big)^{1/2}k^4\frac{4\sigma\cos\Phi}{n_1\cosh f_1 + n_2\cosh f_2}, \quad (18)$$

with $\Phi = \Big[k + (a-p)\frac{k^2}{\sqrt{\alpha_1}}\Big]x + (a+p)k^2 y + \Big[2(\sigma^2 + \lambda^2 - a^2 - p^2)\frac{\beta_1}{\alpha_1}k^4 - (a-p)c_g\frac{k^2}{\sqrt{\alpha_1}}\Big]t + \varphi_\rho$, $f_1 = (\sigma+\lambda)\frac{k^2}{\sqrt{\alpha_1}}x + (\sigma-\lambda)k^2 y - \Big[(\sigma+\lambda)c_g\frac{k^2}{\sqrt{\alpha_1}} + 4(a\sigma - \lambda p)\frac{\beta_1}{\alpha_1}k^4\Big]t$, $f_2 = (\sigma-\lambda)\frac{k^2}{\sqrt{\alpha_1}}x + (\sigma+\lambda)k^2 y - \Big[(\sigma-\lambda)c_g\frac{k^2}{\sqrt{\alpha_1}} + 4(a\sigma+\lambda p)\frac{\beta_1}{\alpha_1}k^4\Big]t + \varphi_\gamma$ and $B_0 = 4\sigma\Big(\frac{4\beta_1}{\alpha_2\beta_4}\Big)^{1/2}\frac{k^5}{\sqrt{k^2+4c^2}}$. The function D_0 is given by

$$\begin{aligned} D_0 = &\{\sigma\exp(\eta_1+\eta_1^*)[1+\gamma\exp(\eta_2+\eta_2^*)] \\ &+ \lambda\exp(\eta_2+\eta_2^*)[1+\gamma\exp(\eta_1+\eta_1^*)]\} \\ &\cdot\big[1+\exp(\eta_1+\eta_1^*)+\exp(\eta_2+\eta_2^*) \\ &+ \gamma\exp(\eta_1+\eta_1^*+\eta_2+\eta_2^*)\big]^{-1} \end{aligned} \quad (19)$$

with $\eta_1+\eta_1^* = 2\sigma\Big\{\frac{k^2}{\sqrt{\alpha_1}}x + k^2 y - \Big[\frac{k^2}{\sqrt{\alpha_1}}c_g + 4a\frac{\beta_1}{\alpha_1}k^4\Big]t\Big\}$ and $\eta_2+\eta_2^* = 2\lambda\Big\{-\frac{k^2}{\sqrt{\alpha_1}}x + k^2 y + \Big[\frac{k^2}{\sqrt{\alpha_1}}c_g - 4p\frac{\beta_1}{\alpha_1}k^4\Big]t\Big\}$. From the expression (17) we see that the excitation is a *grey dromion* created from the background (the ground-state condensate). The parameter B_0 characterizes its *greyness*. The phase correction of the order parameter, i.e., φ given by Eq. (18), includes two parts. The first part is a mean flow (represented by D_0) describing an oblique interaction of two plane kinks. The second part is a dromion decaying exponentially in all spatial directions.

We have investigated the dynamics of (2+1)D weak nonlinear matter-wave pulses in a disk-shaped BEC. By means of a method of multiple scales, the Davey–Stewartson equations, which describe the time evolution of an envelope superposed by short-wavelength collective modes and a long-wavelength mean field generated from the self-interaction of the short-wavelength collective modes, have been derived from the order parameter equation of the condensate. Our results show that the (2+1)-D nonlinear localized structures, i.e., dark dromions, may be excited in the BEC with a disk-shaped trap for repulsive atom–atom interactions. We note that disk-shaped traps have been used to realize the first BEC in 1995[18] and observe linear excitations lately.[19] We hope the results given above can be used to guide new experimental findings of high-D solutions in BECs.

The author is grateful to Jacob Szeftel for warm hospitality received at LPTMC, Université Paris-VII, where part of this work was carried out.

References

[1] Dalfovo F et al 1999 *Rev. Mod. Phys.* **71**463
Leggett A J 2001 *Rev. Mod. Phys.* **73** 307
[2] Deng L et al 1999 *Nature* **398** 218
Wu Y et al 2000 *Phys. Rev.* A **61** 043604
Trippenbach M et al 2000 *Phys. Rev.* A **62** 023608
Yang Q et al 2003 *Phys. Rev.* A **67** 013603
[3] Burger S et al 1999 *Phys. Rev. Lett.* **83** 5198
Denschlag J et al 2000 *Science* **287** 97
Dutton Z et al 2001 *Science* **293** 663
Strecker K E et al 2002 *Nature* **417** 150
Khaykovich L et al 2002 *Science* **296** 1290
[4] Madison K W et al 2000 *Phys. Rev. Lett.* **84** 806
Abo-Shaeer J R et al 2001 *Science* **292** 476
[5] Inouye S et al 1999 *Science* **285** 571
[6] Inouye S et al 1999 *Nature* **402** 641
Kozuma M et al 1999 *Science* **286** 2309
[7] Rolston S L and Phillips W D 2002 *Nature* **416** 219
[8] Perez-Garcia V M et al 1998 *Phys. Rev.* A **57** 3837
Hong T et al 1998 *Phys. Rev.* A **58** 3128
Liu W-M et al 2000 *Phys. Rev. Lett.* **84** 2294
Busch Th and Anglin J R 2000 *Phys. Rev. Lett.* **84** 2298
Wu B et al 2002 *Phys. Rev. Lett.* **88** 034101
[9] Huang G et al 2001 *Phys. Rev.* A **64** 013617
[10] Huang G et al 2002 *Phys. Rev.* A **65** 053605
[11] Huang G et al 2003 *Phys. Rev.* A **67** 023604
[12] Davey A and Stewartson K 1974 *Proc. R. Soc. London* Ser. A **338** 101
[13] Cui W et al 2003 *Chin. Phys. Lett.* **20** 246
[14] Huang G et al 2001 *Phys. Rev.* E **64** 056619
[15] Khismatulin D B and Akhatov I Sh 2001 *Phys. Fluids* **13** 3582
[16] Ablowitz M J et al 2001 *Phys. Rev.* E **63** 046605
[17] Huang G et al *in preparation*
[18] Anderson M H et al 1995 *Science* **269** 198
[19] Jin D S et al 1996 *Phys. Rev. Lett.* **77** 420