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Vectorial coupled-mode solitons in one-dimensional photonic crystals*

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We study the dynamics of vectorial coupled-mode solitons in one-dimensional photonic crystals with quadratic and cubic nonlinearities. Starting from Maxwell's equations, the vectorial coupled-mode equations for the envelopes of two fundamental-frequency optical mode and one low-frequency mode components due to optical rectification are derived by means of the method of multiple scales. A set of coupled soliton solutions of the vectorial coupled-mode equations is provided. The results show that a modulation of the fundamental-frequency optical modes occurs due to the optical rectification field resulting from the quadratic nonlinearity. The optical rectification field disappears when the frequency of the fundamental-frequency optical fields approaches the edge of the photonic bands.

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1. Introduction

In recent years, much attention has been paid to the study of optical solitons.[1] The reason is that, in addition to some fundamental interest, by using them as information carriers, optical solitons have promising applications in all-optical processing technology. For instance, optical solitons can be used to realize stable super-long distance information transmission and to design all-optical switches with good performance.[1−4] There have been many investigations on optical solitons not only in homogeneous optical materials,[5−7] but also in photonic crystals.[8−10] Photonic crystals are optical materials artificially made with a periodic distribution of dielectric function. Due to this property the eigenfrequency spectrum (dispersion relation) of a photonic crystal displays band structures. There is a frequency gap (also called energy gap) between two adjacent bands. Because of Bragg reflection, the photons with frequency in bandgaps are forbidden. However, if the light intensity is large enough, localized electromagnetic modes may form with their frequencies in the bandgaps resulting from the nonlinear effect. Such localized modes are called photonic bandgap solitons. The study of photonic bandgap solitons has become a rapidly developing field and many possible practical applications may be found in the near future. Unlike the formation mechanism of optical solitons in uniform materials, where the production of a soliton is due to the balance between nonlinearity and the material dispersion of the system, the formation of a soliton in a photonic crystal results from the balance between the nonlinearity and the geometric dispersion coming from the periodic distribution of the dielectric function. Since the geometric dispersion can be artificially controlled, photonic crystals can be designed according to practical needs. Thus, the study of photonic crystals is an interesting and promising research field, both in theory and in application.

There has been a series of researches on optical solitons in one-dimensional (1D) photonic crystals. But these researches are concentrated mainly on materials with cubic ($\chi^{(3)}$) nonlinearity, and a scalar approximation and a narrow bandgap assumption have been made.[11] Arraf and Sterke[12] have investigated the coupling of band-edge modes in the case of quadratic ($\chi^{(2)}$) nonlinearity. Their results are valid only for the case of small bandgap, and the optical rectification resulting from the quadratic nonlinearity has been also disregarded. Note that a light field is a vectorial field. Different components of the vector

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field will couple and interact with each other due to the nonlinear effect. Thus, it is necessary to give a vectorial description to the light field, especially when the nonlinear effect is significant. Although there have been some investigations in this aspect,[2,5,6,13] the effort has been made mainly on the optical materials with a centro-symmetry (i.e. $\chi^{(2)}=0$).

In the present work, we study the dynamics of vectorial coupled-mode solitons in a 1D photonic crystal with quadratic and cubic nonlinearities. We show that an optical rectification field resulting from the quadratic nonlinearity produces a nonlinear modulation for fundamental-frequency optical fields. And inversely, the optical rectification field results in a change for the fundamental-frequency solitons into different types. The optical rectification field disappears when the vibrating frequency of the fundamental-frequency optical fields approaches the band-edge of the photonic crystal.

This paper is organized as follows. Section 2 introduces the vector model and makes an asymptotic expansion based on a method of multiple scales. The coupled-mode equations for the optical rectification field and the fundamental-frequency fields are derived in section 3. In section 4, the optical soliton solutions for the coupled-mode envelope equations are provided. The final section contains a discussion and summary of our results.

2. Vector model and asymptotic expansion

In a non-magnetic dielectric without a source, the electric field intensity $\boldsymbol{E}$, the electric polarization vector $\boldsymbol{P}$ and the electric displacement vector $\boldsymbol{D}$ satisfy the Maxwell equations

$$\nabla^2\boldsymbol{E}-\nabla(\nabla\cdot\boldsymbol{E})-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left(\boldsymbol{E}+\frac{\boldsymbol{P}}{\varepsilon_0}\right)=0, \quad (1)$$

$$\nabla\cdot\boldsymbol{D}=0, \quad (2)$$

where c is the light speed in vacuum and ε_0 is permittivity in vacuum. In a non-resonance case and under the condition of neglecting the intrinsic dispersion, i.e. materials dispersion, and the dissipation of the dielectric, the constitutive relation of the system reads

$$\boldsymbol{P}=\varepsilon_0(\chi^{(1)}\cdot\boldsymbol{E}+\chi^{(2)}:\boldsymbol{E}\boldsymbol{E}+\chi^{(3)}\vdots\boldsymbol{E}\boldsymbol{E}\boldsymbol{E}), \quad (3)$$

where $\chi^{(i)}$ (i=1, 2, 3) is the (i+1)th-order tensor of the electric susceptibility, which is a real number under the above dissipation-free assumption.

We consider uniaxial dielectric crystals with 4mm symmetry. There are many optical materials with such symmetry, e.g. $BaTiO_3$, SBN, KT, etc.[6] For simplicity, we consider the TE wave in a 1D photonic crystal. In the crystallographic-axis coordinate system ($oxyz$), we take the y-axis as the optical axis and assume that the dielectric is uniform in the x- and y-directions. But the susceptibility is a periodic function in the z-direction with a periodicity d, i.e. one has $\chi^{(i)}(z)=\chi^{(i)}(z+d)$ (i=1,2,3). We take the light field as $\boldsymbol{E}=(E_x(z,t),\ E_y(z,t),0)$. Then by using Eqs.(1)–(3) one obtains the vector equations

$$c^2\frac{\partial^2E_x}{\partial z^2}-n_x^2\frac{\partial^2E_x}{\partial t^2}-\frac{\partial^2}{\partial t^2}\times(\chi_{x1}^{(2)}E_xE_y+\chi_{x1}^{(3)}E_x^3+\chi_{x2}^{(3)}E_y^2E_x)=0, \quad (4)$$

$$c^2\frac{\partial^2E_y}{\partial z^2}-n_y^2\frac{\partial^2E_y}{\partial t^2}-\frac{\partial^2}{\partial t^2}\times(\chi_{y1}^{(2)}E_x^2+\chi_{y2}^{(2)}E_y^2+\chi_{y1}^{(3)}E_y^3+\chi_{y2}^{(3)}E_x^2E_y)=0, \quad (5)$$

where $n_j^2=n_j^2(z)=1+\chi_{jj}^{(1)}(z)=n_j^2(z+d)$ ($j=x,y$), $\chi_{x1}^{(2)}=\chi_{xyx}^{(2)}+\chi_{xxy}^{(2)}$, $\chi_{x1}^{(3)}=\chi_{xxxx}^{(3)}$, $\chi_{x2}^{(3)}=\chi_{xxyy}^{(3)}+\chi_{xyxy}^{(3)}+\chi_{xyyx}^{(3)}$, $\chi_{y1}^{(2)}=\chi_{yxx}^{(2)}$, $\chi_{y2}^{(2)}=\chi_{yyy}^{(2)}$, $\chi_{y1}^{(3)}=\chi_{yyyy}^{(3)}$, and $\chi_{y2}^{(3)}=\chi_{yyxx}^{(3)}+\chi_{yxyx}^{(3)}+\chi_{yxxy}^{(3)}$. Equations (4) and (5) imply that the light field travels in the direction perpendicular to the optical axis (i.e. y-axis), its x-component is also perpendicular to the optical axis, and its y-component is parallel to the optical axis. From Eqs.(4) and (5) it is seen that $\chi_{x1}^{(2)}$, $\chi_{x2}^{(3)}$, $\chi_{y2}^{(3)}$ describe the mutual interaction between different light field components, $\chi_{y1}^{(2)}$ describes the interaction between the x- and y-components. The other terms appearing in Eqs.(4) and (5) represent self-interactions of different optical field components. We must stress that the mutual interaction between different components will be lost when taking a scalar wave approximation.

We consider the weak nonlinear excitations of the system. In this circumstance, one can introduce the asymptotic expansion for different light field components as

$$E_j(z,t)=\sum_{i=1}\mu^iE_j^{(i)}=\mu E_j^{(1)}+\mu^2E_j^{(2)}+\mu^3E_j^{(3)}\cdots, \quad (6)$$

where $j=x,y$ and $0<\mu\ll 1$. μ is a small parameter denoting the relative amplitude of the field intensity, $E_j^{(i)}=E_j^{(i)}(z_0,z_1,z_2,\cdots;t_0,t_1,t_2,\cdots)$ with $z_i=\mu^iz$ and $t_i=\mu^it$ (i=0, 1, 2,$\cdots$). z_0 and t_0 are called fast variables; z_i and t_i (i=1, 2, $\cdots$) are called slow variables. Also, for simplicity, we assume that the electric susceptibility depends only on the fast variables, i.e. one has $n^2(z) = n^2(z_0)$, $\chi^{(2)}(z) = \chi^{(2)}(z_0)$, $\chi^{(3)}(z) = \chi^{(3)}(z_0)$. Substituting Eq.(6) into Eqs.(4) and (5), one obtains a chain of linear but inhomogeneous equations by collecting the same powers of μ

$$c^2 \frac{\partial^2 E_j^{(i)}}{\partial z_0^2} - n_j^2(z_0) \frac{\partial^2 E_j^{(i)}}{\partial t_0^2} = M_j^{(i)} \quad (i = 1, 2, 3, \cdots; j = x, y), \tag{7}$$

with

$$M_x^{(1)} = 0, \tag{8}$$

$$M_x^{(2)} = -2c^2 \frac{\partial^2 E_x^{(1)}}{\partial z_0 \partial z_1} + 2n_x^2(z_0) \frac{\partial^2 E_x^{(1)}}{\partial t_0 \partial t_1} + \chi_{x1}^{(2)}(z_0) \frac{\partial^2 (E_x^{(1)} E_y^{(1)})}{\partial t_0^2}, \tag{9}$$

$$\begin{aligned} M_x^{(3)} = & -c^2 \left[2 \frac{\partial^2 E_x^{(2)}}{\partial z_0 \partial z_1} + \frac{\partial^2 E_x^{(1)}}{\partial z_1^2} + 2 \frac{\partial^2 E_x^{(1)}}{\partial z_0 \partial z_2} \right] \\ & + n_x^2 \left[2 \frac{\partial^2 E_x^{(2)}}{\partial t_0 \partial t_1} + \frac{\partial^2 E_x^{(1)}}{\partial t_1^2} + 2 \frac{\partial^2 E_x^{(1)}}{\partial t_0 \partial t_2} \right] \\ & + \chi_{x1}^{(2)} \left[2 \frac{\partial^2}{\partial t_0^2} (E_x^{(1)} E_y^{(2)} + E_y^{(1)} E_x^{(2)}) \right. \\ & \left. + 2 \frac{\partial^2 E_x^{(1)} E_y^{(1)}}{\partial t_0 \partial t_1} \right] + \chi_{x1}^{(3)}(z_0) \frac{\partial^2 (E_x^{(1)})^3}{\partial t_0^2} \\ & + \chi_{x2}^{(3)}(z_0) \frac{\partial^2 E_x^{(1)} (E_y^{(1)})^2}{\partial t_0^2}, \end{aligned} \tag{10}$$

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$$M_y^{(1)} = 0, \tag{11}$$

$$\begin{aligned} M_y^{(2)} = & -2c^2 \frac{\partial^2 E_y^{(1)}}{\partial z_0 \partial z_1} + 2n_y^2(z_0) \frac{\partial^2 E_y^{(1)}}{\partial t_0 \partial t_1} \\ & + \chi_{y1}^{(2)}(z_0) \frac{\partial^2 (E_x^{(1)})^2}{\partial t_0^2} + \chi_{y2}^{(2)}(z_0) \frac{\partial^2 (E_y^{(1)})^2}{\partial t_0^2}, \end{aligned} \tag{12}$$

$$\begin{aligned} M_y^{(3)} = & -c^2 \left[2 \frac{\partial^2 E_y^{(2)}}{\partial z_0 \partial z_1} + \frac{\partial^2 E_y^{(1)}}{\partial z_1^2} + 2 \frac{\partial^2 E_y^{(1)}}{\partial z_0 \partial z_2} \right] \\ & + n_y^2 \left[2 \frac{\partial^2 E_y^{(2)}}{\partial t_0 \partial t_1} + \frac{\partial^2 E_y^{(1)}}{\partial t_1^2} + 2 \frac{\partial^2 E_y^{(1)}}{\partial t_0 \partial t_2} \right] \\ & + \chi_{y1}^{(2)} \left[2 \frac{\partial^2 E_x^{(1)} E_x^{(2)}}{\partial t_0^2} + 2 \frac{\partial^2 (E_x^{(1)})^2}{\partial t_0 \partial t_1} \right] \\ & + \chi_{y2}^{(2)} \left[2 \frac{\partial^2 E_y^{(1)} E_y^{(2)}}{\partial t_0^2} + 2 \frac{\partial^2 (E_y^{(1)})^2}{\partial t_0 \partial t_1} \right] \\ & + \chi_{y1}^{(3)}(z_0) \frac{\partial^2 (E_y^{(1)})^3}{\partial t_0^2} + \chi_{y2}^{(3)}(z_0) \frac{\partial^2 E_y^{(1)} (E_x^{(1)})^2}{\partial t_0^2}, \end{aligned} \tag{13}$$

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The expressions of higher-order $M_x^{(i)}$ and $M_y^{(i)}$ are omitted here.

3. Vectorial coupled-mode equations

Now we solve Eq.(7) with Eqs.(8)–(13) order by order. At leading order (i=1) it is a homogeneous equation since $M_x^{(1)} = 0$, $M_y^{(1)} = 0$. The solution can be taken as a linear superposition of many eigenmodes $\phi_{jm_jk}(z_0)\exp(-\mathrm{i}\omega_j t_0)$, where $\phi_{jm_jk}(z_0)$ satisfies the equation

$$c^2 \frac{\mathrm{d}^2 \phi_{jm_jk}}{\mathrm{d}z_0^2} + \omega_j^2 n_j^2(z_0) \phi_{jm_jk} = 0. \tag{14}$$

Because $n_j^{(2)}(z_0)$ is a periodic function, the eigenmode solution of Eq.(14) is the Bloch function $\phi_{jm_jk}(z_0) = \exp(\mathrm{i}kz_0)u_{jm_jk}(z_0)$, where $u_{jm_jk}(z_0) = u_{jm_jk}(z_0 + d)$. The eigenvalue $\omega_j = \omega_{jm_j}(k)$ has a band structure with m_j being the band index and k being a reduced wave vector. As j=x and y, the dispersion relation displays two branches, i.e. ω_x and ω_y, called the x and y branches, respectively . The appearance of two branches of dispersion relation comes from the vector property of the model considered. Obviously, each branch of the dispersion relation has a band structure. Generally speaking, the band structure for each different branch is different from each other in non-centro-symmetric dielectric crystals. The function set of $\{\phi_{jm_jk}(z_0)\}$ is complete, orthogonal and normalized, i.e.

$$\begin{aligned} & \int_0^L n_j^2(z_0) \phi_{jm'_jk'}^*(z_0) \phi_{jm_jk}(z_0) \mathrm{d}z_0 \\ = & N \int_0^d n_j^2(z_0) \phi_{jm'_jk'}^*(z_0) \phi_{jm_jk}(z_0) \mathrm{d}z_0 \\ = & N \delta_{m'_j, m_j} \delta_{k', k}, \end{aligned} \tag{15}$$

where L is the length of the system and $N = L/d$ is an integer.

We assume that, at the leading-order approximation, the excitation of the light field takes the form

$$E_x^{(1)} = E_1^{(1)} \phi_{x1}(z_0) \exp(-\mathrm{i}\omega_{x1} t_0) + \mathrm{c.c.}, \tag{16}$$

$$E_y^{(1)} = E_2^{(1)} \phi_{y1}(z_0) \exp(-\mathrm{i}\omega_{y1} t_0) + \mathrm{c.c.}, \tag{17}$$

where $E_i^{(1)} = E_i^{(1)}(z_1, z_2, \cdots; t_1, t_2, \cdots)$ (i=1,2) are two undetermined functions denoting the complex envelopes (also called amplitudes) of the excitation, and c.c. represents the corresponding complex conjugate term. ϕ_{x1} (z_0) $(\phi_{y1}(z_0))$ is the eigenfunction corresponding to the eigenvalue ω_{x1} (ω_{y1}), which can be chosen arbitrarily in this order. We assume $\omega_{x1} \neq \omega_{y1}$ in the following discussion. Note that a mean flow (also called a dc field) is also a possible solution, corresponding to an excitation with a long wavelength. We do not take into account this kind of mean flow in the present work.

Using the leading-order solution given by Eqs.(16) and (17), one obtains the expressions of $M_x^{(2)}$ and $M_y^{(2)}$ (see Eqs.(9) and (12)). Using the two solvability conditions of Eq.(7) (i.e. the conditions of eliminating secular terms for the solution of $E_j^{(2)}$), we obtain

$$v_{gx}\frac{\partial E_1^{(1)}}{\partial z_1}+\frac{\partial E_1^{(1)}}{\partial t_1}=0, \tag{18}$$

$$v_{gy}\frac{\partial E_2^{(1)}}{\partial z_1}+\frac{\partial E_2^{(1)}}{\partial t_1}=0, \tag{19}$$

where $\hat{\beta} = -\mathrm{i}c\partial/\partial z_0$, and $v_{gj} = (\mathrm{d}\omega_j/\mathrm{d}k)|_{j=j1} = c\langle j1|\hat{\beta}|j1\rangle/\omega_{j1}$ is the group velocity of the wave packets (16) and (17). The matrix element $\langle ji|\hat{\beta}|jl\rangle$ is defined as

$$\langle ji|\hat{\beta}|jl\rangle = -\int_0^d \mathrm{i}c\phi_{ji}^*(z_0)(\partial\phi_{jl}(z_0)/\partial z_0)\mathrm{d}z_0.$$

The second-order approximation solution is found to be

$$E_x^{(2)} = \left(\sum_l E_{1l}^{(2)}\phi_{xl}(z_0)\exp(-\mathrm{i}\omega_{x1}t_0)+\mathrm{c.c.}\right)+\cdots, \tag{20}$$

$$\begin{aligned}E_y^{(2)} =&E_0^{(2)}+\Bigg[\Bigg(\sum_l E_{2l}^{(2)}\phi_{yl}(z_0)\\&\times\exp(-\mathrm{i}\omega_{y1}t_0)+\mathrm{c.c.}\Bigg)+\cdots\Bigg],\end{aligned} \tag{21}$$

where $E_0^{(2)} = E_0^{(2)}(z_1,z_2,\cdots;t_1,t_2,\cdots)$ is an optical rectification field originating from the self-interaction of the fundamental-frequency light fields $E_i^{(1)}$ (i=1,2). $E_{il}^{(2)} = E_{il}^{(2)}$ $(z_1,z_2,\cdots;t_1,t_2,\cdots)$ results from the non-uniform of the dielectric function of the system. In Eqs.(20) and (21) there exist double-, sum- and difference-frequency components, which are not given explicitly here. We do not consider the three-wave resonance process in the present work.

Using the results of the first-order and second-order solutions given above, one can calculate $M_x^{(3)}$ and $M_y^{(3)}$. When considering the solution at the third-order approximation (i=3), the solvability conditions of Eq.(7) give rise to the equations for $E_i^{(1)}$ (i=1,2) as follows

$$\begin{aligned}&\frac{\Lambda_{11}}{2}\frac{\partial^2E_1^{(1)}}{\partial z_1^2}-\frac{1}{2\omega_{x1}}\frac{\partial^2E_1^{(1)}}{\partial t_1^2}+\mathrm{i}\left(v_{gx}\frac{\partial E_1^{(1)}}{\partial z_2}+\frac{\partial E_1^{(1)}}{\partial t_2}\right)\\&+[\Lambda_{12}E_0^{(2)}+\Lambda_{13}|E_1^{(1)}|^2+\Lambda_{14}|E_2^{(1)}|^2]E_1^{(1)}=0,\end{aligned} \tag{22}$$

$$\begin{aligned}&\frac{\Lambda_{21}}{2}\frac{\partial^2E_2^{(1)}}{\partial z_1^2}-\frac{1}{2\omega_{y1}}\frac{\partial^2E_2^{(1)}}{\partial t_1^2}+\mathrm{i}\left(v_{gy}\frac{\partial E_2^{(1)}}{\partial z_2}+\frac{\partial E_2^{(1)}}{\partial t_2}\right)\\&+[\Lambda_{22}E_0^{(2)}+\Lambda_{23}|E_2^{(1)}|^2+\Lambda_{24}|E_1^{(1)}|^2]E_2^{(1)}=0,\end{aligned} \tag{23}$$

where

$$\Lambda_{11}=\frac{c^2\langle x1|x1\rangle}{\omega_{x1}}+\frac{4c^2}{\omega_{x1}}\sum_{l\neq1}\frac{|\langle x1|\hat{\beta}|xl\rangle|^2}{\omega_{x1}^2-\omega_{xl}^2},$$

$$\Lambda_{21}=\frac{c^2\langle y1|y1\rangle}{\omega_{y1}}+\frac{4c^2}{\omega_{y1}}\sum_{l\neq1}\frac{|\langle y1|\hat{\beta}|yl\rangle|^2}{\omega_{y1}^2-\omega_{yl}^2},$$

$$\begin{aligned}\Lambda_{12}=&\frac{\omega_{x1}}{2}C_{x1;x1,x1}^{(2)},\\ \Lambda_{13}=&\omega_{x1}\Bigg(\sum_l\frac{2\omega_{x1}^2C_{y1;yl,x1x1}^{(2)}C_{x1;x1x1,yl}^{(2)}}{\omega_{yl}^2-4\omega_{x1}^2}\\&+\frac{3}{2}C_{x1;x1x1,x1x1}^{(3)}\Bigg),\end{aligned}$$

$$\begin{aligned}\Lambda_{22}=&\omega_{y1}C_{y2;y1,y1}^{(2)},\\ \Lambda_{23}=&\omega_{y1}\Bigg(\sum_l\frac{4\omega_{y1}^2|C_{y2;y1y1,yl}^{(2)}|^2}{\omega_{yl}^2-4\omega_{y1}^2}\\&+\frac{3}{2}C_{y1;y1y1,y1y1}^{(3)}\Bigg),\end{aligned}$$

$$\begin{aligned}\Lambda_{14}=&\frac{\omega_{x1}}{2}\Bigg(\sum_l\frac{(\omega_{y1}+\omega_{x1})^2|C_{x1;xl,x1y1}^{(2)}|^2}{\omega_{xl}^2-(\omega_{y1}+\omega_{x1})^2}\\&+\sum_l\frac{(\omega_{y1}-\omega_{x1})^2|C_{x1;xlx1,y1}^{(2)}|^2}{\omega_{xl}^2-(\omega_{y1}-\omega_{x1})^2}\\&+2C_{x2;x1y1,x1y1}^{(3)}\Bigg),\end{aligned}$$

$$\begin{aligned}\Lambda_{24}=&\omega_{y1}\Bigg(\sum_l\frac{2(\omega_{x1}+\omega_{y1})^2C_{y1;y1x1,xl}^{(2)}C_{x1;xl,x1y1}^{(2)}}{\omega_{xl}^2-(\omega_{x1}+\omega_{y1})^2}\\&+\sum_l\frac{(\omega_{y1}-\omega_{x1})^2C_{y1;y1,xlx1}^{(2)}C_{x1;xlx1,y1}^{(2)}}{\omega_{xl}^2-(\omega_{y1}-\omega_{x1})^2}\\&+C_{y2;y1x1,y1x1}^{(3)}\Bigg),\end{aligned}$$

$$\langle x1|x1\rangle = \int_0^d \phi_{x1}^*(z_0)\phi_{x1}(z_0)\mathrm{d}z_0,$$

$$C_{x1;x1,x1}^{(2)} = \int_0^d \chi_{x1}^{(2)}(z_0)\phi_{x1}^*(z_0)\phi_{x1}(z_0)\mathrm{d}z_0,$$

$$C_{y2;y1,y1}^{(2)} = \int_0^d \chi_{y2}^{(2)}(z_0)\phi_{y1}^*(z_0)\phi_{y1}(z_0)\mathrm{d}z_0,$$

$$C_{x1;x1x1,x1x1}^{(3)} = \int_0^d \chi_{x1}^{(3)}\phi_{x1}^*\phi_{x1}^*\phi_{x1}\phi_{x1}\mathrm{d}z_0,$$

$$C_{y1;yl,x1x1}^{(2)} = \int_0^d \chi_{y1}^{(2)}(z_0)\phi_{yl}^*(z_0)\phi_{x1}^2(z_0)\mathrm{d}z_0,$$

$$C_{y1;y1y1,y1y1}^{(3)} = \int_0^d \chi_{y1}^{(3)}\phi_{y1}^*\phi_{y1}^*\phi_{y1}\phi_{y1}\mathrm{d}z_0.$$

From Eqs.(22) and (23) we find that there is a nonlinear coupling among the optical rectification field $E_0^{(2)}$ and two fundamental-frequency fields $E_1^{(1)}$ and $E_2^{(1)}$.

In order to gain a set of closed equations for $E_0^{(2)}$, $E_1^{(1)}$ and $E_2^{(1)}$, we still need another equation. This equation can be obtained from Eq.(7) at the fourth order (i=4). One obtains

$$v_{\mathrm{p}y}^2\frac{\partial^2 E_0^{(2)}}{\partial z_1^2} - \frac{\partial^2 E_0^{(2)}}{\partial t_1^2} - 2\left(\frac{C_{y1;x1,x1}^{(2)}}{\overline{n}_y^2 d}\frac{\partial^2 |E_1^{(1)}|^2}{\partial t_1^2} + \frac{C_{y2;y1,y1}^{(2)}}{\overline{n}_y^2 d}\frac{\partial^2 |E_2^{(1)}|^2}{\partial t_1^2}\right) = 0, \quad (24)$$

where $v_{\mathrm{p}y} = c/\overline{n}_y$, $\overline{n}_y^2 = \int_0^d n_y^2(z_0)\mathrm{d}z_0/d$. Obviously, $v_{\mathrm{p}y}$ is the phase velocity of the low-frequency optical rectification field. From Eq.(24) we see that the low-frequency optical rectification field $E_0^{(2)}$ is due to the contribution of the self-interaction of two fundamental-frequency optical fields $E_1^{(1)}$ and $E_2^{(1)}$. And inversely, the optical rectification field $E_0^{(2)}$ gives a nonlinear modulation to two fundamental-frequency optical fields $E_1^{(1)}$ and $E_2^{(1)}$ (see Eqs.(22) and (23)).

4. Vector soliton solutions

Equations (22)–(24) are a set of coupled-mode equations determining the nonlinear evolution of $E_0^{(2)}$, $E_1^{(1)}$ and $E_2^{(1)}$. Note that the solutions of the coupled-mode equations must fulfil Eqs.(18) and(19). From Eqs.(18) and (19) we see that, since $E_1^{(1)}$ and $E_2^{(1)}$ have been selected from different branches, principally one can choose ω_{x1} and ω_{y1} so that the two modes have nearly equal group velocity. In this situation two fundamental-frequency optical fields have small walk-off effect and hence the interaction will be strong. For example, one can choose two band-edge modes which have the same group velocity, i.e. zero. If this group-velocity matching condition can be satisfied, one has $v_{\mathrm{g}x} \approx v_{\mathrm{g}y} = v_{\mathrm{g}}$ and thus we can assume that $E_i^{(1)} = E_i^{(1)}(\xi, z_2, t_2)$ (i=1, 2) with $\xi = z_1 - v_{\mathrm{g}}t_1$. Then Eqs.(18) and (19) are satisfied automatically. By using Eq.(24) we obtain

$$E_0^{(2)} = \frac{2v_{\mathrm{g}}^2(C_{y1;x1,x1}^{(2)}|E_1^{(1)}|^2 + C_{y2;y1,y1}^{(2)}|E_2^{(1)}|^2)}{(v_{\mathrm{p}y}^2 - v_{\mathrm{g}}^2)\overline{n}_y^2 d}. \quad (25)$$

From this result we see that the optical rectification field disappears if the group velocity v_{g} vanishes. Consequently, when the vibrating frequency of the fundamental-frequency optical fields approaches the edge of the bands (typically when $k = 0$ or $k = \pi/d$), the optical rectification field goes to zero.

By taking $E_i = \mu E_i^{(1)}$ (i=1, 2), $\mu^2\zeta_2 = z_2 - v_{\mathrm{g}}t_2$ and noting that $\xi = \mu\zeta$, $\zeta = z - v_{\mathrm{g}}t$ and $t_2 = \mu^2 t$, when returning to the original variables z and t, Eqs.(22) and (23) become

$$\mathrm{i}\frac{\partial E_1}{\partial t} + \frac{\omega_{x1}''}{2}\frac{\partial^2 E_1}{2\zeta^2} + b_{11}|E_2|^2 E_1 + b_{12}|E_1|^2 E_1 = 0, \quad (26)$$

$$\mathrm{i}\frac{\partial E_2}{\partial t} + \frac{\omega_{y1}''}{2}\frac{\partial^2 E_2}{2\zeta^2} + b_{21}|E_1|^2 E_2 + b_{22}|E_2|^2 E_2 = 0, \quad (27)$$

where

$$b_{11} = \frac{\omega_{x1}v_{\mathrm{g}}^2 C_{x1;x1,x1}^{(2)}C_{y2;y1,y1}^{(2)}}{(v_{\mathrm{p}y}^2 - v_{\mathrm{g}}^2)\overline{n}_y^2 d} + \varLambda_{14},$$

$$b_{21} = \frac{2\omega_{y1}v_{\mathrm{g}}^2 C_{y2;y1,y1}^{(2)}C_{y1;x1,x1}^{(2)}}{(v_{\mathrm{p}y}^2 - v_{\mathrm{g}}^2)\overline{n}_y^2 d} + \varLambda_{24}, \quad (28)$$

$$b_{12} = \frac{\omega_{x1}v_{\mathrm{g}}^2 C_{x1;x1,x1}^{(2)}C_{y1;x1,x1}^{(2)}}{(v_{\mathrm{p}y}^2 - v_{\mathrm{g}}^2)\overline{n}_y^2 d} + \varLambda_{13},$$

$$b_{22} = \frac{2\omega_{y1}v_{\mathrm{g}}^2 C_{y2;y1,y1}^{(2)}C_{y2;y1,y1}^{(2)}}{(v_{\mathrm{p}y}^2 - v_{\mathrm{g}}^2)\overline{n}_y^2 d} + \varLambda_{23}, \quad (29)$$

$$\omega_{j1}'' = \left.\frac{\mathrm{d}^2\omega_j}{\mathrm{d}k^2}\right|_{j=j1} = \frac{c^2\langle j1|j1\rangle}{\omega_{j1}} + \frac{4c^2}{\omega_{j1}} \times \sum_{l\neq 1}\frac{|\langle j1|\hat{\beta}|jl\rangle|^2}{\omega_{j1}^2 - \omega_{jl}^2} - \frac{(v_{\mathrm{g}j})^2}{\omega_{j1}}, \quad (30)$$

with j=x and y, where ω''_{j1} is the group-velocity dispersion of the fundamental-frequency optical fields. Note that in our system the materials dispersion is not taken into account. The contribution to dispersion only comes from the periodic distribution of the linear dielectric function ($\varepsilon(z) = \varepsilon_0 n^2(z)$), i.e. the geometric dispersion. In addition, it is noted that, from expressions (28) and (29), when the group velocity of the fundamental-frequency optical fields $v_{\rm g}$ approaches the phase velocity of the optical rectification field $v_{{\rm p}y}$, b_{i1} and b_{i2} (i=1, 2) tend to infinity and hence Eqs.(26) and (27) lose their validity for a reasonable description for the nonlinear dynamics of the system. This interesting phenomenon corresponds to a resonance between a long wave and a short wave, requiring a detailed discussion not given in this paper.

From Eqs.(26) and (27) it is seen that, due to the group-velocity matching, two fundamental-frequency optical fields E_1 and E_2 possess stronger coupling. In addition to a self-phase modulation represented by the term $|E_i|^2E_i$, there is also a cross-phase modulation denoted by the term $|E_i|^2E_{3-i}$ (i=1, 2). The existence of the cross-phase modulation will bring a significant change in the type and property of coupled solitons. Depending on the value of the coefficients, Eqs.(26) and (27) are completely integrable in some particular cases, i.e. they can be solved exactly by the inverse scattering method. In general, they are not integrable and the types of the solutions depend on b_{il} and ω''_{j1}.

Equations (26) and (27) are similar to the coupled-mode equations obtained in Ref.[14] where a scalar model is employed. Some exact coupled soliton solutions have been given in Ref.[14] and obviously these solutions are also valid in our present case. The types of solutions include bright–bright, dark–dark and bright–dark coupled solitons, depending on particular cases by taking different values of the coefficients appearing in the equations. It is not necessary to repeat these solutions here. But we should note that, although their mathematical forms are similar, the physical meanings are different. In Ref.[14] a scalar model is considered and hence only one branch of dispersion relation is possible. The coupling discussed there is of two solitons excited in the same electric field component but from different bands. In the present work, a vector model is taken into account. The system displays two branches of dispersion relation and the soliton coupling discussed here is of two solitons excited in different electric field components and from different bands.

It is easy to show that Eqs.(26) and (27) allow a coupled grey–bright soliton solution. Under the conditions $\omega''_{y1} \neq 0$ and $b_{11}b_{21} > b_{12}b_{22}$, $\omega''_{x1}b_{22} > \omega''_{y1}b_{11}$, $\omega''_{x1}b_{21} > \omega''_{y1}b_{12}$ (or inverse), one has

$$E_1 = W_1[\tanh[K_0\zeta - \omega''_{x1}K_0(K_0\lambda + K_1)t] + {\rm i}\lambda] \times \exp({\rm i}(K_1\zeta - \Omega_1 t)), \quad (31)$$

$$E_2 = W_2 {\rm sech}\,[K_0\zeta - \omega''_{x1}K_0(K_0\lambda + K_1)t] \times \exp({\rm i}(K_2\zeta - \Omega_2 t)), \quad (32)$$

where K_0, K_1 and λ are arbitrary constants, and

$$K_2 = \omega''_{x1}(K_0\lambda + K_1)/\omega''_{y1}, \quad (33)$$

$$|W_1|^2 = K_0^2(\omega''_{x1}b_{22} - \omega''_{y1}b_{11})/(b_{11}b_{21} - b_{12}b_{22}), \quad (34)$$

$$\Omega_1 = (K_1^2\omega''_{x1}/2) - (1+\lambda^2)b_{12}|W_1|^2, \quad (35)$$

$$|W_2|^2 = K_0^2(\omega''_{x1}b_{21} - \omega''_{y1}b_{12})/(b_{11}b_{21} - b_{12}b_{22}), \quad (36)$$

$$\Omega_2 = ((K_2^2+K_0^2)\omega''_{y1}/2) - b_{22}|W_2|^2 - \lambda^2 b_{21}|W_1|^2. \quad (37)$$

Noting that $\tanh u + {\rm i}\lambda = [(1+\lambda^2) - {\rm sech}^2 u]^{1/2} \exp({\rm i}\varphi(u))$ with $\varphi(u) = \arctan(\lambda/\tanh u)$, and when taking $\phi_{j1}(z) = |\phi_{j1}(z)|\exp({\rm i}\varphi_j(z))$ ($j = x, y$) and $W_l = |W_l|\exp({\rm i}\theta_l)$ (l=1,2), in the leading-order approximation we obtain the expression of the optical field as follows

$$E_x = 2|W_1||\phi_{x1}(z)|[(1+\lambda^2) - {\rm sech}^2[K_0z - K_0(v_{\rm g} + \omega''_{x1}K_0\lambda + \omega''_{x1}K_1)t]]^{1/2} \times \cos[K_1z - (\omega_{x1} + v_{\rm g}K_1 + \Omega_1)t + \varphi(z,t) + \varphi_x(z) + \theta_1], \quad (38)$$

$$E_y = 2|W_2||\phi_{y1}(z)|{\rm sech}\,[K_0z - K_0(v_{\rm g} + \omega''_{x1}K_0\lambda + \omega''_{x1}K_1)t] \times \cos[K_2z - (\omega_{y1} + v_{\rm g}K_2 + \Omega_2)t + \varphi_y(z) + \theta_2]. \quad (39)$$

For band-edge modes one has $K_1 = v_{\rm g} = \lambda = 0$, and the condition of group-velocity matching can be satisfied. In this situation Eqs.(38) and (39) represent two coupled non-propagating optical solitons. If $v_{\rm g} + \omega''_{x1}K_0\lambda + \omega''_{x1}K_1 = 0$ can be fulfilled, Eqs.(38) and (39) describe also two coupled non-propagating optical solitons. According to the symmetry of Eqs.(26) and (27), one can also obtain bright–grey soliton solutions under appropriate conditions, which are not discussed here.

5. Summary and discussion

From expressions (38) and (39) we can see that, for the grey–bright soliton pair, the phases of the two light field components are complicated functions of time and space. The vibrating frequencies of E_x and E_y are determined by

$$\omega_x = \omega_{x1} + K_1 v_{\mathrm{g}} + \frac{\omega''_{x1} K_1^2}{2} - (1+\lambda^2) b_{12} |W_1|^2 + \frac{\lambda K_0 [v_{\mathrm{g}} + \omega''_{x1}(K_1 + K_0\lambda)] \operatorname{sech}^2 u}{\lambda^2 + \tanh^2 u}, \quad (40)$$

$$\omega_y = \omega_{y1} + K_2 v_{\mathrm{g}} + \frac{\omega''_{y1}}{2}(K_2^2 + K_0^2) - b_{22}|W_2|^2 - \lambda^2 b_{21} |W_1|^2, \quad (41)$$

respectively, where $u = K_0 z - K_0[v_{\mathrm{g}} + \omega''_{x1}(K_1 + K_0\lambda)]t$. For photonic crystals, eigenfrequency displays a band structure, and we are interested in their band-edge modes. For such modes, one has $v_{\mathrm{g}} = 0$. In this case, if K_0, K_1 and λ are chosen appropriately, the vibrating frequency of the optical solitons may be located in the frequency bands or in gaps. From expression (40) we know that ω_x may be different for different time and space coordinates.

In summary, in this paper we have studied the dynamics of vectorial coupled-mode solitons in 1D photonic crystals with quadratic and cubic nonlinearities. For a weak nonlinear effect, we have presented an asymptotic expansion for the nonlinear Maxwell equations and derived nonlinear coupled-mode equations for the envelopes of fundamental-frequency fields and the optical rectification field by using the method of multiple scales. Due to the quadratic nonlinearity of the system, the self-interaction of the fundamental-frequency fields result in the appearance of the optical rectification field, and the produced optical rectification field contributes inversely a modulation to the fundamental-frequency fields. We have provided some coupled soliton solutions for the vectorial coupled-mode equations. The results show that, although the form of the envelope equations obtained in the case of vectorial coupled-mode excitations considered is the same as that obtained in the case of scalar coupled-mode excitations discussed in Ref.[14], the physics in the two models are different. The former (i.e. the vectorial coupling case discussed in the present work) has considered the vectorial property of the optical field and thus the system displays two dispersion relations, and the soliton coupling corresponds to the interaction between two eigenmodes chosen from different dispersion branches. The coupling appearing in the vectorial description comes from the self-interaction and the mutual interaction between different vector components. The latter (i.e. the scalar coupling case discussed in Ref.[14]) disregarded the vectorial property of optical field, and hence only one dispersion relation is obtained for the system, and the soliton coupling is between two eigenmodes of the same dispersion branch. The coupling appearing in the scalar description results from the interaction between the modes in the same electric field component. The condition for a stronger coupling between two different modes requires group-velocity matching. As the band structure of the dispersion relation appears in the photonic crystals, the group-velocity matching can be satisfied easily if one chooses band-edge modes, all of which have equal group velocity, i.e. zero. Our results also show that the optical rectification field disappears when the frequency of the fundamental-frequency fields approaches the band-edge of photonic crystals.

References

[1] Hasegawa A and Kodama Y 1995 *Solitons in Optical Communications* (Oxford: Oxford University Press)

[2] Lee S and Ho S T 1993 *Opt. Lett.* **18** 962
Perera S and Sipe J E 2000 *Phys. Rev.* E **62** 5745

[3] Joannopoulos J D, Villeneuve P R and Fan S 1997 *Nature* **386** 143

[4] Li Z Y *et al* 1998 *Phys. Rev. Lett.* **81** 2574
Zhu S Y *et al* 2000 *Phys. Rev. Lett.* **85** 2136

[5] Bang O 1997 *J. Opt. Soc. Am.* B **14** 51

[6] Ablowitz M J, Biondini G and Blair S 2001 *Phys. Rev.* E **63** 046605-1

[7] Huang G X and Velarde M G 1996 *Phys. Rev.* E **54** 3048
Huang G X 2001 *Chin. Phys.* **10** 418

[8] Martorell J *et al* 1997 *Appl. Phys. Lett.* **70** 702
Conti C *et al* 1997 *Phys. Rev. Lett.* **78** 2341
Conti C *et al* 2000 *Phys. Rev. Lett.* **85** 2502

[9] Vlasov R A and Smirnov A G 2000 *Phys. Rev.* E **61** 5808
Andreev A V *et al* 2001 *Phys. Rev.* E **63** 016602

[10] Mingaleev S F and Kivshar Y S 2001 *Phys. Rev. Lett.* **86** 5474
Li S M *et al* 2001 *Acta. Phys. Sin.* **50** 489 (in Chinese)

[11] Sterke C M and Sipe J E 1994 *Prog. Opt.* **XXXIII** 203

[12] Arraf A and Sterke C M 1998 *Phys. Rev.* E **58** 7951
Iizuka T and Kivshar Y S 2000 *Phys. Rev.* E **61** 4491
Iizuka T and Kivshar Y S 2000 *Phys. Rev.* E **62** 4246

[13] Konotop V V 1995 *Phys. Rev.* E **51** R3422

[14] Zhu S H, Cui W N and Huang G X 2002 *Acta. Phys. Sin.* **51** 789 (in Chinese)