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A New Numerical Solution of Fluid Flow in Stratigraphic Porous Media*

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Abstract *A new numerical technique based on a lattice-Boltzmann method is presented for analyzing the fluid flow in stratigraphic porous media near the earth's surface. The results obtained for the relations between porosity, pressure, and velocity satisfy well the requirements of stratigraphic statistics and hence are helpful for a further study of the evolution of fluid flow in stratigraphic media.*

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Key words: fluid flow, stratigraphic porous media, lattice-Boltzmann method

1 Introduction

The study of the fluid flows in stratigraphic porous media near the earth's surface has attracted much attention, due to its wide applications in petroleum resources, gas production, geophysical fluid dynamics, and other related areas.[1,2] Although much work has been done, to our knowledge a satisfactory description for the fluid flows in stratigraphic porous media, like that done in nonporous ones, is still lacking. One of the reasons for this is the complicated physical structure in the flows. In particular, our ability to understand the physical processes in such systems is limited by the lack of a suitable mathematical representation for the geometry. The models proposed up to now are usually restricted to the cases in which the geometry and the boundary conditions are highly simplified and thus idealized. Some approximated analytical solutions[2,3] have been presented, but real systems of porous media tend to have more intricate pore structures and a wider distribution of pore sizes. It is necessary to develop powerful numerical methods to treat such problems.

In recent years, as an efficient numerical tool Lattice Boltzmann method (LBM) has been proposed to investigate the fluid flows with highly complex geometries, such as porous media.[4−8] LBM allows a detailed discretization of the porous geometry and hence one can have an exact simulation of the flows without using any semi-empirical homogenization models. In some sense the LBM may be considered as a "numerical experiment" with an additional advantage because more information about local flow properties can be obtained in comparison with the numerical methods conventionally used. In this work we develop an external LBM to simulate the fluid flow in a stratigraphic porous medium near the earth's surface. The two-dimensional (2D) projection of a 3D flow and a 3D simulation show that our numerical solutions agree well with the analytical result obtained in Ref. [9]

The paper is organized as follows. In Sec. 2, we give a brief introduction for the LBM and describe the simulation frame for the fluid flow in stratigraphic porous media near the earth's surface. Section 3 deals with the scheme for treatment of boundary conditions. The numerical results of our simulations are presented in Sec. 4. The last section contains a summary of our results.

2 Model and Lattice Boltzmann Method

The dynamics of fluid flows in stratigraphic porous media near the earth's surface, as well as the pore fluid dynamics, are associated with the accumulation of sediments on the earth's surface, the thermal and chemical processes determining the rheological properties of the medium beneath the earth's surface, and the actual flow mechanism under the action of gravity. In order to understand this problem, we need an idea of how a fluid flows in a stratigraphic porous medium near the earth's surface. Based on the hypothesis in Ref. [9], the relation between the pressure p of the fluid and the effective pressure of matrix $p_{\rm matrix}$ can be written as

$$p_{\rm matrix} = -(\rho + \triangle\rho)gy - p\,, \qquad (1)$$

where ρ is the fluid density, g is gravity acceleration, y is the positive vertical coordinate in the upward direction, and $\triangle\rho = \rho_{\rm matrix} - \rho$. For simulating the fluid flow in the stratigraphic porous medium we need its equations of motion with corresponding boundary conditions, which

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are obtained by combining Eq. (1) with the LBM, as described as follows.

Generally, the Boltzmann equation with the single relation time approximation can be written as[10]

$$\frac{\partial f}{\partial t}+\vec{\xi}\cdot\nabla f=-\frac{1}{\lambda}(f-f^{\rm eq})\,, \tag{2}$$

where $\vec{\xi}$ is particle velocity, f is particle distribution function, $f^{\rm eq}$ is equilibrium particle distribution function, and λ is relaxation time. Discretizing Eq. (2) in the velocity space $\vec{\xi}$ and using a finite set of discrete velocities $\vec{e}_i$, one obtains

$$\frac{\partial f_i}{\partial t}+\vec{e}_i\cdot\nabla f_i=-\frac{1}{\lambda}(f_i-f_i^{\rm eq})\,. \tag{3}$$

In this work, we use the $D3Q19$ LBM, developed in Ref. [11]. The lattices with nineteen velocities are $e_0=(0,0)$, $e_{1,2},e_{3,4},e_{5,6}=(\pm1,0,0)$, $(0,\pm1,0)$, $(0,0,\pm1)$, and $e_{7,\ldots,10},e_{11,\ldots,14},\ e_{15,\ldots,18}=(\pm1,\pm1,0)$, $(\pm1,0,\pm1)$, $(0,\pm1,\pm1)$. The local equilibrium distribution function in Eq. (2) can be written as

$$f_i^{\rm eq}=t_p\rho\Big\{1+\frac{\vec{e}_{i\alpha}\cdot\vec{u}_\alpha}{c^2}+\frac{\vec{u}_\alpha\cdot\vec{u}_\beta}{2c^2}\Big(\frac{\vec{e}_{i\alpha}\cdot\vec{e}_{i\beta}}{c^2}-\delta_{\alpha\beta}\Big)\Big\}\,, \tag{4}$$

where α and β represent the component of Cartesian coordinates (with implied summation for repeated indices), $c=\delta x/\delta t$ is the lattice speed (where δx and δt are the lattice constant and the time step, respectively), the index p is the square modulus of particle's velocity, and t_p is the corresponding equilibrium distribution for $\vec{u}=0$, which is determined to achieve isotropy of the fourth-order tensor of velocities and Galilean invariance.[11] We take the values of t_p as $t_0=1/3, t_1=1/18,\ t_2=1/36,\ t_3=0$. The density ρ and the velocity u of the fluid are defined by

$$\rho=\sum_i f_i \qquad \text{and} \qquad \vec{u}=\sum_i f_i\vec{e}_i/\rho\,. \tag{5}$$

The lattice Boltzmann equation[12] is obtained by further discretizing Eq. (3) in space $\vec{x}$ and time t,

$$f_i(x+\delta\vec{x}\cdot\vec{e}_i,t+\delta t)-f_i(\vec{x},t)=-\frac{1}{\tau}(f_i-f_i^{\rm eq})\,, \tag{6}$$

where $\tau=\lambda/\delta t$. Using a multiscale technique we can obtain the macroscopic equations for the motion of the fluid in the second-order approximation, which are the continuity equation,

$$\partial_t\rho+\partial_\alpha(\rho\vec{u}_\alpha)=0\,, \tag{7}$$

and the Navier–Stokes equations,

$$\partial_t(\rho\vec{u}_\alpha)+\partial_\alpha(\rho\vec{u}_\alpha\vec{u}_\beta) =-\partial_\alpha(c^2\rho)+\mu\partial_\beta[\partial_\beta(\rho\vec{u}_\alpha)+\partial_\alpha(\rho\vec{u}_\beta)]\,, \tag{8}$$

where the viscosity μ is given by

$$\mu=\frac{(2\tau-1)}{6}c^2\delta t\,. \tag{9}$$

Without loss of generality, we set $c=1$.

Combining our previous discussions in Ref. [7], we obtain the macroscopic dynamic equations for the fluid in a stratigraphic porous medium near the earth's surface as

$$\frac{\partial(\phi\rho)}{\partial t}+\nabla\cdot(\phi\rho\vec{u})=0\,, \tag{10}$$

$$\phi(\vec{u}-\vec{u}_{\rm matrix})=-\frac{k}{\mu}(\nabla p+\rho\vec{g})\,, \tag{11}$$

$$\nabla\cdot\vec{u}_{\rm matrix}=-\frac{1}{\mu}p_{\rm matrix}\,. \tag{12}$$

In the above equations, ϕ represents porosity. By introducing the characteristic length, pressure, and time scales

$$L=\sqrt{U\mu_0/\triangle\rho g},\quad P=\triangle\rho gL,\quad T=L/U\,, \tag{13}$$

we define the non-dimensional quantities

$$y'=y/L,\quad t'=t/T,\quad \phi'=\phi/\Phi\,,$$
$$u'=u/\Phi U,\quad p'=p/P\,, \tag{14}$$

where U is the velocity of boundary, and Φ is the porosity on the upper boundary. The boundary conditions can be written as

$$y'=0:\qquad \vec{u}\,'=0,\qquad \vec{u}'_{\rm matrix}=0\,, \tag{15}$$

$$y'=Ut:\qquad \phi'=0.176,\qquad p'=0\,. \tag{16}$$

The initial conditions are

$$\vec{u}'=0,\qquad \vec{u}'_{\rm matrix}=0,\qquad p=0\,. \tag{17}$$

The values of other parameters are $U=10^{-6}$ m/s, $\mu=0.5\times10^{21}$ kg/(m·s), $k=0.5\times10^{-14}$ m^2, $\rho=10^3$ kg/m^3, $\rho_{\rm matrix}=1.6\times10^3$ kg/m^3, $g=9.8$ m/s^2.

3 Analysis of Boundary Conditions

The boundary conditions[13] are extremely important to obtain an accurate results for the numerical simulation. Bouncing-back boundary condition[14] is a primary method in the lattice Boltzmann simulation and has been proved to be have first-order accuracy. More accurate boundary conditions have been proposed in the past few years.[15−18] In this work we use the scheme for the treatment of the boundary condition by considering a curved boundary lying between the nodes of the equidistant lattice of space $\Delta\delta x$ for a 2D projection of a 3D body, as shown in Fig. 1.

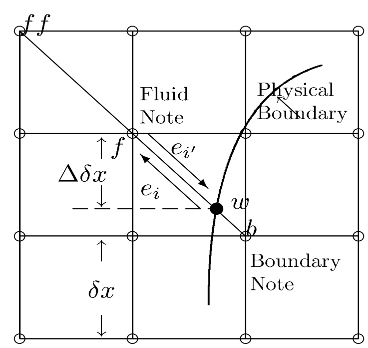


Fig. 1 Layout of the regularly spaced lattice and curved wall boundary.

The lattice node on the solid and fluid side are denoted by $\vec{x}_b$ and $\vec{x}_f$ respectively. We assume $\vec{e}_i = \vec{x}_b - \vec{x}_f$ and $\vec{e}_{i'} = -\vec{e}_i$. The filled small circle at $\vec{x}_w$ is the intersection with the physical boundary on the link between $\vec{x}_b$ and $\vec{x}_f$. The fraction of an intersected link in the fluid is Θ, defined by

$$\Theta = \frac{|\vec{x}_f - \vec{x}_w|}{|\vec{x}_f - \vec{x}_b|}, \qquad 0 \le \Theta \le 1\,. \tag{18}$$

After a collision step, the distribution functions at $\vec{x}_f$ and t are known as the following streaming step,

$$f_i(\vec{x}_f, t + \delta t) = f_i(\vec{x}_{ff}, t)\,, \tag{19}$$

while $f_{i'}(\vec{x}_f)$ can be obtained by

$$f_{i'}(\vec{x}_f, t + \delta t) = f_{i'}(\vec{x}_b, t)\,. \tag{20}$$

However, the distribution function $f_{i'}(\vec{x_b}, t)$ at boundary node is unknown. According to Ref. [16], we assume that the $f_{i'}(\vec{x}_b, t)$ can be satisfied with the following linear interpolation formula,

$$f_{i'}(\vec{x}_b, t) = (1-\chi) f_i(\vec{x}_f, t) + \chi f_i^*(\vec{x}_b, t) + 6\alpha_i \vec{e}_{i'} \cdot \vec{u}_w\,, \tag{21}$$

where $\vec{u}_w = \vec{u}(\vec{x}_w, t)$ is the velocity at the physical boundary and χ is a parameter. f_i^* is a fictitious equilibrium distribution function given by

$$f_i^*(\vec{x}_b, t) = t_p \rho \Big\{ 1 + \vec{e}_{i\alpha} \cdot \vec{u}_{\alpha bf} + \frac{\vec{u}_{\alpha f} \cdot \vec{u}_{\beta f}}{2} \times (\vec{e}_{i\alpha} \cdot \vec{e}_{i\beta} - \delta_{\alpha\beta}) \Big\}\,, \tag{22}$$

where $\vec{u}_{\alpha f} = \vec{u}_\alpha(\vec{x}_b, t)$ and $\vec{u}_{\beta f} = \vec{u}_\beta(\vec{x}_b, t)$ are the fluid velocity near the solid and $\vec{u}_{bf}$. In Ref. [18], Fillipova and Hanel proposed

$$\vec{u}_{bf} = (\Theta - 1)\vec{u}_f/\Theta + \vec{u}_w/\Theta \quad \text{and}$$
$$\chi = (2\Theta - 1)/\tau \quad \text{for} \quad \Theta \ge \frac{1}{2}\,, \tag{23}$$

and

$$\vec{u}_{bf} = \vec{u}_f \quad \text{and}$$
$$\chi = (2\Theta - 1)/(\tau - 1), \quad \text{for} \quad \Theta \le \frac{1}{2}\,, \tag{24}$$

in order to obtain a second-order scheme for the "slow flow". Mei *et al.*[19] improved the stability of the scheme by replacing Eq. (23) by

$$\vec{u}_{bf} = \vec{u}_{ff} \quad \text{and}$$
$$\chi = (2\Theta - 1)/(\tau - 2), \quad \text{for} \quad \Theta \le \frac{1}{2}\,. \tag{25}$$

They have used this improved technique to study several flow problems such as the fully developed flow in a square duct, 3D lid-driven cavity flows, fully developed flows inside a circular pipe and a uniform flow over a sphere to demonstrate its accuracy and robustness.

4 Computational Results and Discussion

In our simulation the computational domain is chosen to be $x \times y \times z = 30 \times 500 \times 30$ lattice units. A very good convergence of the numerical solution is achieved.

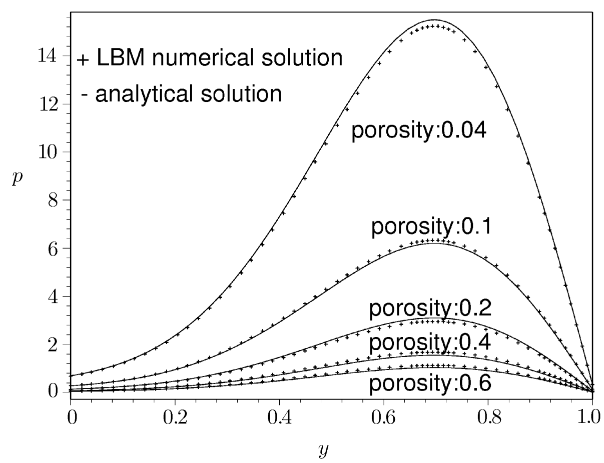


Fig. 2 A comparison between the analytical solution with numerical solution in 2D at $t = 3.6$.

Figure 2 shows a comparison between our numerical solution in 2D at $t = 3.6$ and the result from an analytical solution given in Ref. [9]. One sees that both results agree very well.

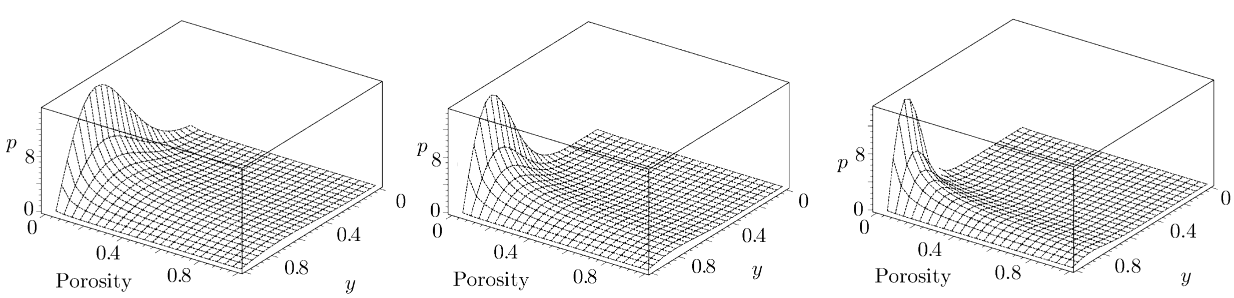
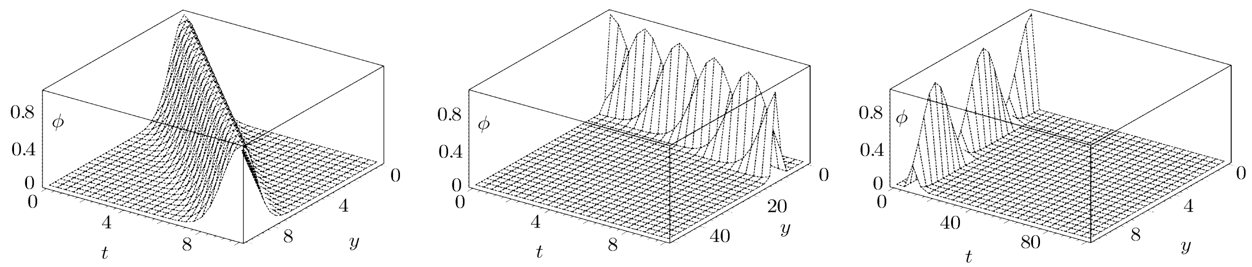


Fig. 3 The relation between effective pressure, porosity, and vertical depth at $t = 3.6, 5.2$, and 7.6.

Shown in Fig. 3 are the numerical solutions for the porosity and effective pressure (i.e. the pressure in the matrix) for three different time values. In this case, it is clear that the result for the porosity almost remains steady as the vertical coordinate and the time increase, but the qualitative difference between the results is appreciable. From Fig. 3(a) we see that there is a slow growth for the effective pressure as the vertical depth of stratum increases. With the geologic age increasing, the numerical values of the effective pressure near the earth surface almost remain steady but it shows a sharp growth in deep medium as the vertical depth of stratum increases (see Figs. 3(b) and 3(c)).



(a) $0\le y\le 10$ and $0\le t\le 10$ (b) $0\le y\le 50$ and $0\le t\le 10$ (c) $0\le y\le 10$ and $0\le t\le 100$

Fig. 4 The numerical value of porosity (ϕ) decreases sharply while t and y increase.

Figure 4 displays the porosity for three difference parameters: (a) $0 \le y \le 10$ and $0 \le t \le 10$, (b) $0 \le y \le 50$, and $0 \le t \le 10$, and (c) $0 \le y \le 10$ and $0 \le t \le 100$. The numerical result shows that the porosity decreases sharply when t and y increase. This is consistent with the hydrostatic and lithostatic principles.[9]

5 Conclusion

We have proposed a new numerical technique based on a lattice Boltzmann method for analyzing the fluid flow in stratigraphic porous media near the earth's surface. A series of results of the fluid flow through stratigraphic porous media are constructed. The properties of the solutions with the dimensionless time t are described. The simplicity of our model will make it possible to determine the porosity, pressure, and velocities for particular geological conditions using simple numerical calculations. The method and the results presented here are helpful for a further study for the evolution of the fluid flows in stratigraphic media.

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