Wireless Communications

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Brief Introduction

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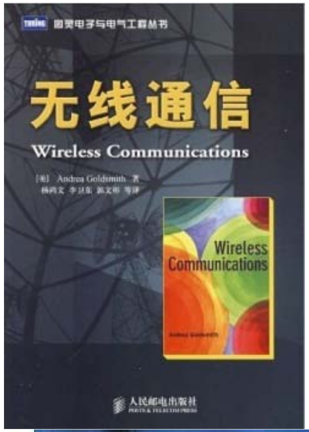
Related Textbooks

- A. Goldsmith, ***Wireless Communications***



- D.N.C. Tse and P. Viswanath,

Fundamentals of Wireless Communications

Grades

- **No Test!**

- **No Homework !**

- **Course Project + Presentation!**

Overview of the Course

- Wireless communication systems
  - Flexibility to support roaming
  - Limitations: Geographical coverage, transmission rate, and transmission errors
- Wireless communication technology
  - Channel Modeling
  - Capacity
  - Multiuser communications
  - MIMO
- Other topics
  - Millimeter wave communications
  - Delay constrained communications

Outline

- Basics
- Architectures of wireless networks
  - Cellular network architecture
  - Satellite systems
  - Wireless LAN/PAN
  - Ad hoc networks
  - Sensor network
- Background
  - FDMA/TDMA/CDMA
  - Connection setup

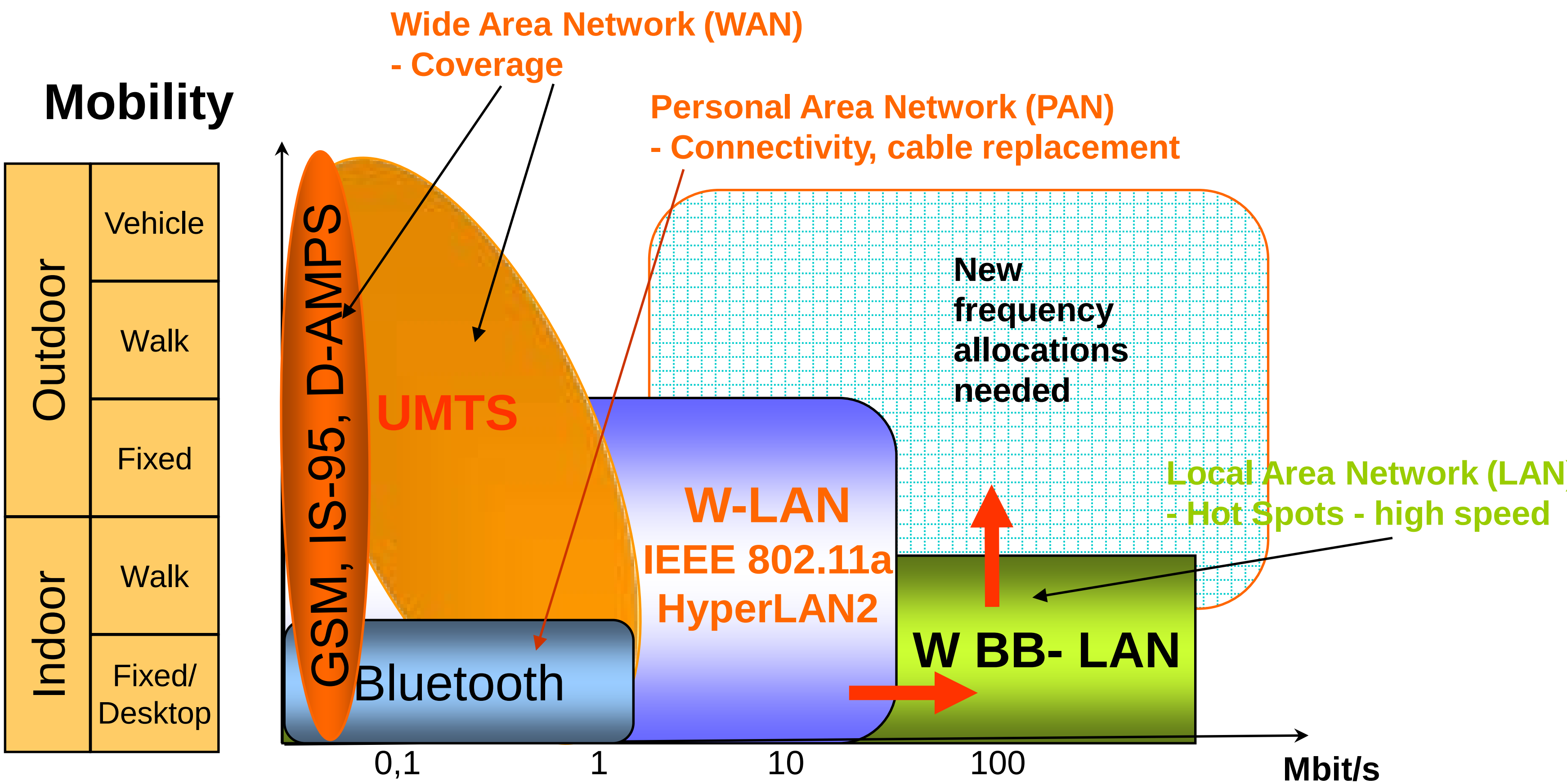
Terminology

- Base station (BS) or Access Point (AP): information distribution center for all mobile devices (MDs) within its signaling coverage area.
- Uplink (Reverse link): Radio channels from an MD to its serving BS/AP.
- Downlink (Forward link): Radio channels from the BS/AP to the MDs.

Current Situation

- Wireless coverage in most highly populated areas
- Insufficient coverage, low system capacity, and low bandwidth
- Numerous overlapping, but incompatible wireless system as the obstacles for inter-system roaming.

Wireless Access: Range of Operation of Different Techniques



Reference to S.R. Treves (Alcatel) presentation in Mobicom'01.Rome Italy

Layer Architecture

- Physical layer
  - Transmission over the propagation channels
  - Modulations, coding/decoding, interferences, multiplexing etc.
- Link layer
  - Radio resource management such as power control, rate control, and error control.
  - Network resource management such as call admission control and service scheduling
- Networking layer
  - Handoff management
  - Location management
  - Traffic management

Influence of Mobile Communication to the Layer Model

Application layer	– service location – new applications, multimedia – adaptive applications
Transport layer	– congestion and flow control – quality of service
Network layer	– addressing, routing, device location – hand-over
Data link layer	– authentication – media access – multiplexing – media access control
Physical layer	– encryption – modulation – interference – attenuation – Frequency

Effects of Portability

- Power consumption
  - Limited computing power, low quality displays, small disks due to limited battery capacity
    - CPU: power consumption
    - Transceiver power consumption
  - Loss of data
  - Higher probability, has to be included in advance into the design (e.g., defects, theft)
- Limited user interfaces
  - compromise between size of fingers and portability
  - integration of character/voice recognition, abstract symbols
- Limited memory
  - limited value of mass memories with moving parts
  - flash-memory as alternative

Difference from Wired Networks

- Higher loss-rates due to interference
  - emissions of, e.g., engines, lightning
- Restrictive regulations of frequencies
  - frequencies have to be coordinated, useful frequencies are almost all occupied
- Lower transmission rate
- Higher delays, higher jitter
  - connection setup time with GSM in the second range, several hundred milliseconds for other wireless systems
- Lower security, simpler active attacking
  - radio interface accessible for everyone, base station can be simulated, thus attracting calls from mobile phones
- Always shared medium: secure access mechanisms

Wireless Technology and Associated Characteristics

- Cellular Networks
- Wireless LAN/MAN/PAN/Bluetooth
- Satellite Based GPS
- Home Networking
- Ad Hoc Networks
- Sensor Networks
- LTE, UWB, UMB, WiMAX, etc …
- The Internet of Things
- …..

First Generation Wireless Systems

1970s	Developments of radio and computer technologies for 800/900 MHz mobile communications
1976	WARC (World Administrative Radio Conference) allocates spectrum for cellular radio
1979	NTT (Nippon Telephone & Telegraph) introduces the first cellular system in Japan
1981	NMT (Nordic Mobile Telephone) 900 system introduced by Ericsson Radio System AB and deployed in Scandinavia
1984	AMPS (Advanced Mobile Phone Service) introduced by AT&T in North America

Second Generation Wireless Systems

1982	CEPT (Conference Europeenne des Post et Telecommunications) established GSM to define future Pan-European Cellular Radio Standards
1990	Interim Standard IS-54 (USDC) adopted by TIA (Telecommunications Industry Association)
1990	Interim Standard IS-19B (NAMPS) adopted by TIA
1991	Japanese PDC (Personal Digital Cellular) system standardized by the MPT (Ministry of Posts and Telecommunications)
1992	Phase I GSM system is operational
1993	Interim Standard IS-95 (CDMA) adopted by TIA
1994	Interim Standard IS-136 adopted by TIA
1995	PCS Licenses issued in North America
1996	Phase II GSM operational
1997	North American PCS deploys GSM, IS-54, IS-95
1999	IS-54: North America, IS-95: North America, Hong Kong, Israel, Japan, China, etc, GSM: 110 countries

Third Generation of Mobile Telecommunications

1985	Future Public Land Mobile Telecommunication System (FPLMTS) come up by ITU (International Telecommunication Union)
1996	Changed name to International Mobile Telecom System-2000 (IMT-2000) Enhanced Voice-Data Optimized (Ev-DO) was come up
1998	The first pre-commercial 3G network was launched by NTT DoCoMo in Japan, branded as FOMA
2001	FOMA is available as a teat of W-CDMA UMTS (Universal Mobile Telecommunications System) based W-CDMA is opened in Europe
2002	The CDMA2000 system is standardized by 3GPP2 (3rd Generation Partnership Project 2) The CDMA-based 1xEV-DO technology in South Korea
2006	Fast low-latency access with seamless handoff orthogonal frequency division multiplexing (Flash-OFDM) was developed
2008	Evolved HSPA (HSPA+) was released

Fourth Generation of Mobile Telecommunications

2006	The pre-4G systems Mobile WiMAX occurred in South-Korea
2008	International Mobile Telecommunications Advanced (IMT-Advanced) specification was specified by ITU-R (The International Telecommunications Union-Radio communications sector)
2009	The technology proposals were submitted to the International Telecommunication Union (ITU) as 4G candidates
2010	First-release versions of Mobile WiMAX and LTE (Long Term Evolution) was recognized by ITU-R
2011	Mobile WiMAX Release 2 (also known as *WirelessMAN-Advanced or IEEE 802.16m'*) and LTE Advanced (LTE-A) are IMT-Advanced compliant backwards compatible versions LTE-Advanced was standardized by 3GPP
2012	4G systems fully compliant with IMT Advanced were standardized
2013	China-Mobile, China-Telecom, China-Unicom got the licence of TD-LTE

5th Generation Wireless System

2008	NASA (National Aeronautics and Space) partnered with Geoff Brown and Machine-to-Machine Intelligence (M2Mi) Corp to develop 5G communications technology The South Korean IT R&D program of "5G mobile communication systems based on beam-division multiple access and relays with group cooperation" was formed
2012	The UK Government announced the setting up of a 5G Innovation Centre at the University of Surrey
2013	ITU-R Working Party 5D (WP 5D) started two study items Samsung Electronics stated that they have developed the world's first "5G" system
2014	NTT DoCoMo start testing 5G mobile networks with Alcatel Lucent, Ericsson, Fujitsu, NEC, Nokia and Samsung
By now	5G does not describe any particular specification in any official document published by any telecommunication standardization body

Internet

Early 1960s	Research into packet switching started
Early 1970s	The ARPANET led to the development of protocols for internetworking.
1969	The first two nodes were interconnected
1974	RFC 675 used the term internet as a shorthand for internetworking and later RFCs repeat this use.
1982	the Internet Protocol Suite (TCP/IP) was standardized
Late 1980s	Commercial Internet service providers (ISPs) began to emerge The Internet started a rapid expansion to Europe and Australia
Early 1990s	The Internet started a rapid expansion to Asia The Internet was fully commercialized in the U.S.
Mid 1990s	The rise of near instant communication by email, instant messaging, Voice over Internet Protocol (VoIP) "phone calls", two-way interactive video calls, and the World Wide Web
2011	The estimated total number of Internet users was 2.095 billion (30.2% of world population)

Mobile Internet

1971	ALOHAnet connected the Hawaii islands with a UHF wireless packet network
1985	The first access to the mobile web was commercially offered in Finland
1991	NCR Corporation with AT&T Corporation invented the precursor to 802.11 intended for use in cashier systems
1996	The Australian radio-astronomer John O'Sullivan developed a key patent used in Wi-Fi as a by-product in a CSIRO research project
1999	The first commercial launch of a mobile-specific browser-based web service was in Japan when i-mode was launched by NTT DoCoMo The Wi-Fi Alliance formed as a trade association to hold the Wi-Fi trademark
2009	14 technology companies agreed to pay CSIRO $250 million for infringements on CSIRO patents
2012	Approximately 10.5% of all Web traffic occurs through mobile devices

Internet and Mobile Internet

- Impact of Mobile Internet
- 零售业　　　　批发业
- 制造业　　　　广告业
- 新闻业　　　　通信业
- 物流业　　　　酒店业/旅游业
- 餐饮业　　　　金融业
- 保险业　　　　医疗业
- 教育业　　　　电视业
- 电影业　　　　出版业
- 垄断行业 ……

Fundamentals of Cellular Systems

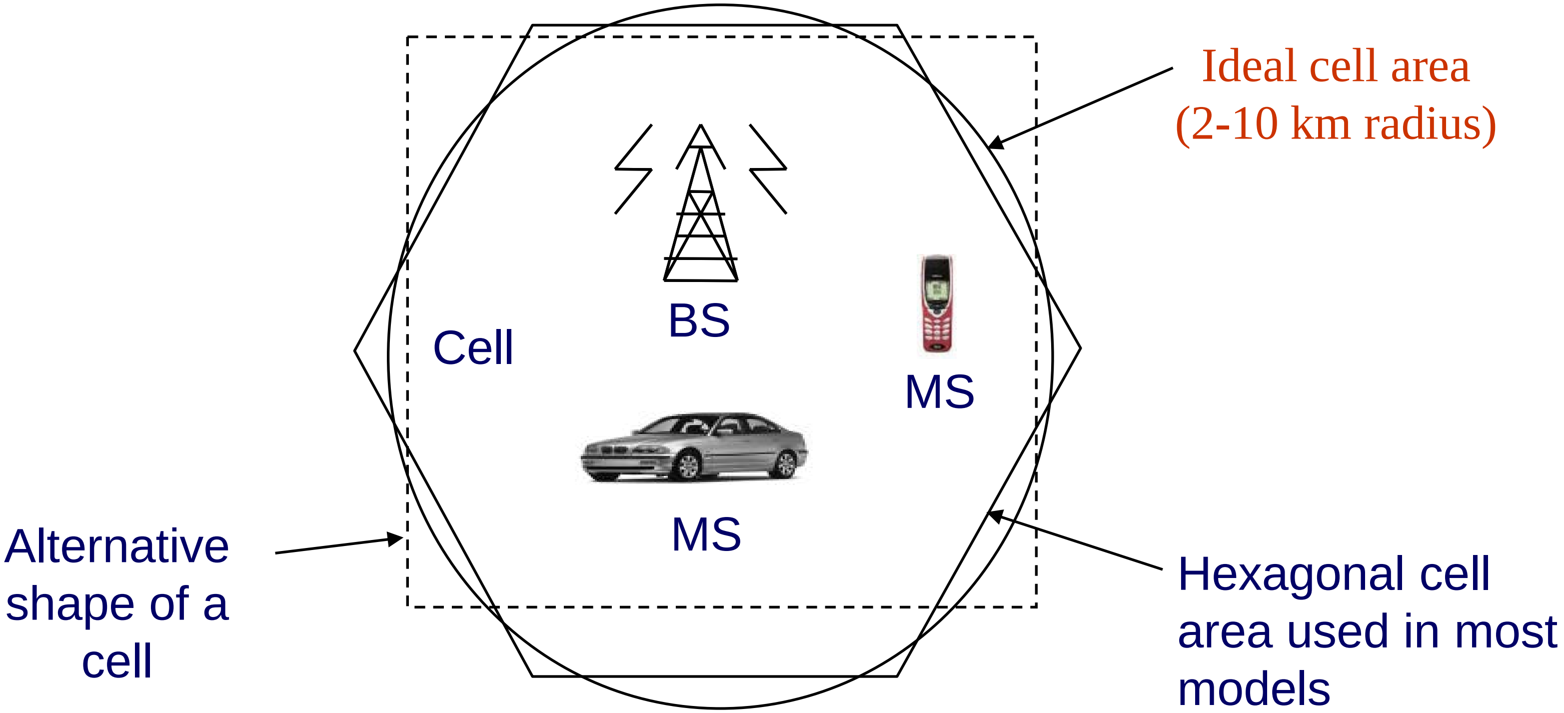
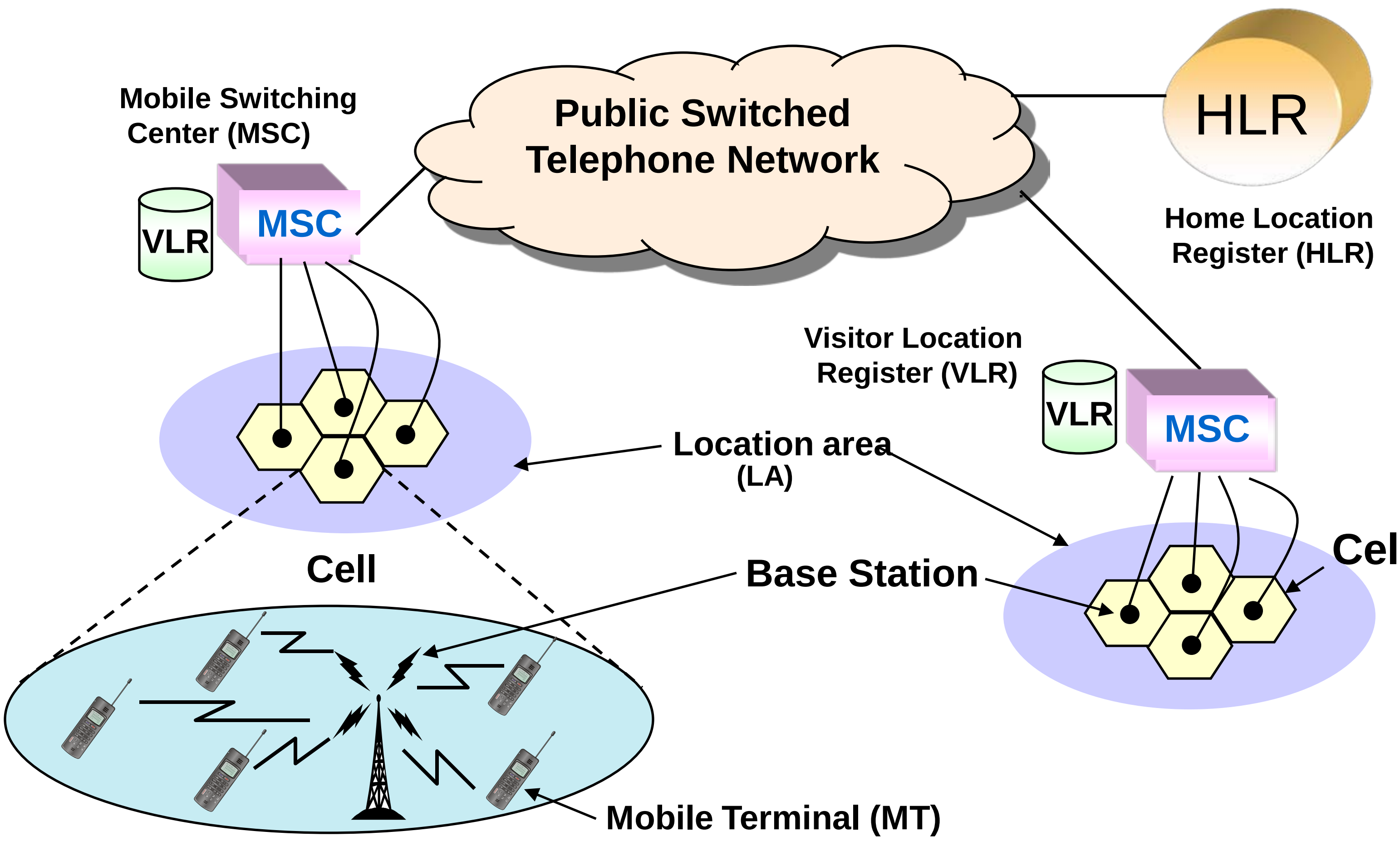


Illustration of a cell with a mobile station and a base station

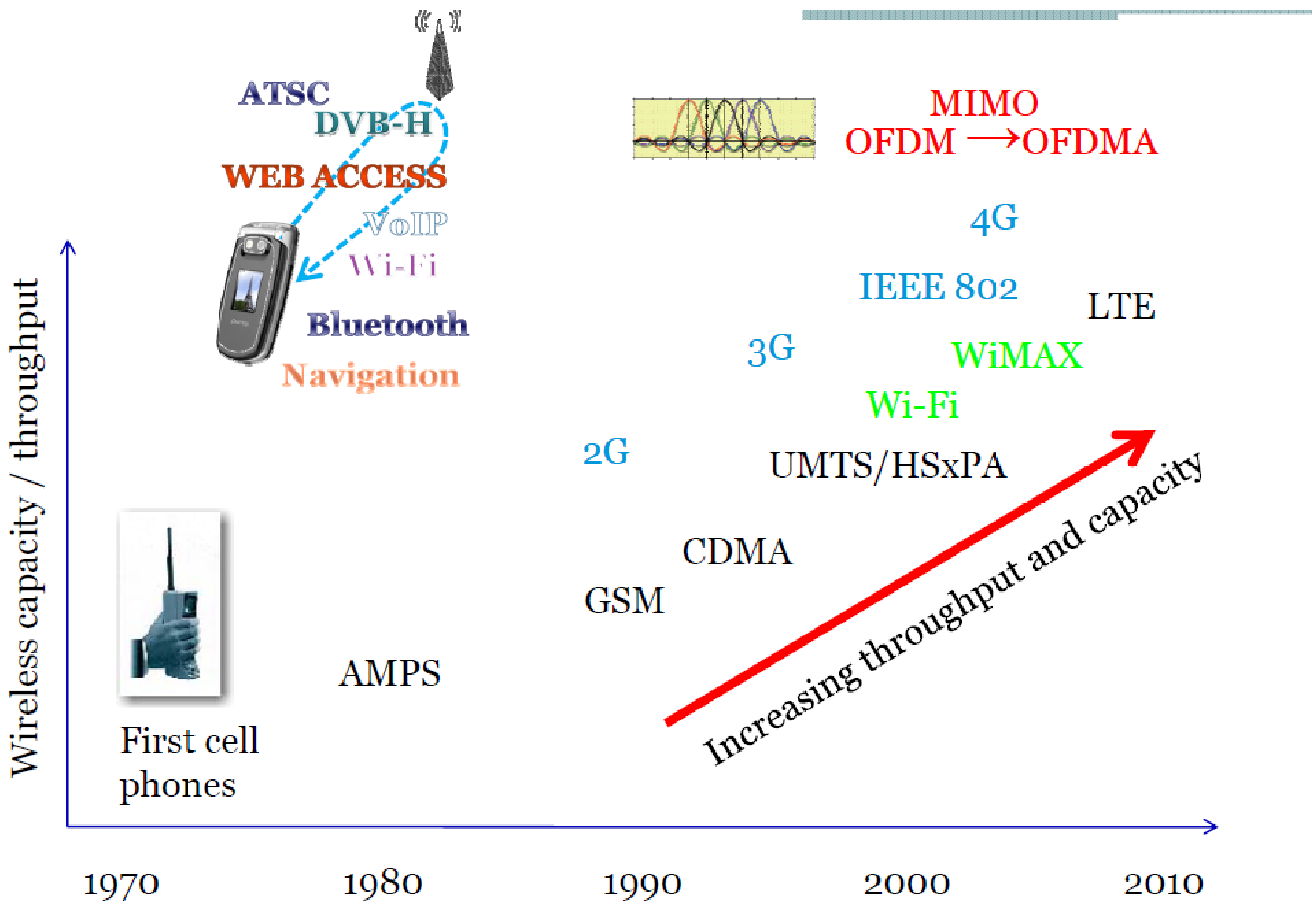
Basic Architecture of Current Wireless Systems



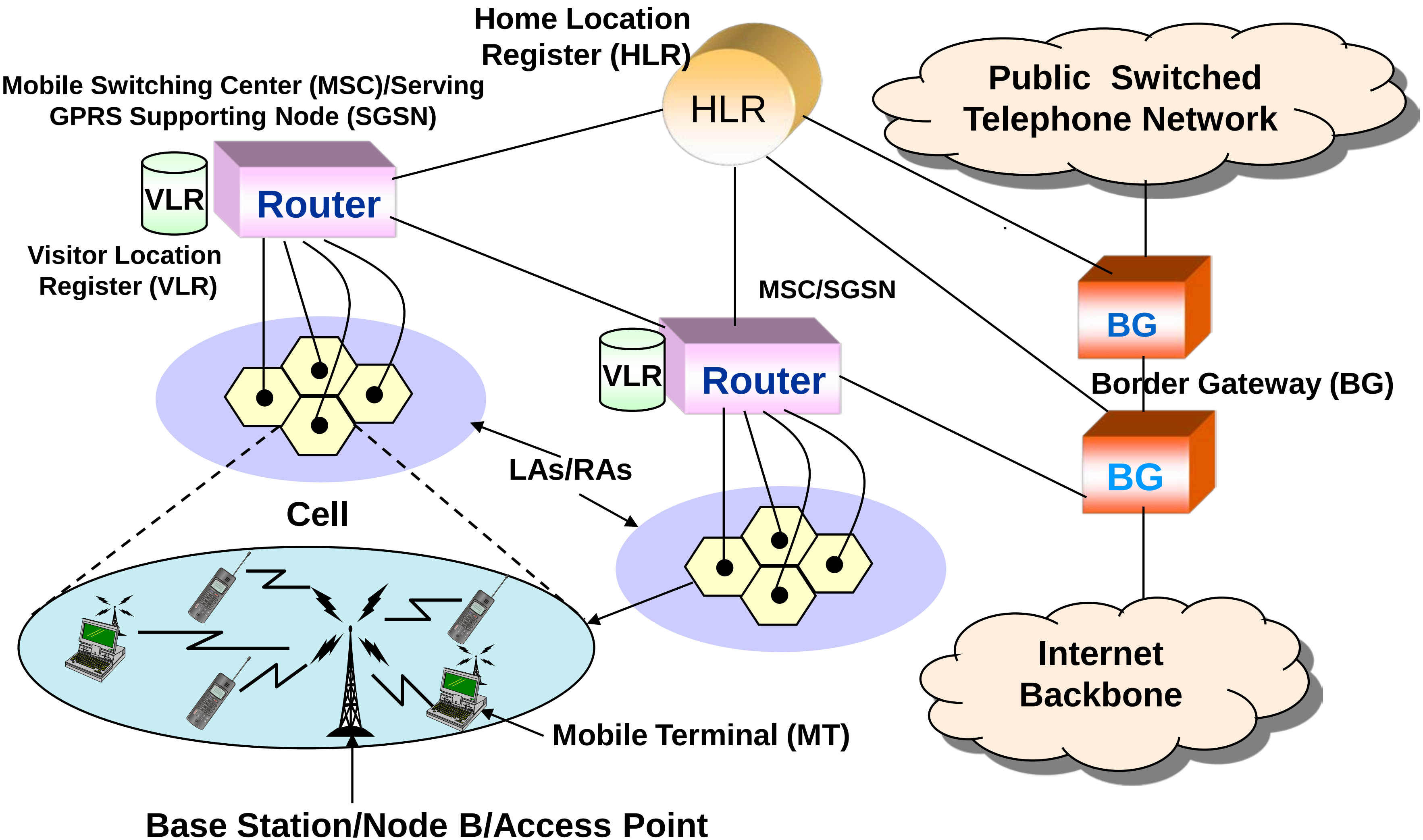
End User needs

- Instant access to information services
  - Anytime, Any Place, Any Device
- Personalized: “My Service, My Applications”
- Location-based: Relevant to wherever I am
- e-Commerce, m-Commerce
- Always On

Technology evolution



Network Architecture of All-IP Wireless Networks



Motivation for 4G Systems

- Increase bandwidth up to 100 Mbps.
- Smooth transition from existing systems
- Make use of existing wireless infrastructure, even privately owned WLANs.
- Roaming has the key role in the success of GSM. How about inter-system roaming?
- Serve the subscribers with required QoS

What's new in 4G

- Entirely packet-switched networks
- All elements are digital
- Higher bandwidths to provide multimedia services at lower cost (~~ 100Mbit/s)
- High network security (security layer)

"Mobile multimedia to all at the same cost as fixed telephony today"

What will be 5G?

- **Faster Speed (10 G+!!)**
- **More Equipment (100 billion+!!)**
- **Lower Latency (1 ms!)**

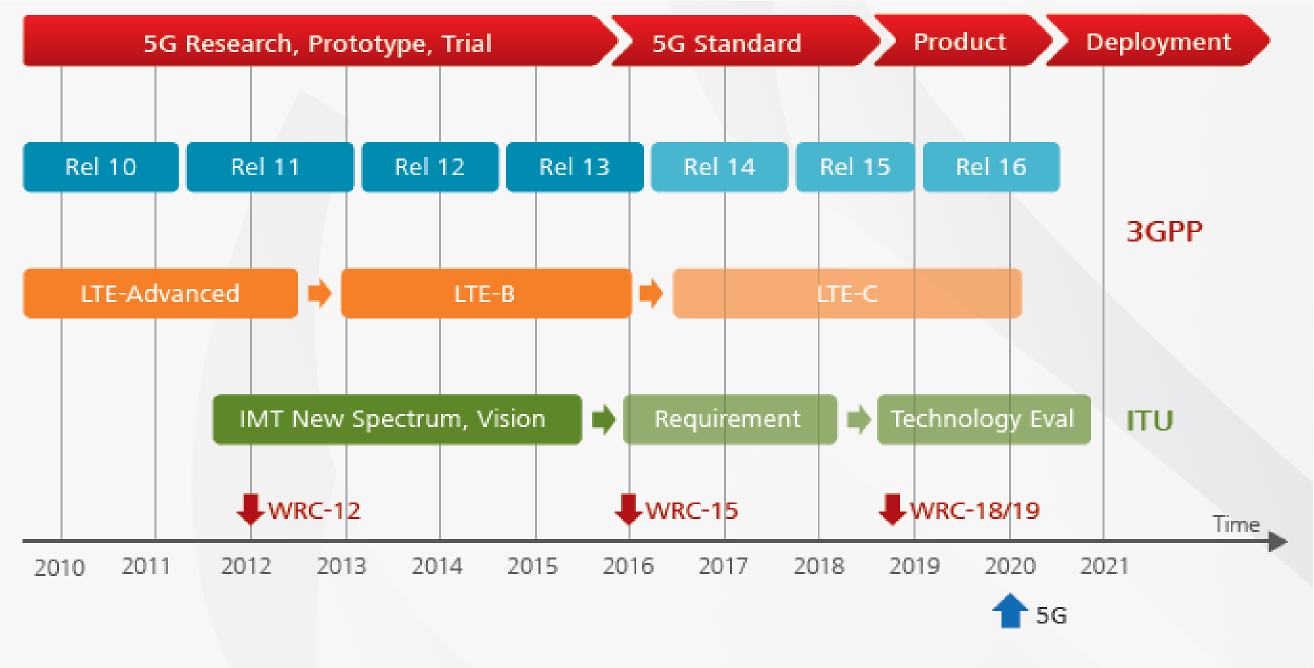
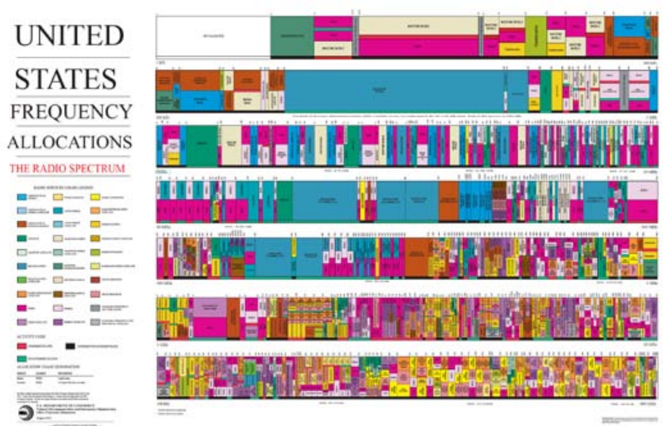
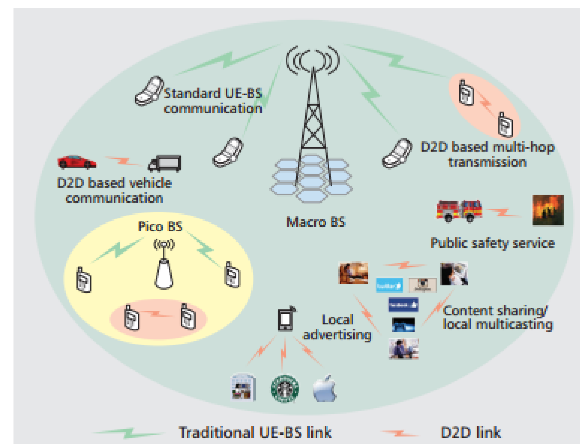


Figure 3 5G roadmap and timeline

What will be 5G?

- **Techniques:**
  - ➢ **Massive MIMO**
  - ➢ **Millimeter Wave**
  - ➢ **D2D, etc.**





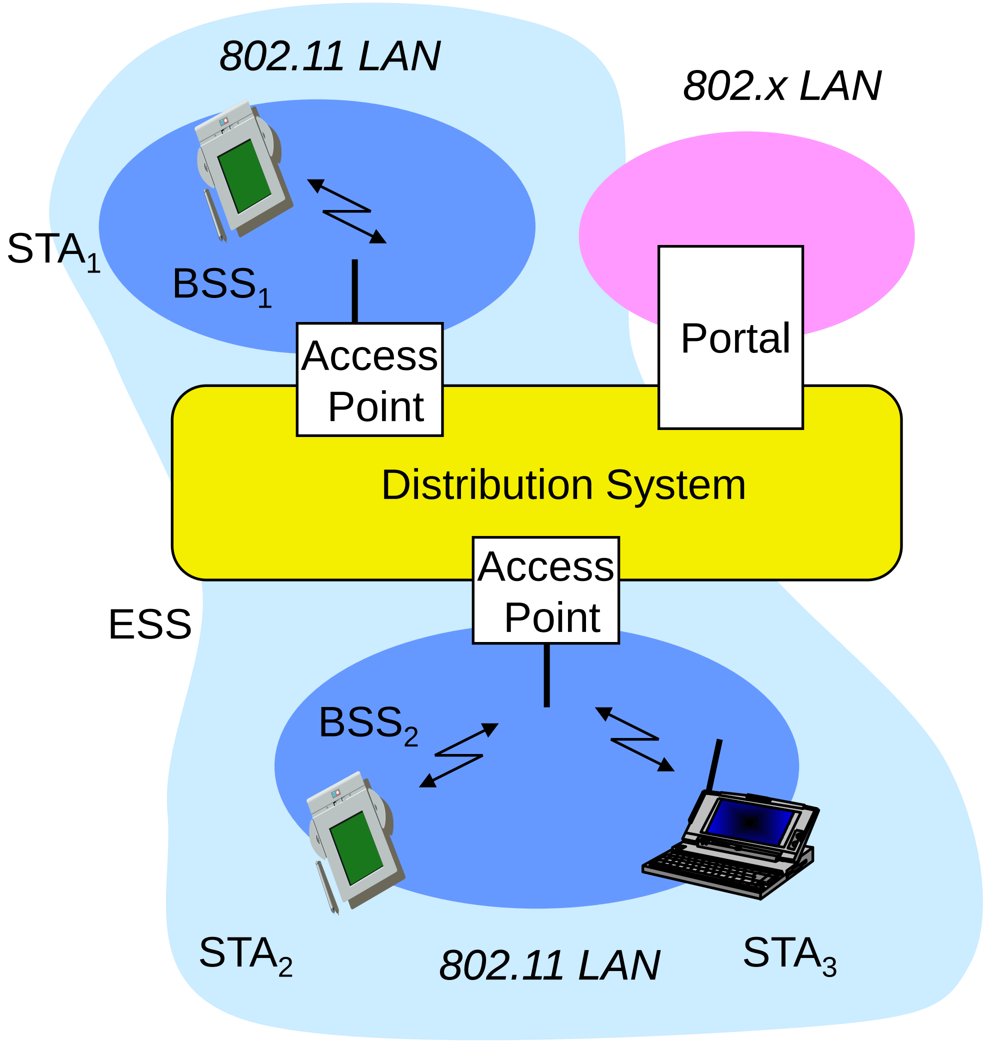
Outline

- Basics
- Architectures of wireless networks
  - Cellular network architecture
  - Satellite systems
  - Wireless LAN/PAN
  - Ad hoc networks: MANET
  - Sensor network
- Background
  - FDMA/TDMA/CDMA
  - Connection setup

Satellite Systems

- Traditional Applications
  - Weather satellite
  - Radio and TV broadcasting
  - Military satellites
- Telecommunication Applications
  - Global telephone connections
  - Backbone for global network
  - GPS

WLAN--Infrastructure Network

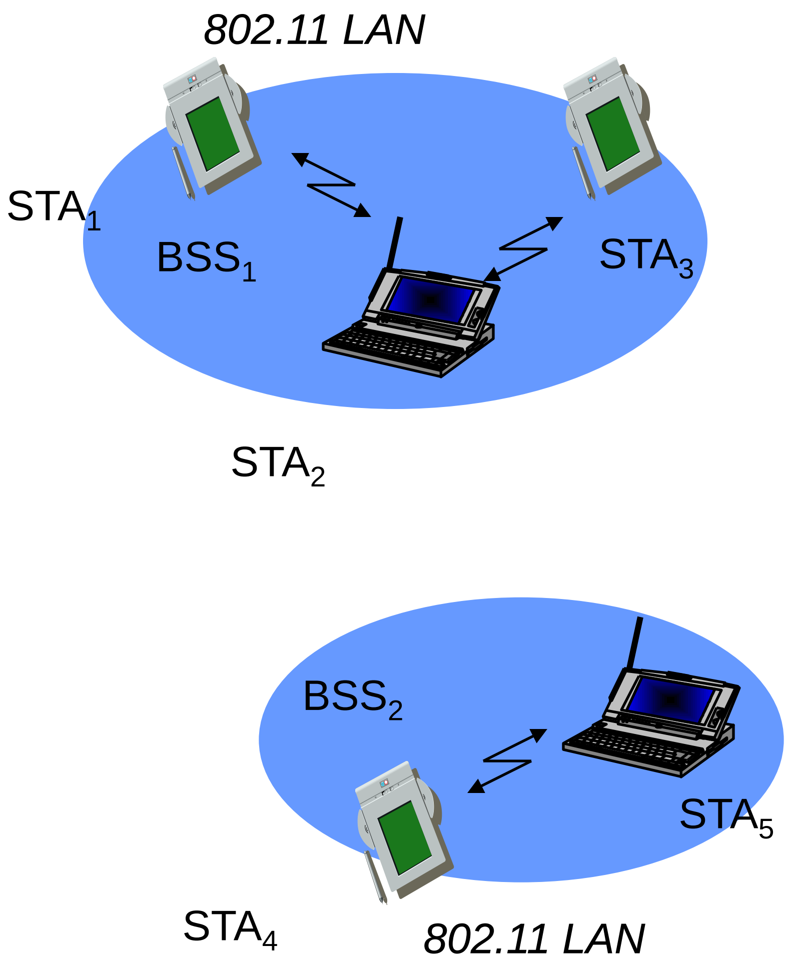


- Station (STA)
  - terminal with access mechanisms to the wireless medium and radio contact to the access point
- Basic Service Set (BSS)
  - group of stations using the same radio frequency
- Access Point
  - station integrated into the wireless LAN and the distribution system
- Portal
  - bridge to other (wired) networks
- Distribution System
  - interconnection network to form one logical network (EES: Extended Service Set) based on several BSS

WLAN -- 802.11

- Advantages
  - very flexible within the reception area
  - (almost) no wiring difficulties (e.g. historic buildings, firewalls)
  - more robust against disasters like, e.g., earthquakes, fire - or users pulling a plug...
  - Ad-hoc networks without previous planning possible
- Disadvantages
  - typically lower bandwidth compared to wired networks (1-54 Mbit/s)
  - many proprietary solutions, especially for higher bit-rates, standards take their time (e.g. IEEE 802.11)
  - products have to follow many national restrictions if working wireless, it takes a very long time to establish global solutions.

Ad hoc Network--802.11



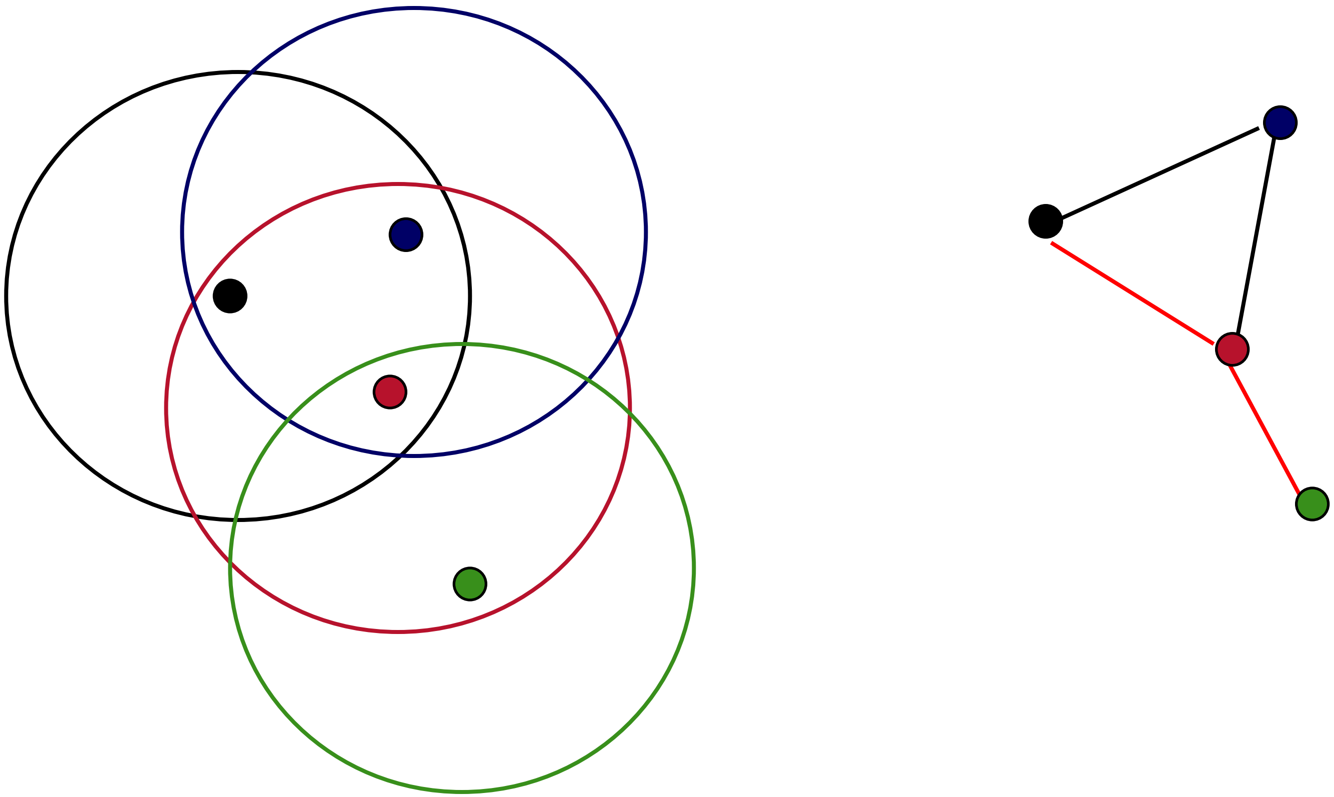
- Direct communication within a limited range
  - Station (STA): terminal with access mechanisms to the wireless medium
  - Basic Service Set (BSS): group of stations using the same radio frequency.
  - Sometimes called Independent BSS (IBSS) mode

What is Mobile Ad hoc Network (MANET)?

- Mobile users with (compatible) wireless communication devices to set up a possible short-lived networks--for a specific need/purpose!
- Without (necessarily) using a pre-existing infrastructure
- Routes between nodes may potentially contain multiple hops

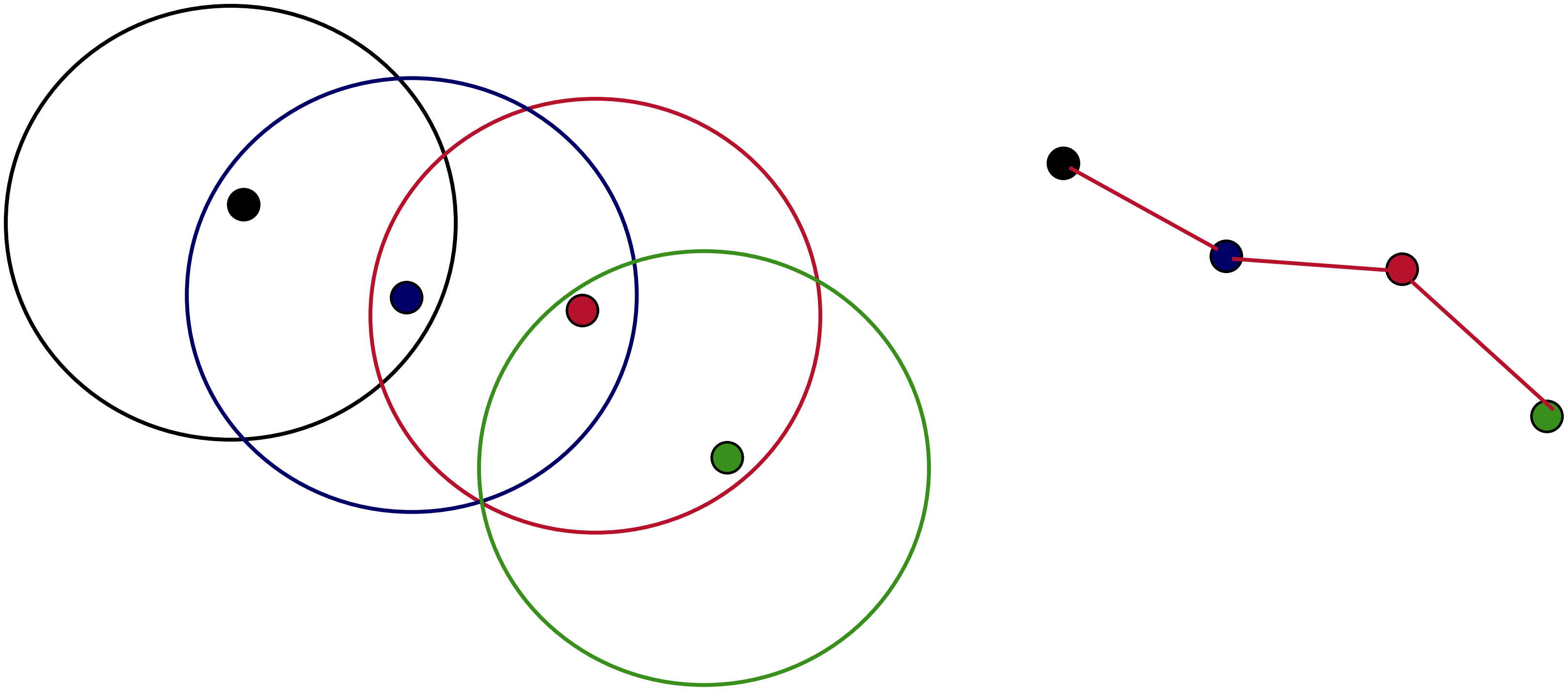
Model of Operation

- Assumption: the nodes are using IP, and they have IP addresses that are assigned by some means.



MANET--Route Changes

- Mobility cause route changes



Challenges in MANET

- Limited wireless transmission range
- Broadcast nature of the wireless medium
  - Hidden terminal problem
- Packet losses due to transmission errors
- Mobility-induced route changes
- Mobility-induced packet losses
- Battery constraints
- Potentially frequent network partitions
- Ease of snooping on wireless transmissions (security hazard)

Comparison of Ad Hoc and Infrastructure Network

- Scalability
  - Ad Hoc: (1) expansion is limited to the coverage of the radio transmitter and receiver for single-hop networks; (2) No simple way to scale up the network coverage or capacity; (3) The potential coverage of the network is increased, but the traffic handling capacity remains the same; (4) use proxy servers with a wireless connection to the backbone wired networks.
  - Infrastructure network: by increasing the number of base stations or access points, the coverage and the capacity will be increased. It is used for *wide area coverage and for variable traffic loads*.

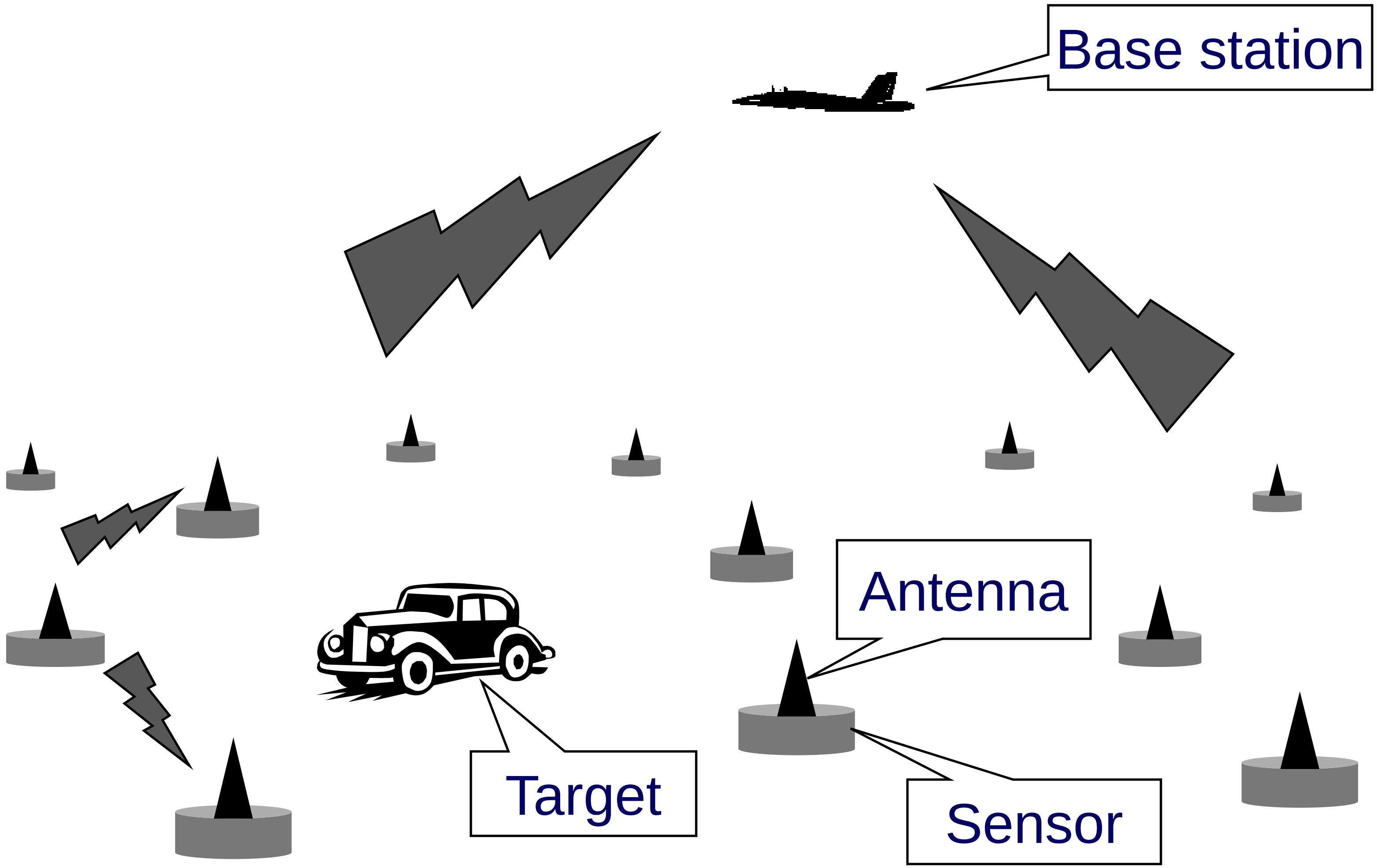
Comparison of Ad Hoc and Infrastructure Network (2)

- Flexibility
  - Ad Hoc network: flexible and can be se up instantly, it is used for *temporary applications.*
  - Infrastructure network: time-consuming and expensive for good design.
- Controllability: time synchronization, power control, authentication and so on.
  - Ad hoc: requires complicated structures demanding changes in all terminals.
  - Infrastructure: *all features are implemented* in BSs or APs.
- Routing complexity
  - Ad hoc: requires each terminal to monitor the existence of others and be able to connect and route messages, needs complicated algorithms.
  - Infrastructure: the *problem does not exist*.

Comparison of Ad Hoc and Infrastructure Network (3)

- Coverage
  - Ad hoc: the maximum distance between two terminals is the range of coverage.
  - Infrastructure: the *maximum distance between two terminals is twice the range of a single wireless terminal* because they communicate through BSs or APs.
- Reliability
  - Ad hoc (s.s. WLAN for military): *single failure point does not exist*.
  - Infrastructure: single failure point networks.
- Store and forward delay and media usage efficiency
  - Ad hoc: information transmitted only once, there is no store and forward procedure for single-hop networks. However, multi-hop networks depend on the topology and number of hops.
  - Infrastructure: information is transmitted twice, first to BS/AP, then from BS/AP to terminals.

Wireless Sensor Networks



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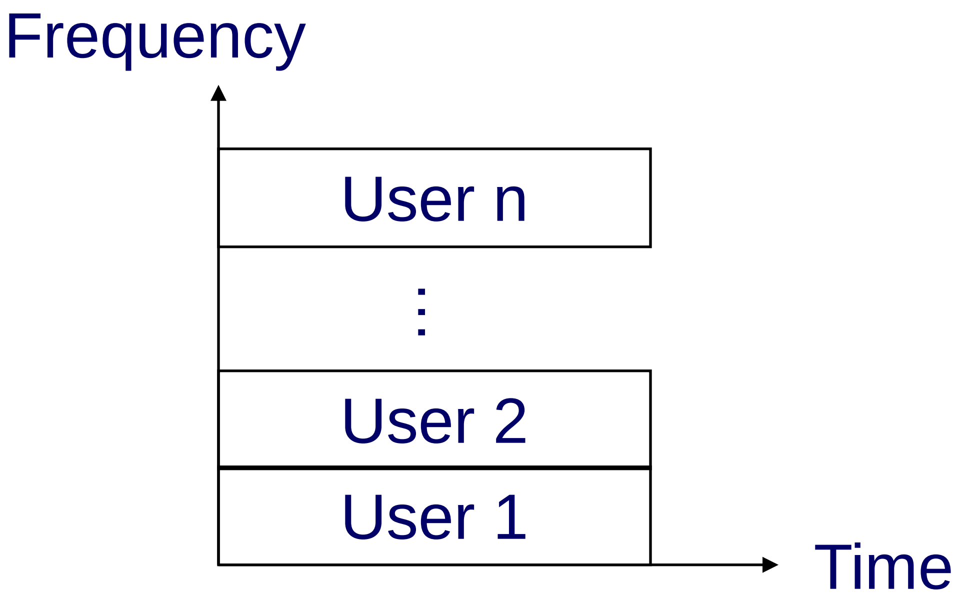
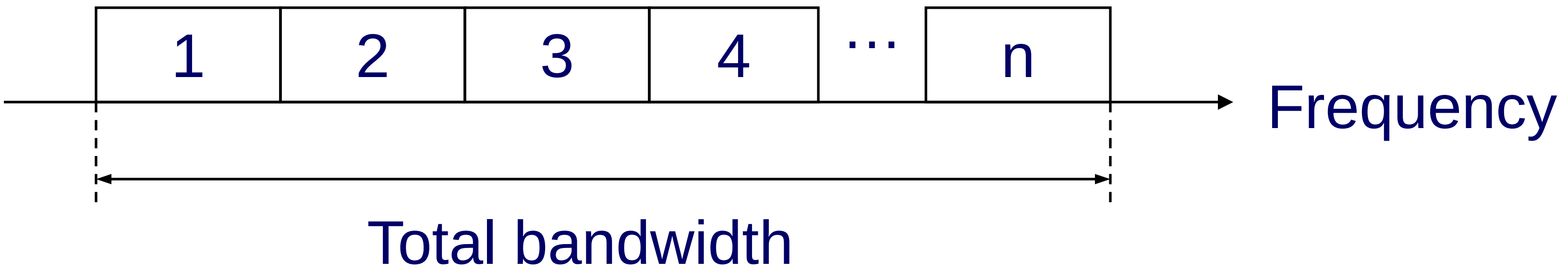
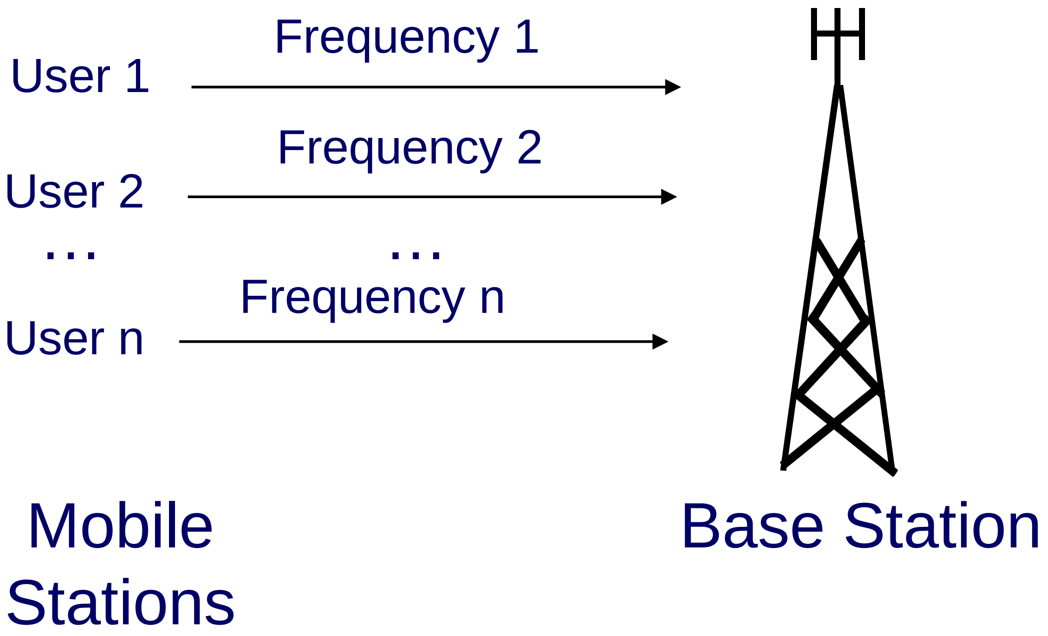
FDMA (Frequency Division Multiple Access)



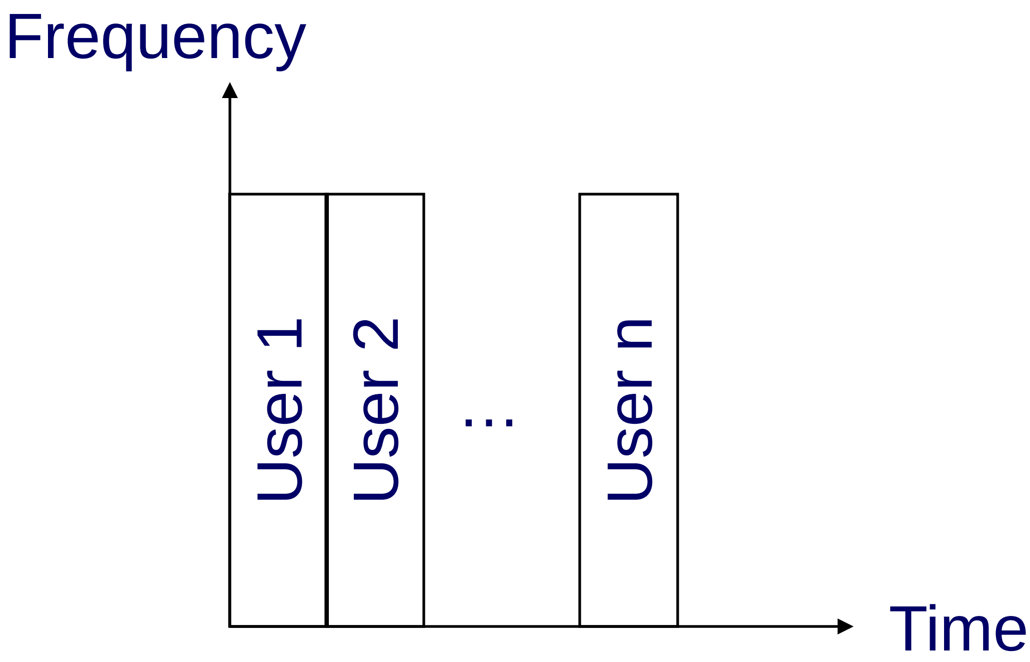
FDMA Bandwidth Structure



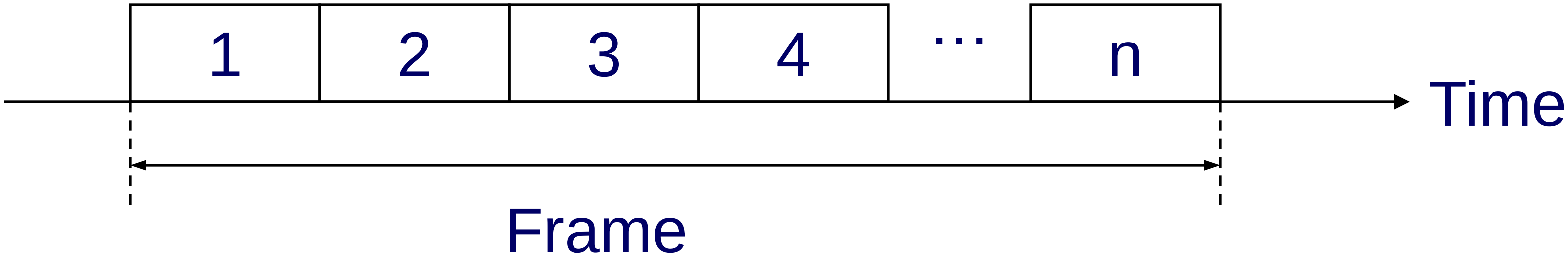
FDMA Channel Allocation



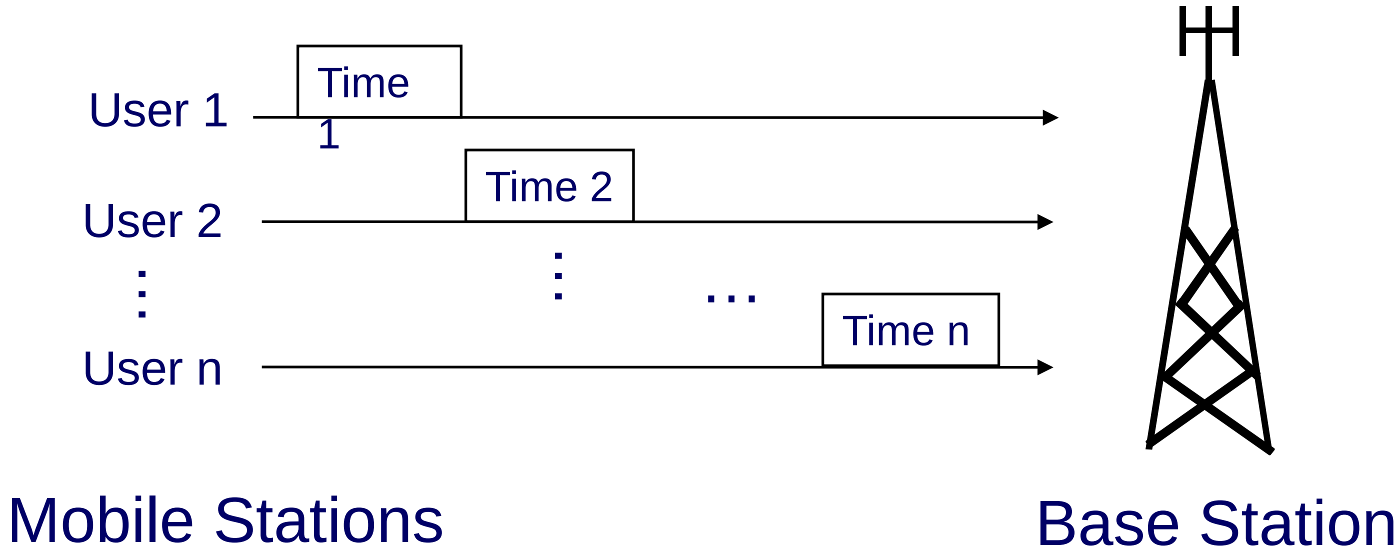
TDMA (Time Division Multiple Access)



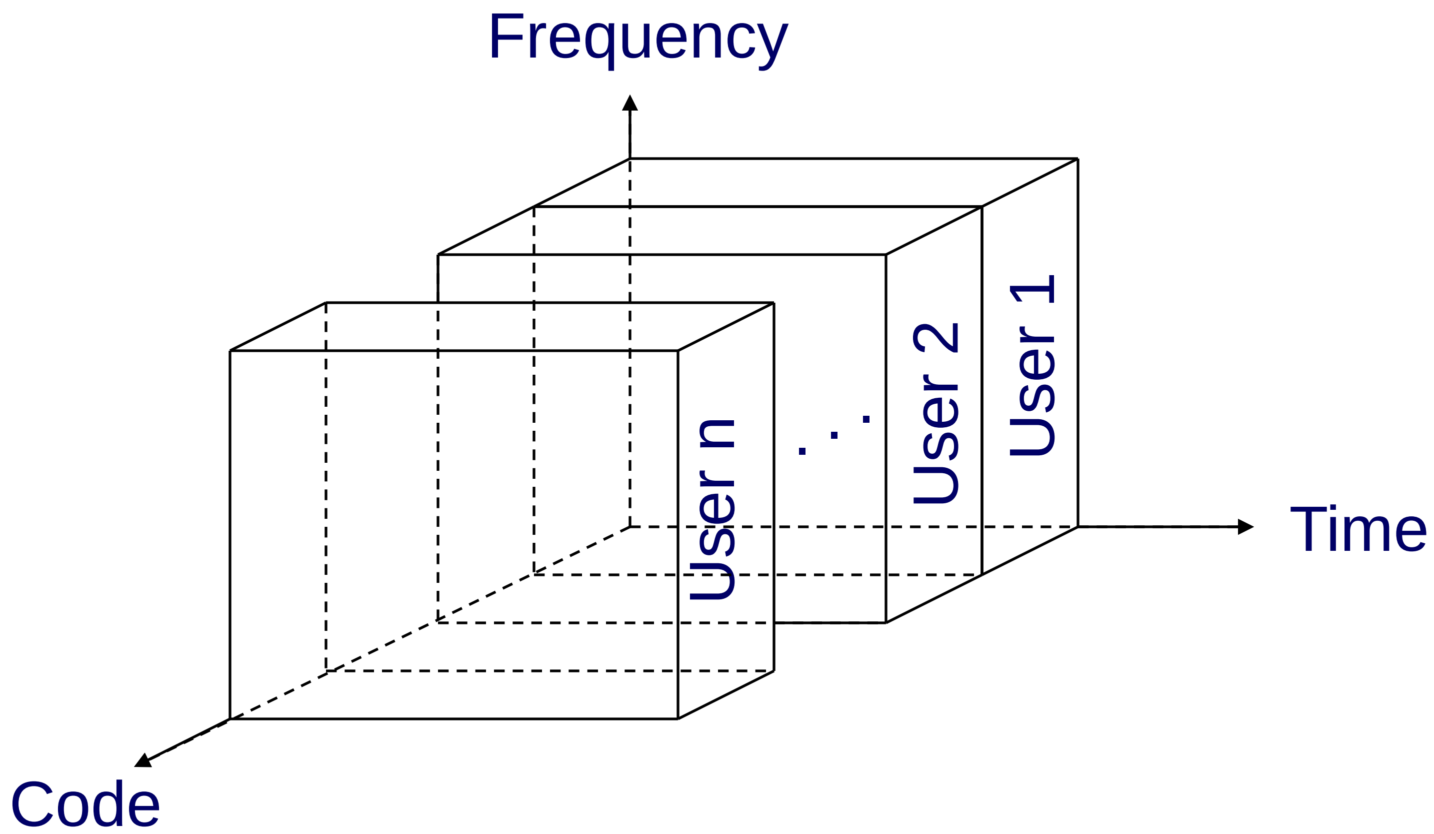
TDMA Frame Structure



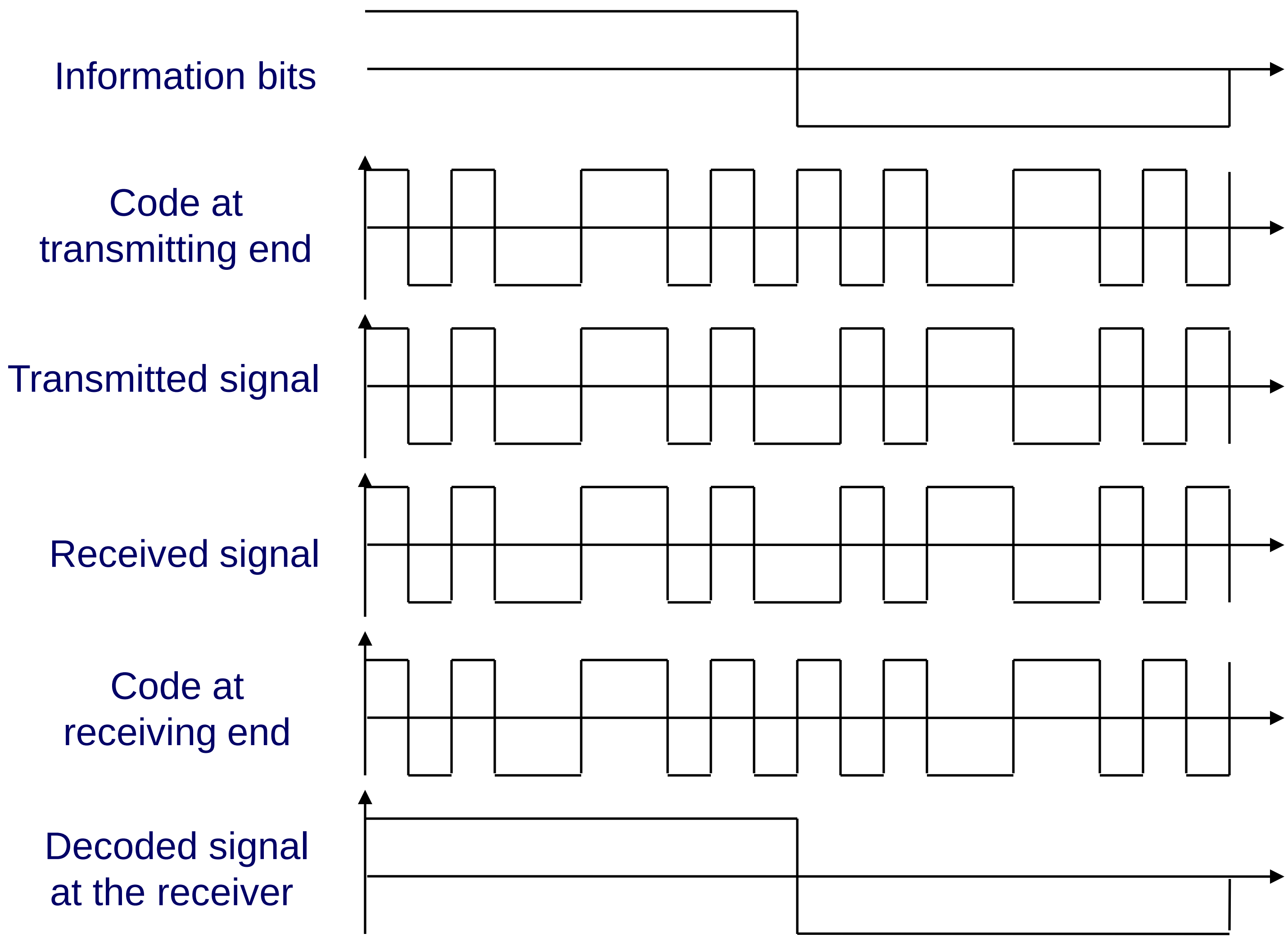
TDMA Frame Allocation



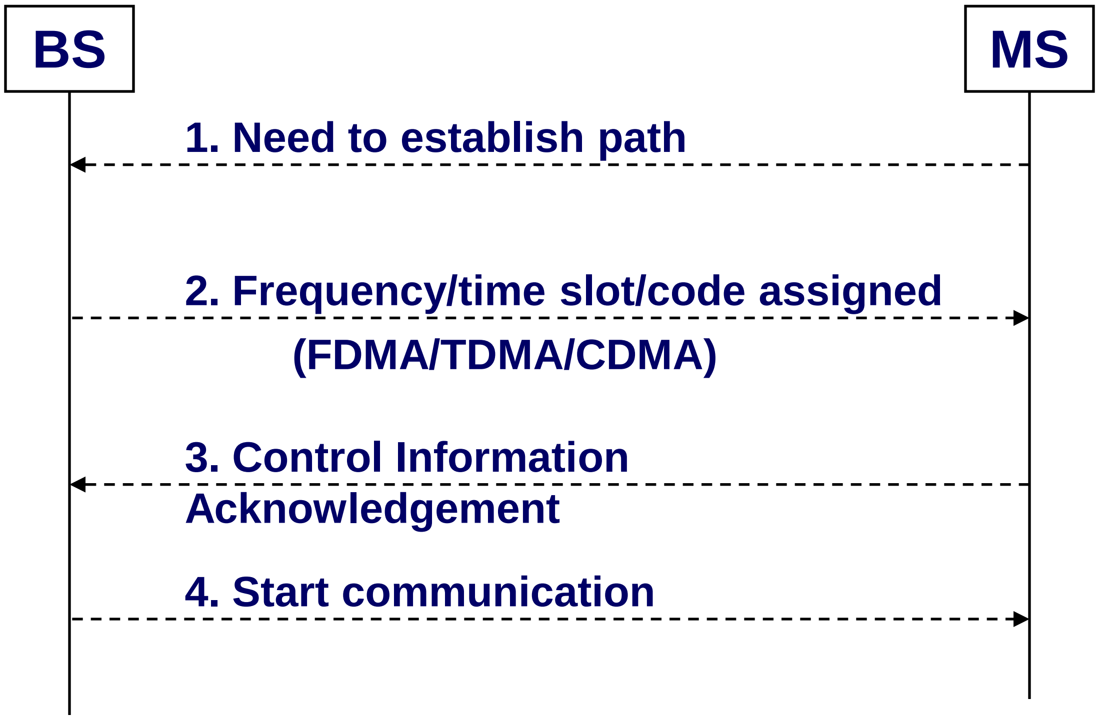
CDMA (Code Division Multiple Access)



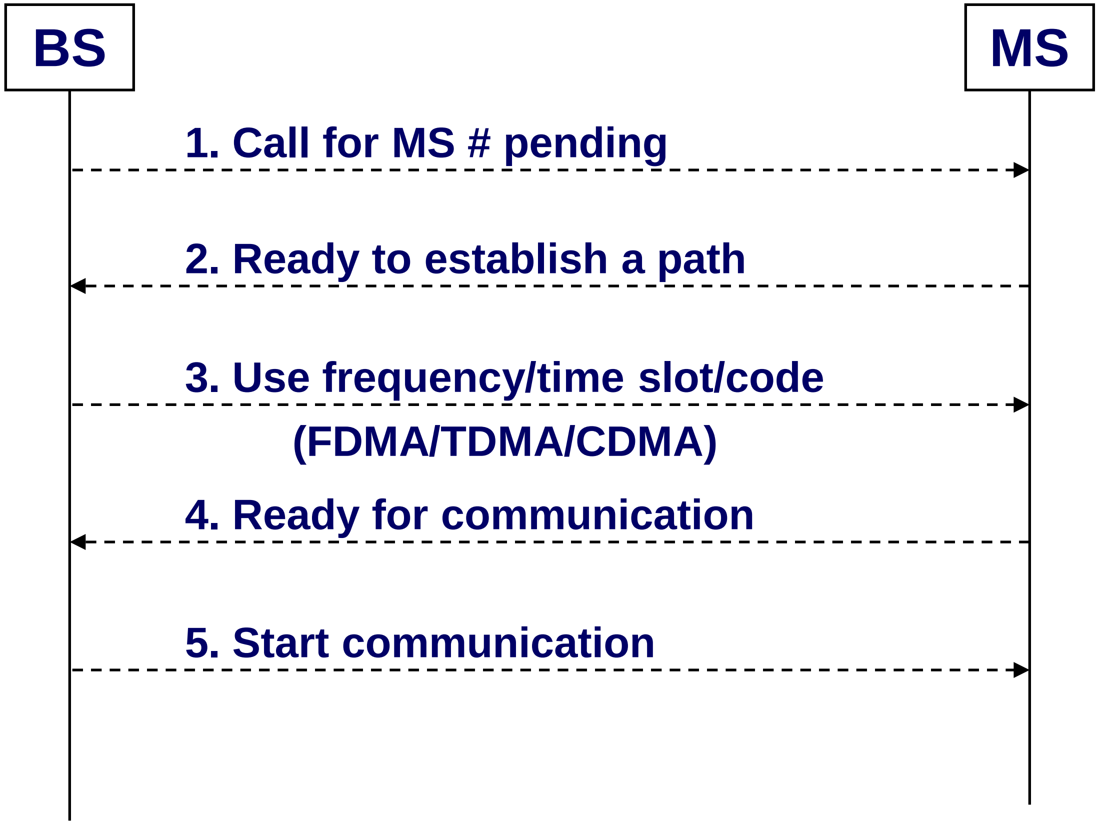
Transmitted and Received Signals in CDMA Systems



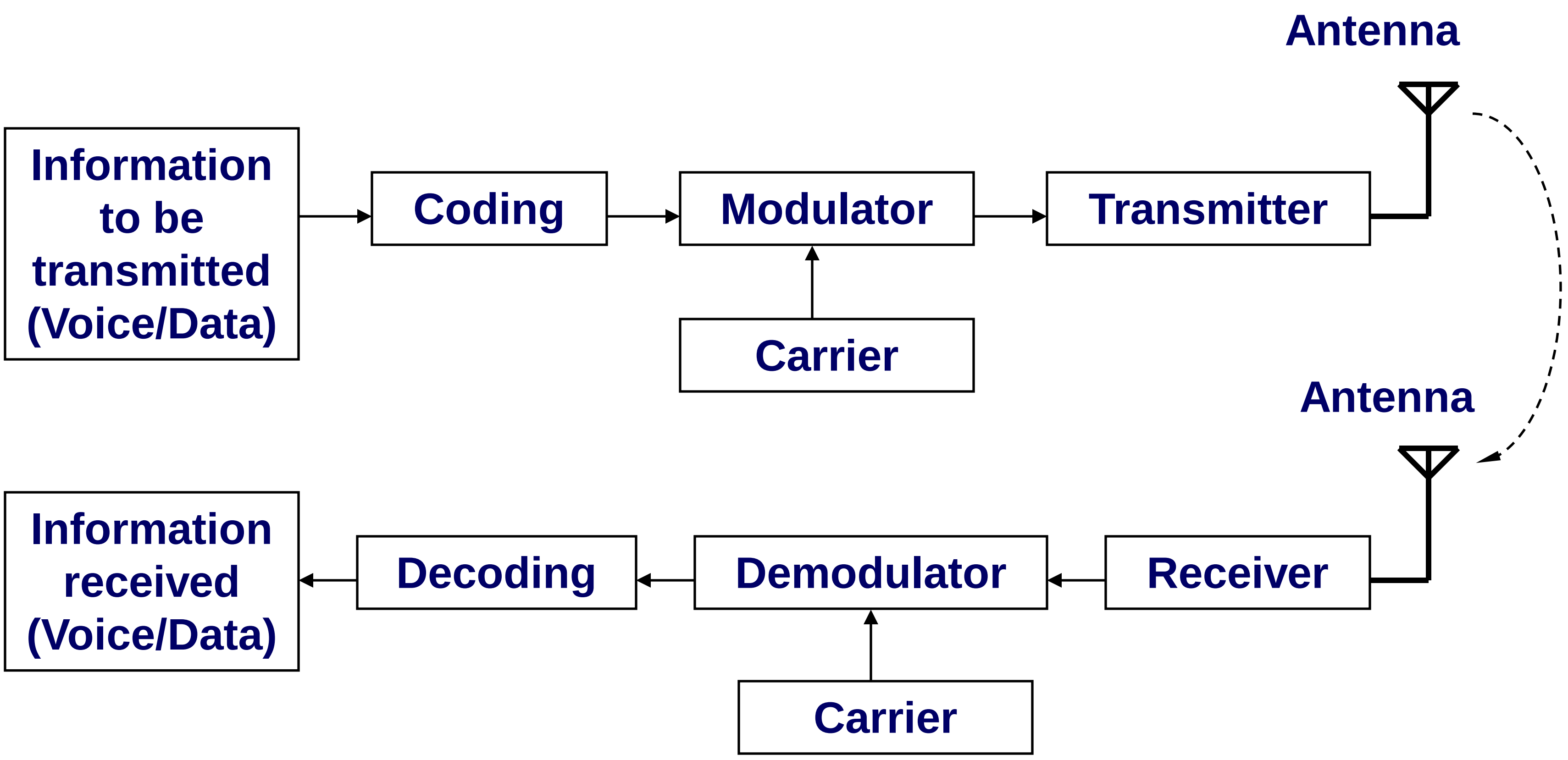
Connection Setup (Service Delivery)



Connection Setup (Service Origination)



Simplified Framework of Wireless Communications



Radio Propagation and Antenna

Radio Propagation and Antenna

- Wired and wireless medium
- Radio propagation mechanism
- Antenna and antenna gain
- Path-loss modeling
  - Free Space model
  - Two-Ray model
  - Shadow fading
  - Different environments
- Effect of Multipath and Doppler
  - Multipath fading
  - Doppler spectrum

Wired and Wireless Media

- Wired media
  - Reliable, guided link: electrical signal associated with the transmission of information from one fixed terminal to another.
  - Like filters that limit the maximum transmitted data rate of the channel because of band limiting frequency response characteristics.
  - Radiates outside of the wire to some extent which can cause interference to close-by radios or other wired transmission.
- Wireless media
  - Relatively unreliable, low bandwidth, and of broadcast nature (unguided medium).
  - All wireless transmissions share the same medium-air, whereas different signals through wired media via different wires.

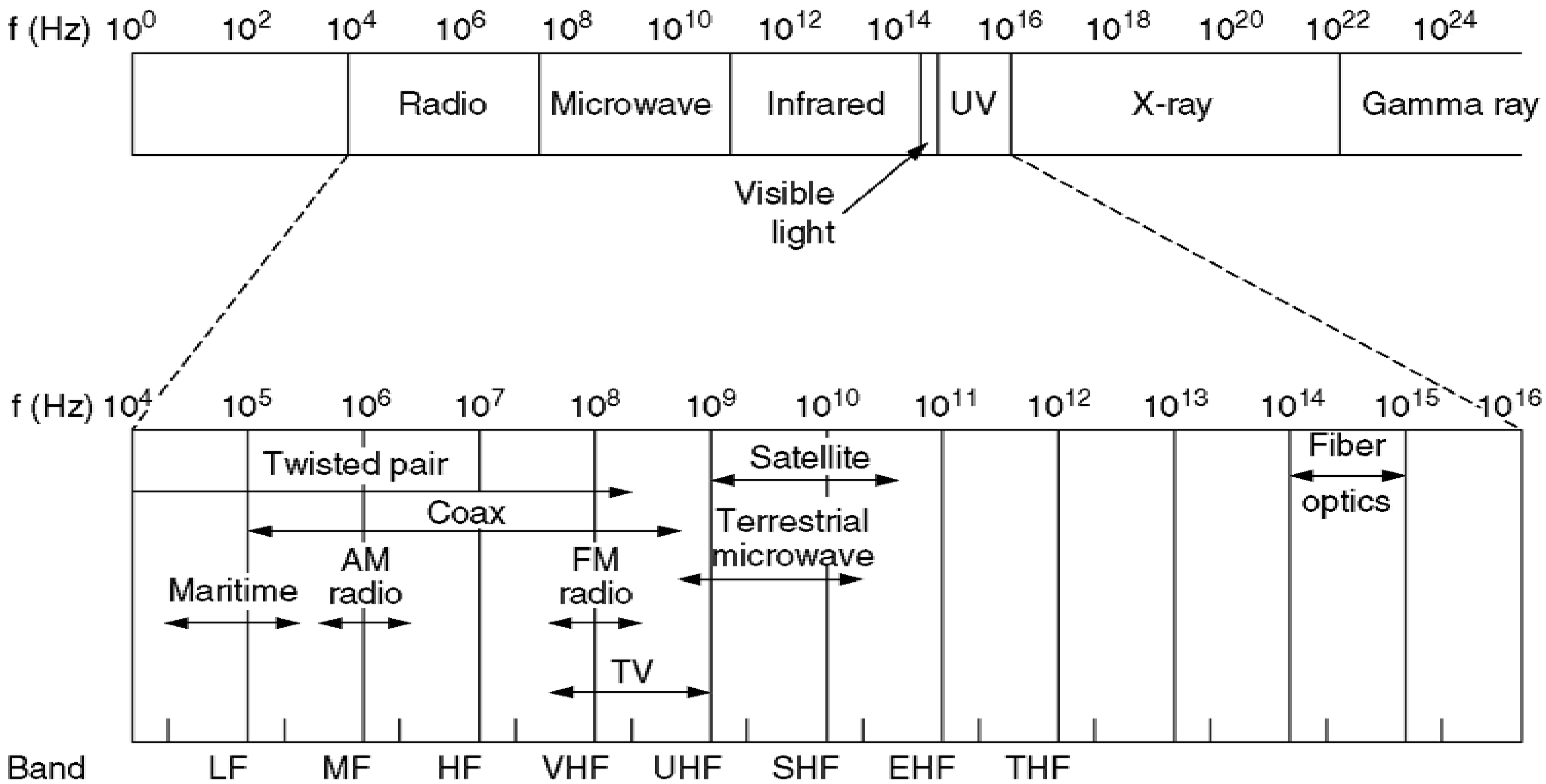
Licensed versus Unlicensed Bands

- Licensed bands
  - Cellular systems operate around 1GHz
  - PCS and WLANs around 2GHz
  - WLAN around 5GHz
  - LMDS (local multipoint distribution service) at 28-60GHz
  - IR (InfraRed) for optical communications
- Unlicensed bands
  - ISM (Industrial, Scientific, and Medical) band
  - U-NII (Unlicensed National Information Infrastructure) bands, were released in 1997, PCS unlicensed bands were released in 1994.

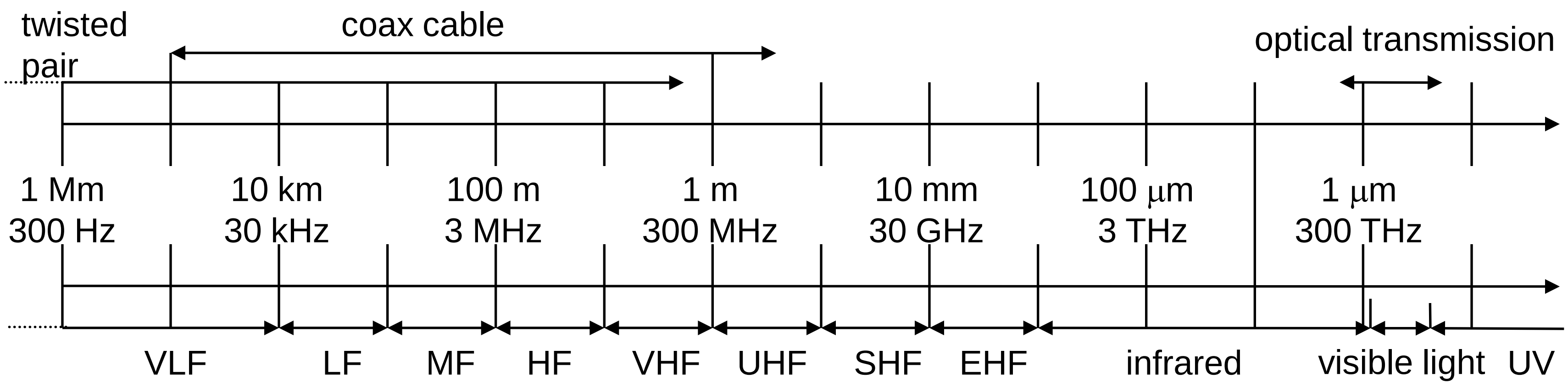
Electromagnetic Spectrum

Relationship between *f*, λ, and *c* (in vacuum) is

$$\lambda f = c$$



Frequencies for communication



- VLF = Very Low Frequency
- LF = Low Frequency
- MF = Medium Frequency
- HF = High Frequency
- VHF = Very High Frequency

UHF = Ultra High Frequency
SHF = Super High Frequency
EHF = Extra High Frequency
UV = Ultraviolet Light

- Frequency and wave length:
- $\lambda = c/f$
- wave length λ, speed of light $c \cong 3 \times 10^8$ m/s, frequency f

Frequencies for mobile communication

- VHF-/UHF-ranges for mobile radio
  - simple, small antenna for cars
  - deterministic propagation characteristics, reliable connections
- SHF and higher for directed radio links, satellite communication
  - small antenna, focusing
  - large bandwidth available
- Wireless LANs use frequencies in UHF to SHF spectrum
  - some systems planned up to EHF
  - limitations due to absorption by water and oxygen molecules (resonance frequencies)
    - weather dependent fading, signal loss caused by heavy rainfall etc.

Frequencies and regulations

- ITU-R holds auctions for new frequencies, manages frequency bands worldwide (WRC, World Radio Conferences)

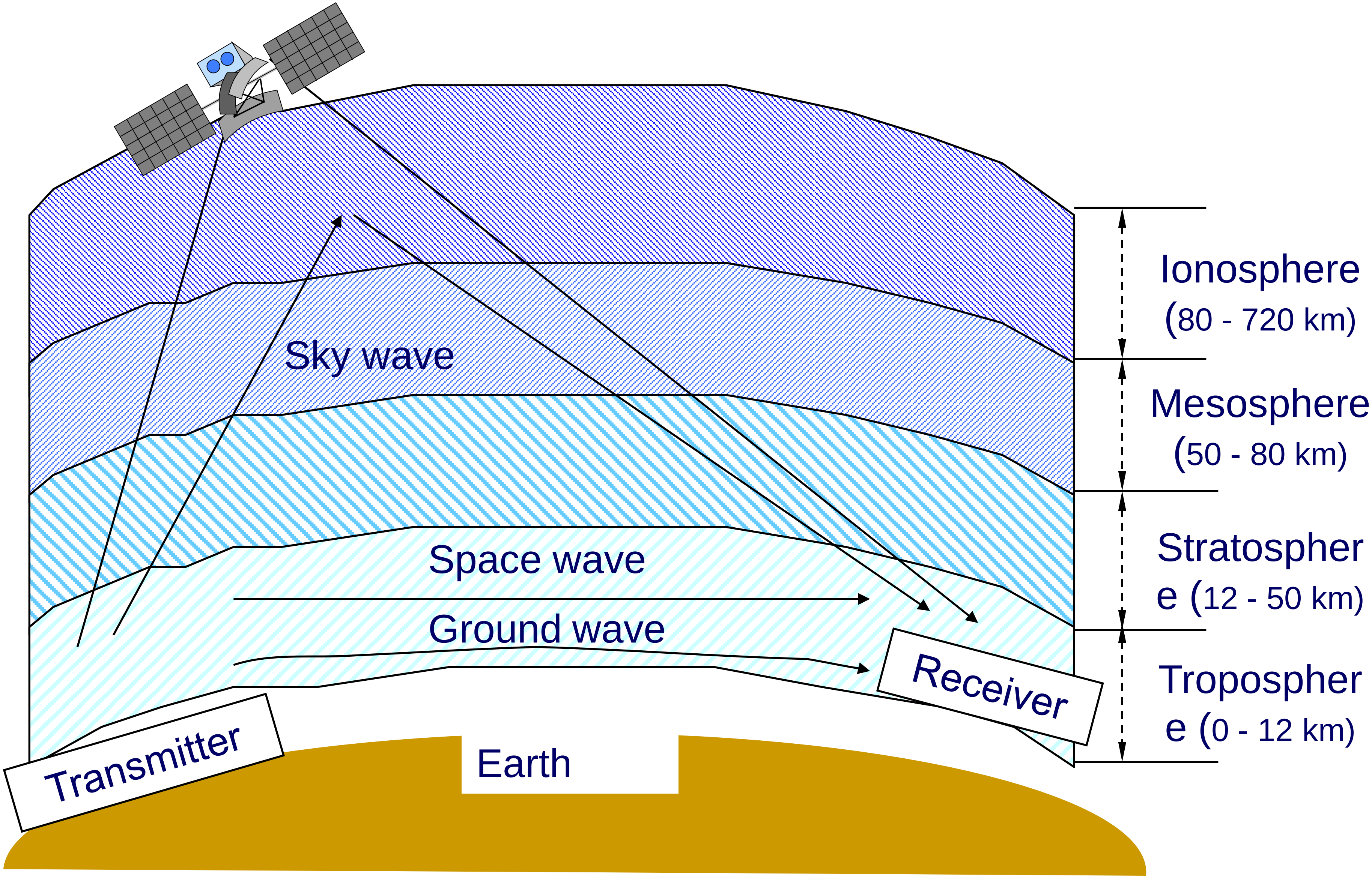
	Europe	USA	Japan
Cellular Phones	**GSM** 450-457, 479-486/460-467,489-496, 890-915/935-960, 1710-1785/1805-1880 **UMTS** (FDD) 1920-1980, 2110-2190 **UMTS** (TDD) 1900-1920, 2020-2025	**AMPS**, **TDMA**, **CDMA** 824-849, 869-894 **TDMA**, **CDMA**, **GSM** 1850-1910, 1930-1990	**PDC** 810-826, 940-956, 1429-1465, 1477-1513
Cordless Phones	**CT1+** 885-887, 930-932 **CT2** 864-868 **DECT** 1880-1900	**PACS** 1850-1910, 1930-1990 **PACS-UB** 1910-1930	**PHS** 1895-1918 **JCT** 254-380
Wireless LANs	**IEEE 802.11** 2400-2483 **HIPERLAN 2** 5150-5350, 5470-5725	902-928 **IEEE 802.11** 2400-2483 5150-5350, 5725-5825	**IEEE 802.11** 2471-2497 5150-5250
Others	**RF-Control** 27, 128, 418, 433, 868	**RF-Control** 315, 915	**RF-Control** 426, 868

Speed, Frequency, and Wavelength

Light speed (c) = Wavelength (λ) *Frequency (f)
= 3 x 108 m/s = 300,000 km/s

System	Frequency	Wavelength
AC current	60 Hz	5,000 km
FM radio	100 MHz	3 m
Cellular	800 MHz	37.5 cm
Ka band satellite	20 GHz	15 mm
Ultraviolet light	10^{15} Hz	10^{-7} m

Wave Types



Sky wave
Space wave
Ground wave
Transmitter
Receiver
Earth
Ionosphere
(80 - 720 km)
Mesosphere
(50 - 80 km)
Stratospher
e (12 - 50 km)
Tropospher
e (0 - 12 km)

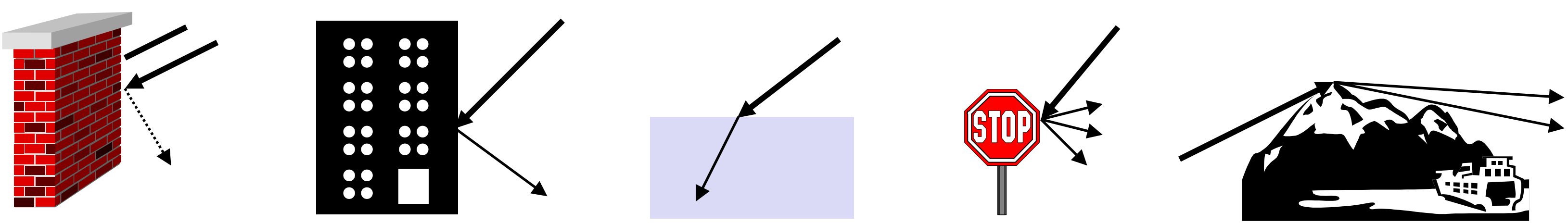
Radio Propagation

- Heavily site-specific and can vary significantly depending on the
  - Terrain (indoor and outdoor)
  - Frequency of operation (low and high)
  - Velocity of the mobile terminal
  - Interference sources
- Performance attributes
  - Signal coverage
  - Reception schemes
  - Interference analysis
  - Optimal location for installing base station antennas

- *Reflection and transmission*: Occur when electromagnetic waves impinge (strike) on obstructions larger than the wavelength.
  - Not dominant outdoors
- *Diffraction*: Rays that are incident upon the edges of buildings, walls, and other large objects can be viewed as exciting the edges to act as a secondary line sources.
  - Shadowed region
  - Relatively weak compared to reflection indoors
- *Scattering*: Occur when objects are of dimensions that are on the order of a wavelength or less of the electromagnetic wave.

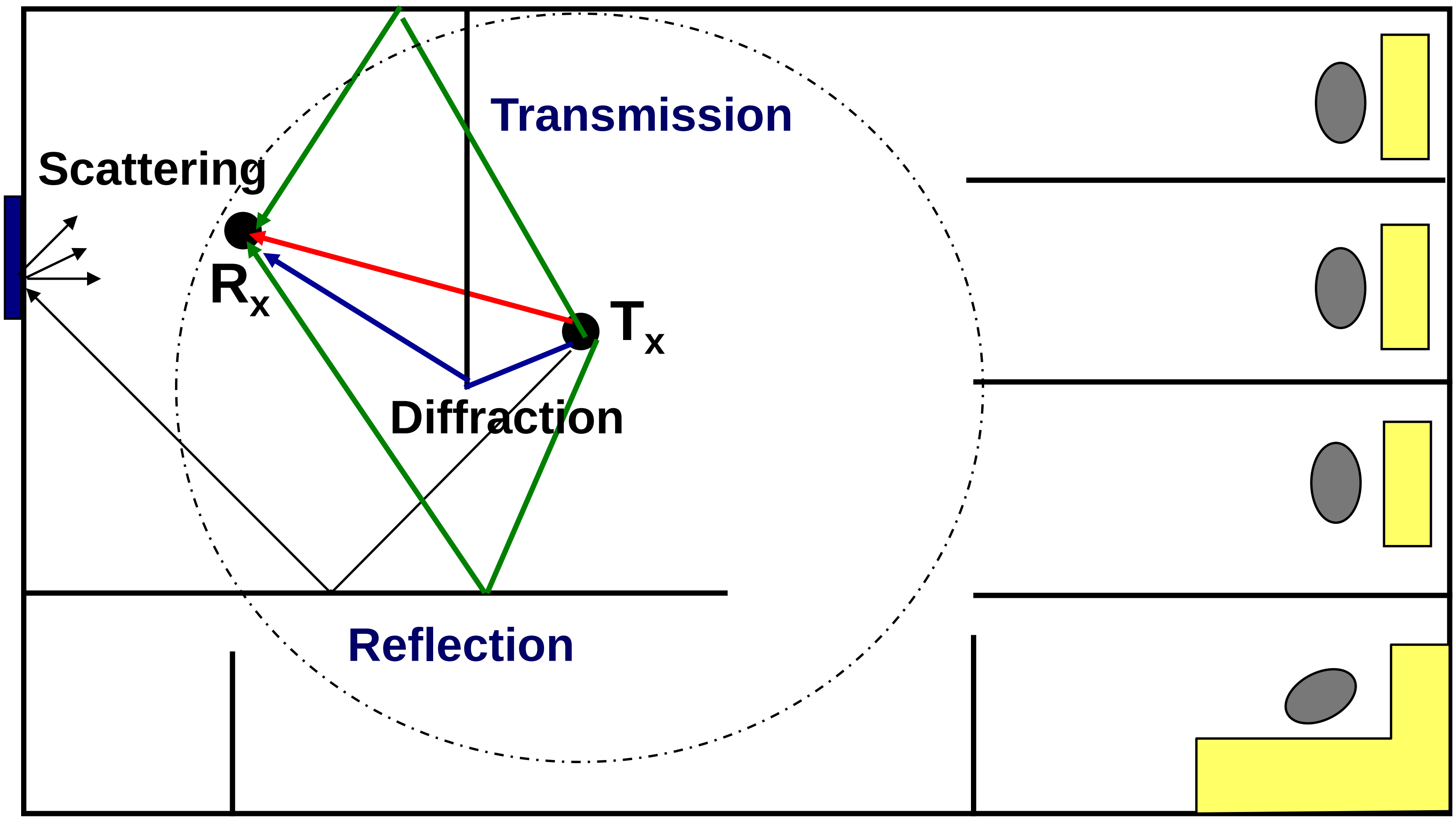
Signal propagation

- Propagation in free space always like light (straight line)
- Receiving power proportional to $1/d^2$
  (d = distance between sender and receiver)
  - Receiving power additionally influenced by
  - fading (frequency dependent)
  - shadowing
  - reflection at large obstacles
  - refraction depending on the density of a medium
  - scattering at small obstacles
  - diffraction at edges

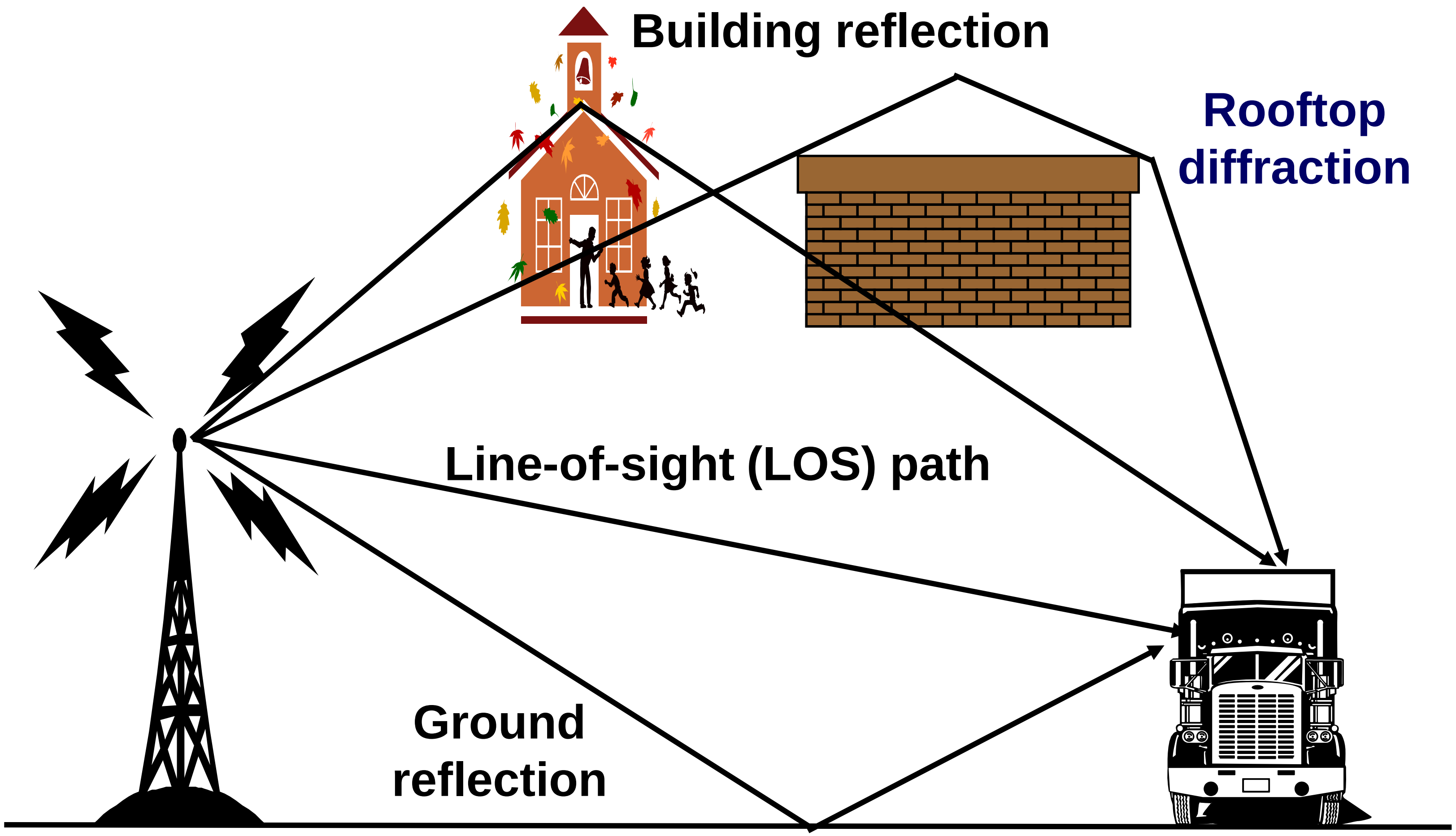


shadowing reflection refraction scattering diffraction

Radio Propagation in an Indoor Area



Radio Propagation in an Outdoor Area

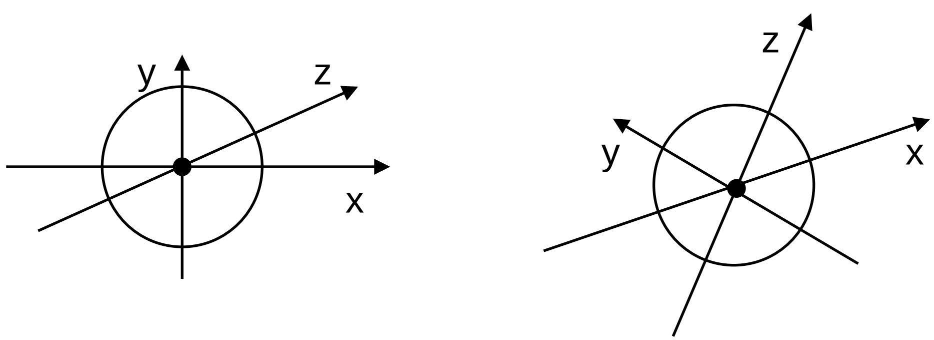


Antenna

- An electrical device
  - An antenna is an electrical conductor or system of conductors
    - Transmission - radiates electromagnetic energy into space
    - Reception - collects electromagnetic energy from space
  - In two-way communication, the same antenna can be used for transmission and reception
- Types of Antenna
  - Isotropic antenna (idealized)
    - Radiates power equally in all directions
  - Dipole antennas
    - Half-wave dipole antenna (or Hertz antenna)
    - Quarter-wave vertical antenna (or Marconi antenna)
  - Parabolic Reflective Antenna

Antennas: isotropic radiator

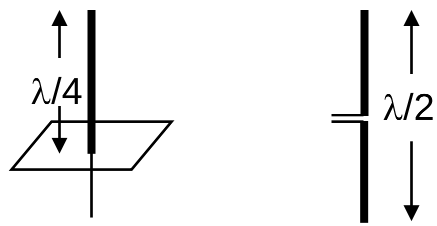
- Radiation and reception of electromagnetic waves, coupling of wires to space for radio transmission
- Isotropic radiator: equal radiation in all directions (three dimensional) - only a theoretical reference antenna
- Real antennas always have directive effects (vertically and/or horizontally)
- Radiation pattern: measurement of radiation around an antenna



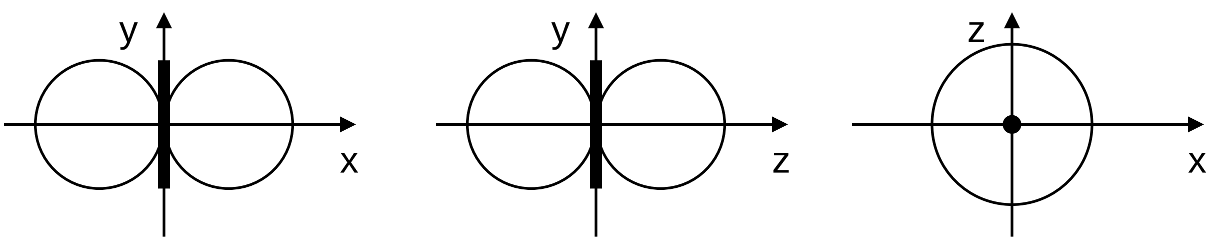
ideal isotropic radiator

Antennas: simple dipoles

- Real antennas are not isotropic radiators but, e.g., dipoles with lengths $\lambda/4$ on car roofs or $\lambda/2$ as Hertzian dipole
  ➔ shape of antenna proportional to wavelength



- Example: Radiation pattern of a simple Hertzian dipole



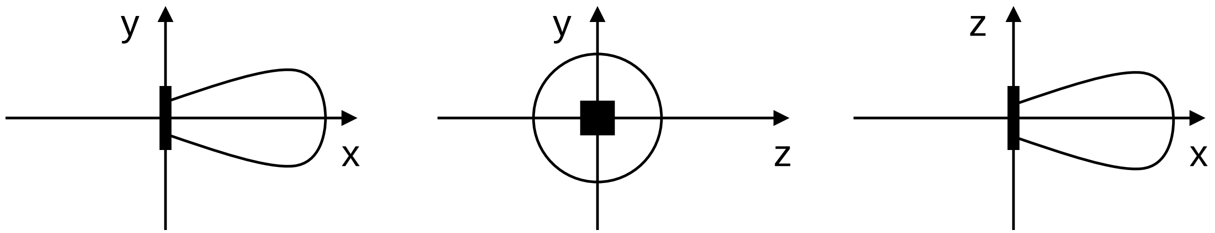
side view (xy-plane) side view (yz-plane) top view (xz-plane)

simple dipole

- Gain: maximum power in the direction of the main lobe compared to the power of an isotropic radiator (with the same average power)

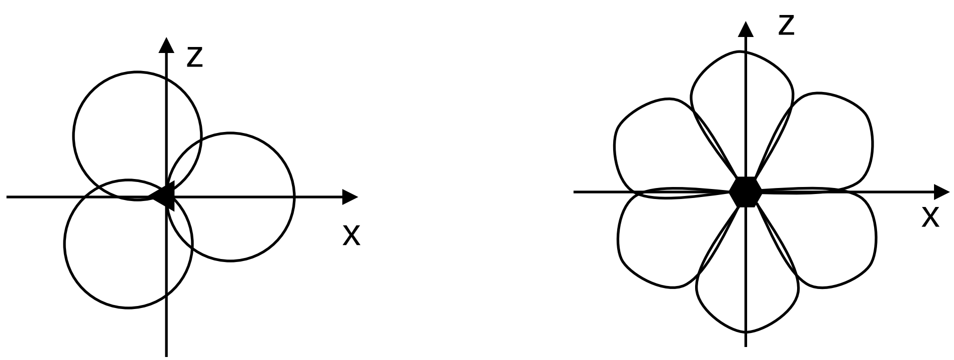
Antennas: directed and sectorized

- Often used for microwave connections or base stations for mobile phones (e.g., radio coverage of a valley)



side view (xy-plane) side view (yz-plane) top view (xz-plane)

directed antenna

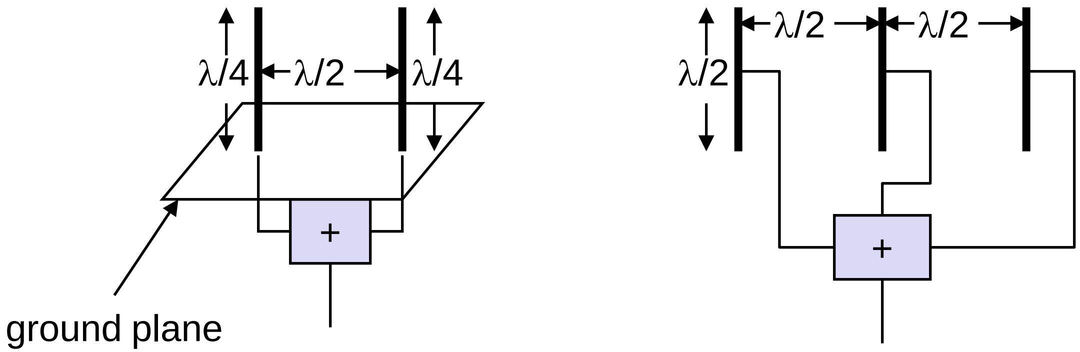


top view, 3 sector top view, 6 sector

sectorized antenna

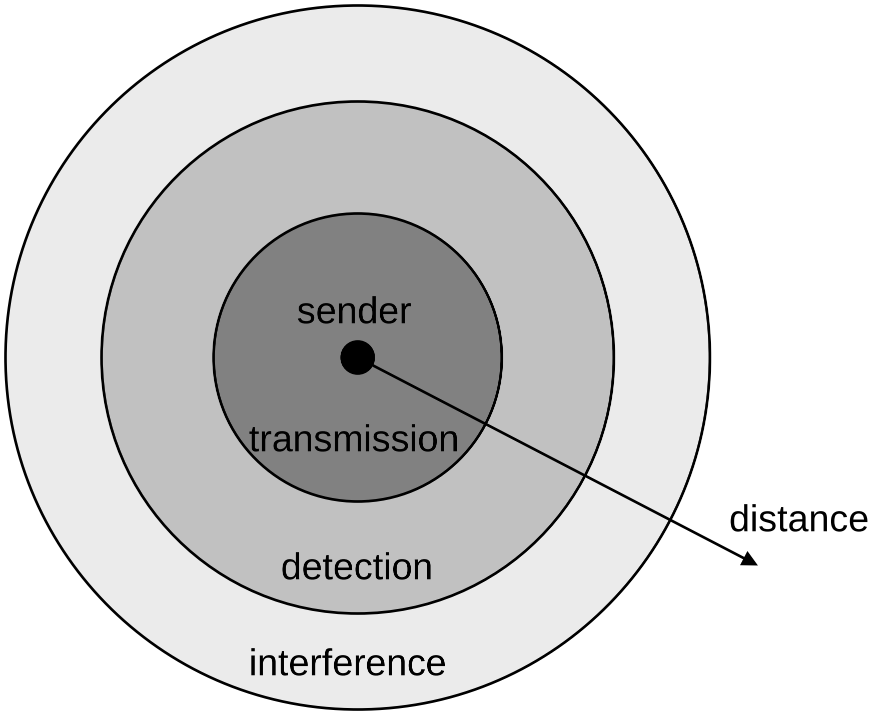
Antennas: diversity

- Grouping of 2 or more antennas
  - multi-element antenna arrays
- Antenna diversity
  - switched diversity, selection diversity
    - receiver chooses antenna with largest output
  - diversity combining
    - combine output power to produce gain
    - cophasing needed to avoid cancellation



Signal propagation ranges

- Transmission range
  - communication possible
  - low error rate
- Detection range
  - detection of the signal possible
  - no communication possible
- Interference range
  - signal may not be detected
  - signal adds to the background noise



Antenna Gain

- Antenna gain
  – Power output, in a particular direction, compared to that produced in any direction by a perfect omnidirectional antenna (isotropic antenna)
- Effective area
  – Related to physical size and shape of antenna
- Relationship between antenna gain and effective area
  - G = antenna gain
  - A_e = effective area
  - f = carrier frequency
  - c = speed of light (≈ 3 ´ 10^8 m/s)
  - λ = carrier wavelength

$$G = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi f^2 A_e}{c^2}$$

Radio Propagation and Antenna (Ch. 5)

- Wired and wireless medium
- Radio propagation mechanism
- Antenna and antenna gain
- Path-loss modeling
  - Free Space model
  - Two-Ray model
  - Shadow fading
  - Different environments
- Effect of Multipath and Doppler
  - Multipath fading
  - Doppler spectrum

Path-Loss Modeling

- Calculation of signal coverage
  - Frequency and terrain for design and deployment of wireless networks
- Several channel models for a variety of environments between the transmitter and receiver.
- What is path-loss model?
  - Relate the loss of signal strength to distance between two terminals.
  - Use path-loss model to calculate distance between a BS and an AP; and maximum distance between two terminals in an ad hoc network.

Free Space Propagation

- *Path-loss gradient*: Radio signal strength falls as some power α of the distance, called the power-distance gradient or ~.
- *Parameters*:
  - If the transmitted power is P_t , after a distance of d in meters, the signal strength will be proportional to $P_t d^{-\alpha}$.
  - Simple case in free space, $\alpha = 2$.
  - When an antenna radiates a signal, the signal propagates in all directions. The signal strength density at a sphere of radius d is the total radiated signal strength divided by the area of the sphere, $4\pi d^2$.
  - Additional losses may be caused depending on the frequencies, G_t and G_r, are the transmitter and receiver antenna gains respectively in the direction from the transmitter to the receiver.

Free Space Propagation

- Let P_t and P_r be the transmitted and received power, then we have

$$\frac{\boldsymbol{P_r}}{\boldsymbol{P_t}} = \boldsymbol{G_t G_r}\left(\frac{\lambda}{4\pi \boldsymbol{d}}\right)^2$$

- If we let $P_0 = P_t G_t G_r (\lambda/4\pi)^2$ be the received signal strength at the first meter (d = 1 m), we can rewrite the equation as:

In decibels (dB), this equation takes the form

$$10log(P_r)=10log(P_0)-20log(d)$$

$$\boldsymbol{P_r} = \frac{\boldsymbol{P}_0}{\boldsymbol{d}^2}$$

- The transmission delay as a function of distance is given by $\tau = d/c = 3d\ ns\ or\ 3\ ns$ per meter of distance.

Antenna Gain

- For a circular reflector antenna
  - Gain $G = \eta (\pi D / \lambda)2$
  - η = net efficiency (depends on the electric field distribution over the antenna aperture, losses, ohmic heating , typically 0.55)
  - D = diameter
  - thus, $G = \eta (\pi D f /c)2$, $c = \lambda f$ (c is speed of light)
- Example: Antenna with diameter = 2 m,
  - Frequency   = 6 GHz, wavelength = 0.05 m, G = 39.4 dB,
  - Frequency = 14 GHz, same diameter, wavelength = 0.021 m, G = 46.9 dB
- Higher the frequency, higher the gain for the same size antenna

Received Signal

- The received signal power:

$$P_r = \frac{G_t G_r P_t}{L}$$

where G_r is the receiver antenna gain,

L is the propagation loss in the channel, i.e.,

$$L = L_P \, L_S \, L_F$$

Fast fading

Slow fading

Path loss

Path Loss in Free Space

- Definition of path loss L_P :

$$L_P = \frac{P_t}{P_r},$$

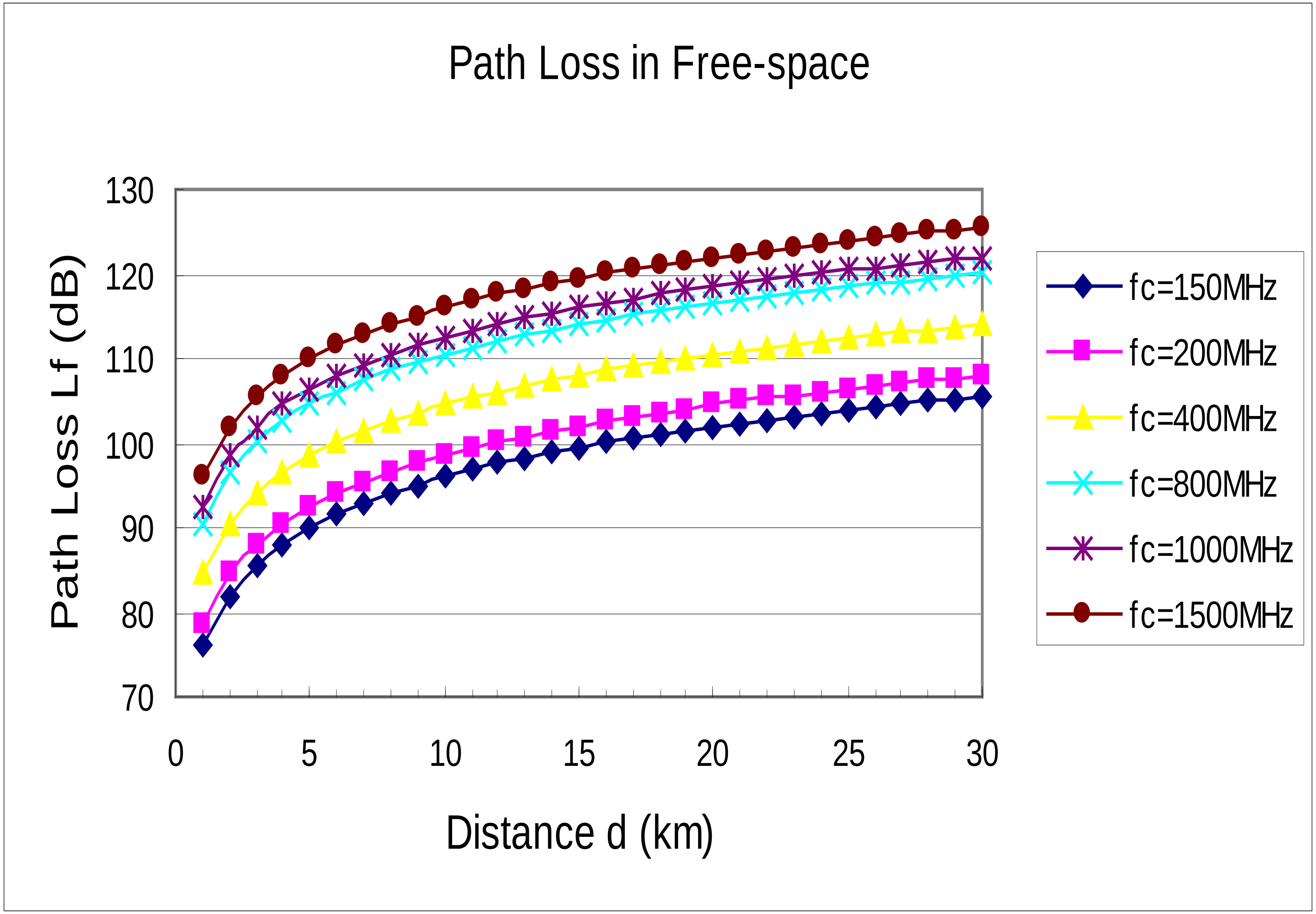
Path Loss in Free-space:

$$L_{PF}(dB) = 32.45 + 20\log_{10} f_c(MHz) + 20\log_{10} d(km),$$

where f_c is the carrier frequency.

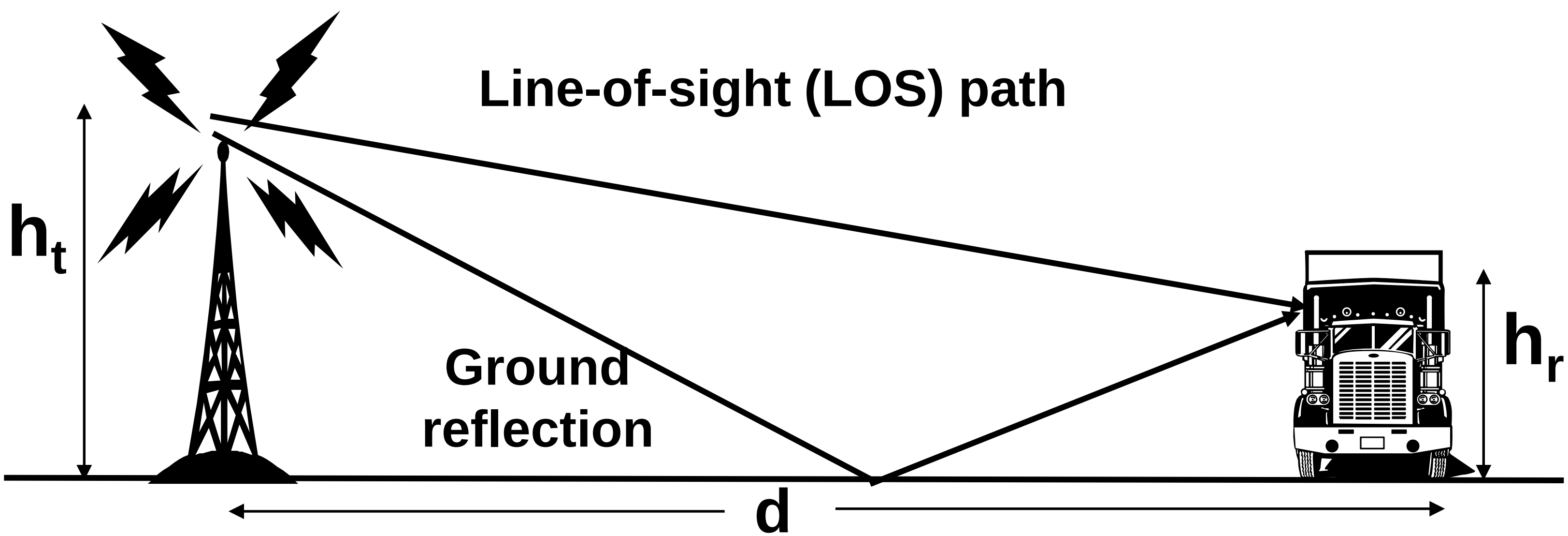
This shows greater the f_c , more is the loss.

Path Loss in Free Space



Two-Ray Model for Mobile Radio Environments (1)

- In all realistic environments, the signal reaches the receiver through several different paths.
- Two –ray or two-path model is popular for land radio.



Two-Ray Model for Mobile Radio Environments (2)

- The relationship between P_t and P_r:

$$\frac{P_r}{P_t} = G_t G_r \frac{h_b^2 h_m^2}{d^4}$$

- The signal strength falls as the fourth power of the distance between the transmitter and the receiver. In other words, there is a loss of 40 dB per decade or 12 per octave.
- The received signal strength can be increased by raising the heights of the transmit and receive antennas.

Distance-Power relationship

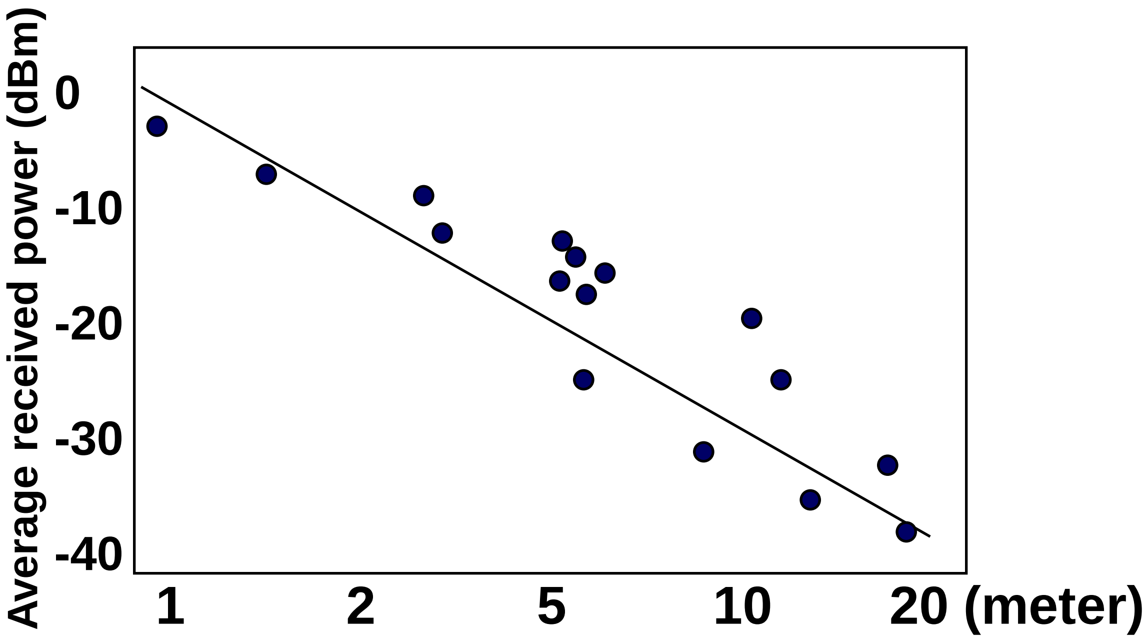
- The simplest method of relating P_t and P_r, is to use distance-power gradient, that is,

$$P_r = P_0 d^{-\alpha} \text{ or } 10\log(P_r) = 10\log(P_0) - 10\,\alpha\,\log(d)$$

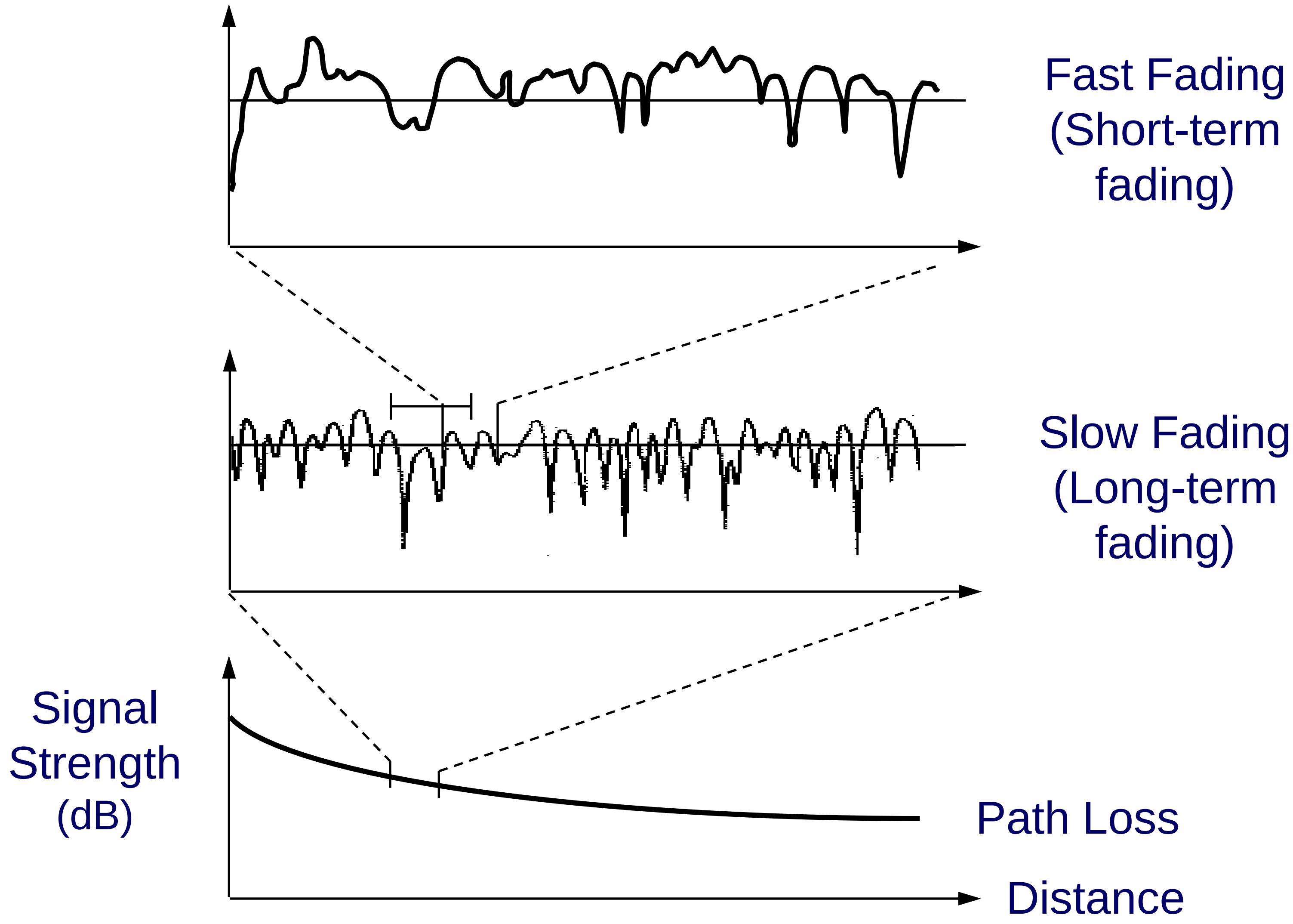
  – P_0 is the received power at a reference distance (usually 1 meter) from the transmitter.
  – $\alpha = 2$ for free-space and $\alpha = 4$ for the simplified two-path model of an urban radio channel.

Measurement of the Distance Power Gradient

- The receiver is fixed at one location, and the transmitter is placed at a number of locations with different distances between the transmitter and the receiver.
- The path loss in dB is plotted against the distance on a logarithmic scale.



Fading



Fast Fading
(Short-term
fading)
Slow Fading
(Long-term
fading)
Signal
Strength
(dB)
Path Loss
Distance

Shadow Fading （Slow Fading）

- The received signal strength for the same distance from the transmitter will be different.
- The variation of the signal strength due to locations is often referred to as *shadow fading* or *slow fading*.
- Reason:
  - Often time, the fluctuations around the mean value are caused due to the signal being blocked from the receiver by buildings or walls and so on.
  - It is called slow fading because the variations are much slower with distance than another fading phenomenon caused due to multipath.

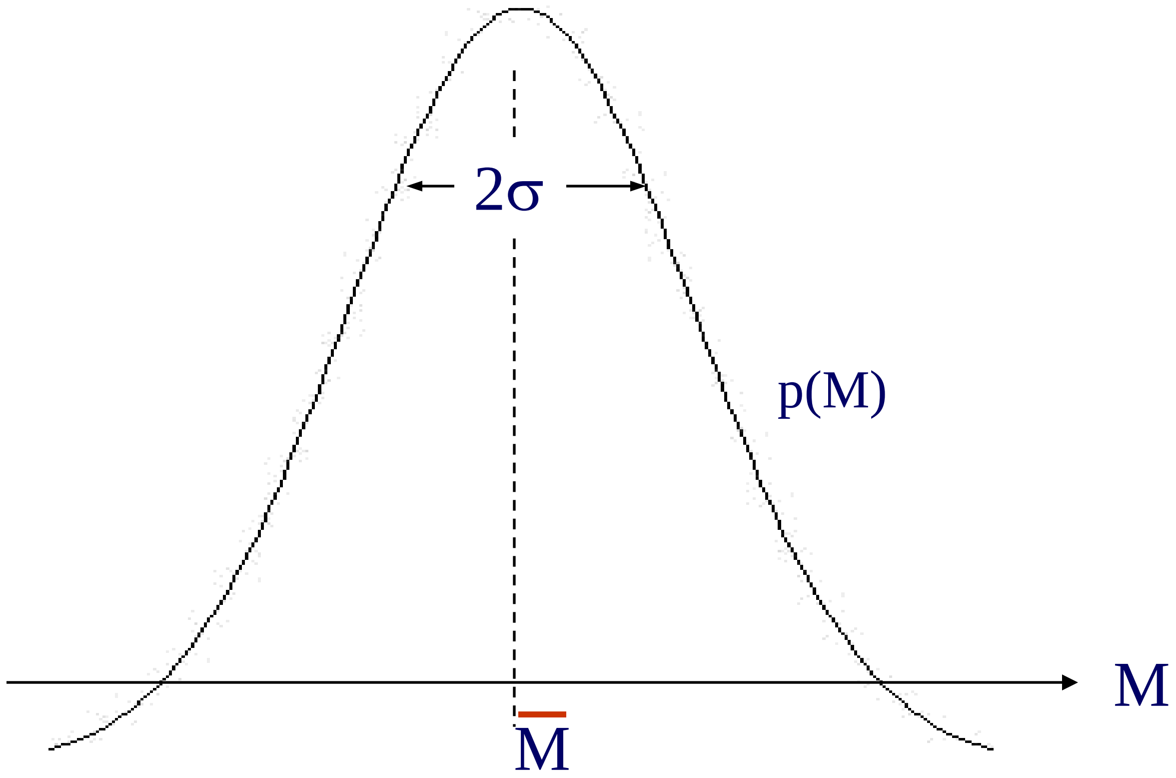
Effect of Shadow Fading

- The long-term variation in the mean level is known as slow fading (shadowing or log-normal fading). This fading is caused by shadowing.
- The path loss equation is modified by adding a random component as follows:

$$L_p = L_0 + 10\, \alpha \log(d) + X$$

  - *X* is a random variable with a distribution that depends on the fading component. Based on measurements and simulations, this variation can be expressed as a log-normal distributed random variable.
  - The problem of shadow fading is that all locations at a given distance may not receive sufficient signal strength for correctly detecting the information.

Log-Normal Distribution



The pdf of the received signal level

Radio Propagation and Antenna

- Wired and wireless medium
- Radio propagation mechanism
- Antenna and antenna gain
- Path-loss modeling
  - Free Space model
  - Two-Ray model
  - Shadow fading
  - Different environments
- Effect of Multipath and Doppler
  - Multipath fading
  - Doppler spectrum

Path Loss in Different Environments

- Path loss in decreasing order:
  - Urban area (large city)
  - Urban area (medium and small city)
  - Suburban area
  - Open area

Path Loss Models for Macro-cellular Areas

- Macro-cellular areas span a few kilometers to tens of kilometers.
- Okumura-Hara Path Loss Model

$$L_p(d) = \begin{cases} A + B\log(d) & \textbf{for urban area} \\ A + B\log(d) - C & \textbf{for suburban area} \\ A + B\log(d) - D & \textbf{for open area} \end{cases}$$

  - The path loss is for carrier frequency f_c between 100MHz and 1920MHz.
  - The heights of base station and mobile station are also identified.
  - *$a(h_m)$ (see next page)* is used as the correction factor for mobile antenna height, which is also dependent on carrier frequency.

Path Loss Models for Macro-cellular Areas (2)

$$A = 69.55 + 26.16\log_{10}(f_c) - 13.82\log_{10}(h_b) - a(h_m)$$

$$B = 44.9 - 6.55\log_{10}(h_b)$$

$$C = 5.4 + 2[\log_{10}(f_c/28)]^2$$

$$D = 40.94 + 4.78[\log_{10}(f_c)]^2 - 18.33\log_{10}(f_c)$$

For small to medium cities,

$$a(h_m) = [1.1\log_{10}(f_c) - 0.7]h_m - [1.56\log_{10}(f_c) - 0.8]$$

For large cities,

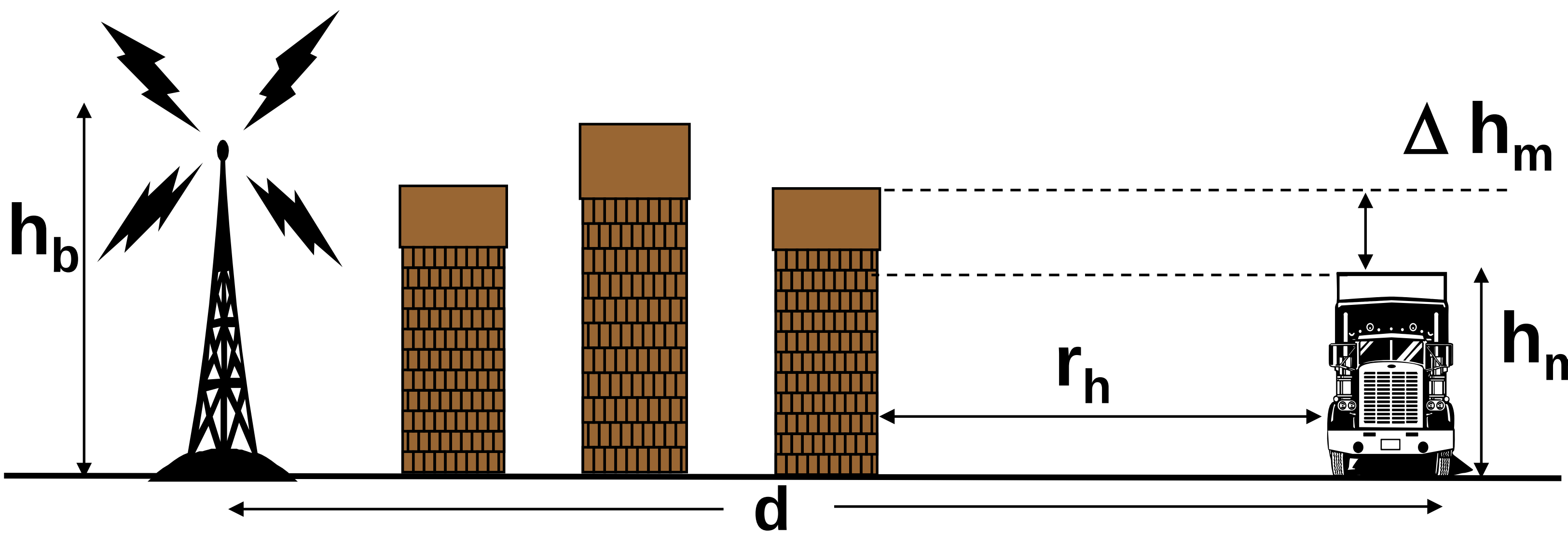
$$a(h_m) = \begin{cases} 8.29[\log_{10}(1.54h_m)]^2 - 1.1 & f_c <= 200MHz \\ 3.2[\log_{10}(1.75h_m)]^2 - 4.97 & f_c <= 200MHz \end{cases}$$

Path Loss Models for Micro-cellular Areas

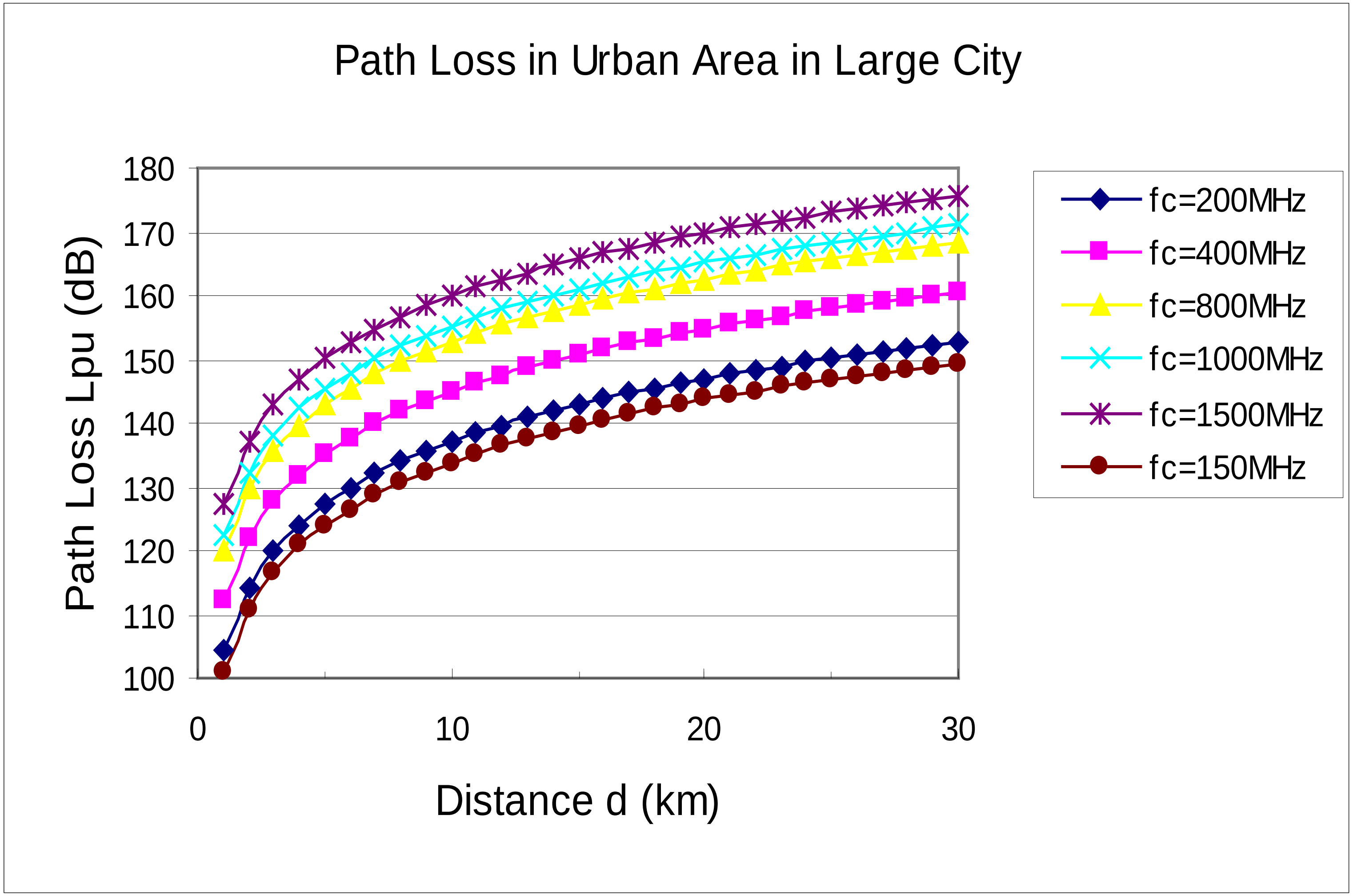
- Microcells span hundreds of meters to a kilometer or so and are usually supported by below rooftop level base station antennas mounted on lampposts or utility poles.
- Usually they are no longer in circular shape because of the streets and buildings in urban areas.
- The propagation characteristics are very complex.

Path Loss Models for Micro-cellular Areas -- Factors

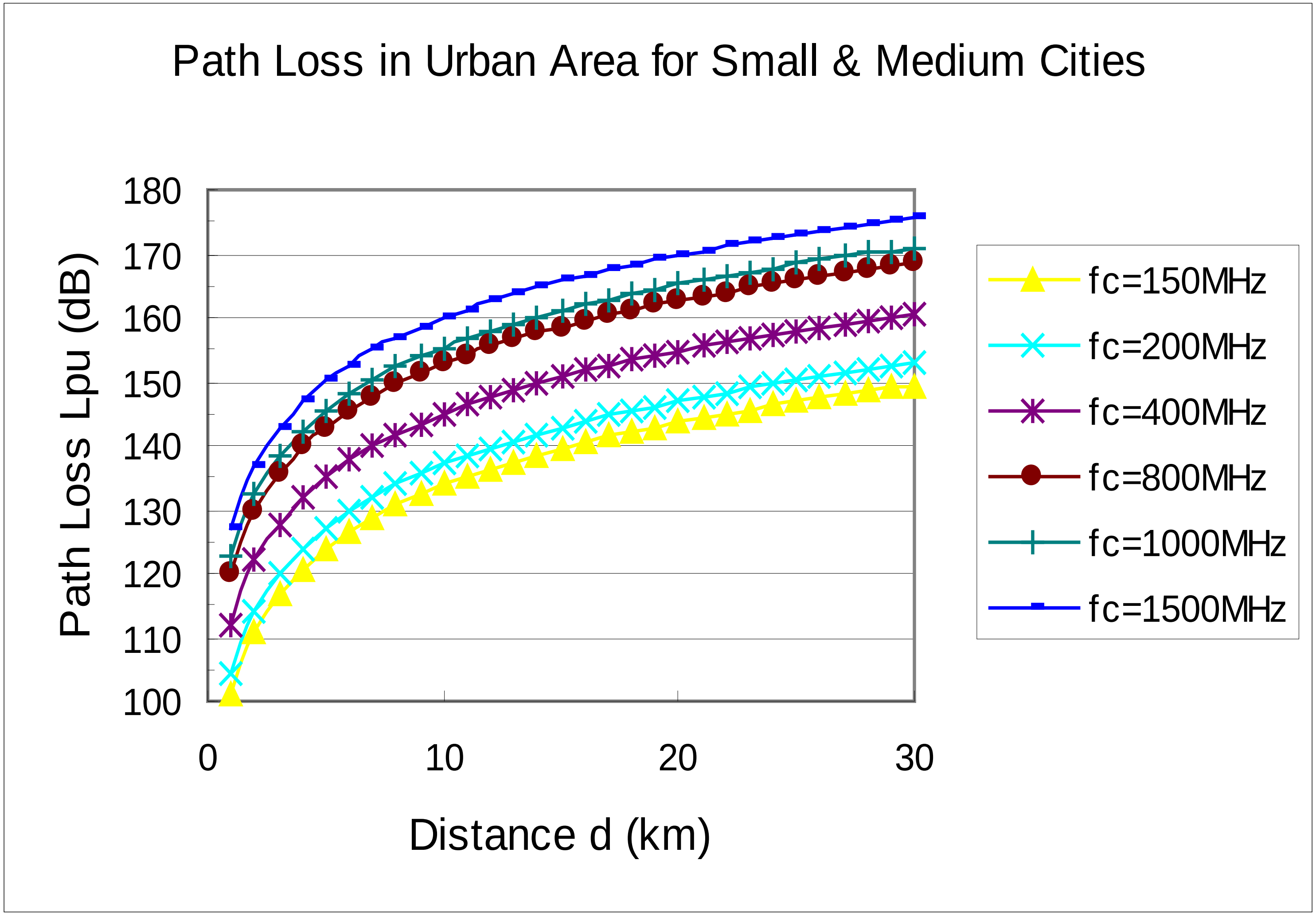
- Distance between the mobile terminal and the transmitter in kilometers, the heights of the base station and mobile terminal, carrier frequency.
- The distance of the mobile terminal from the last rooftop – A rooftop acts as a diffracting screen, and distance from the closet, such rooftop becomes important in NLOS situations.



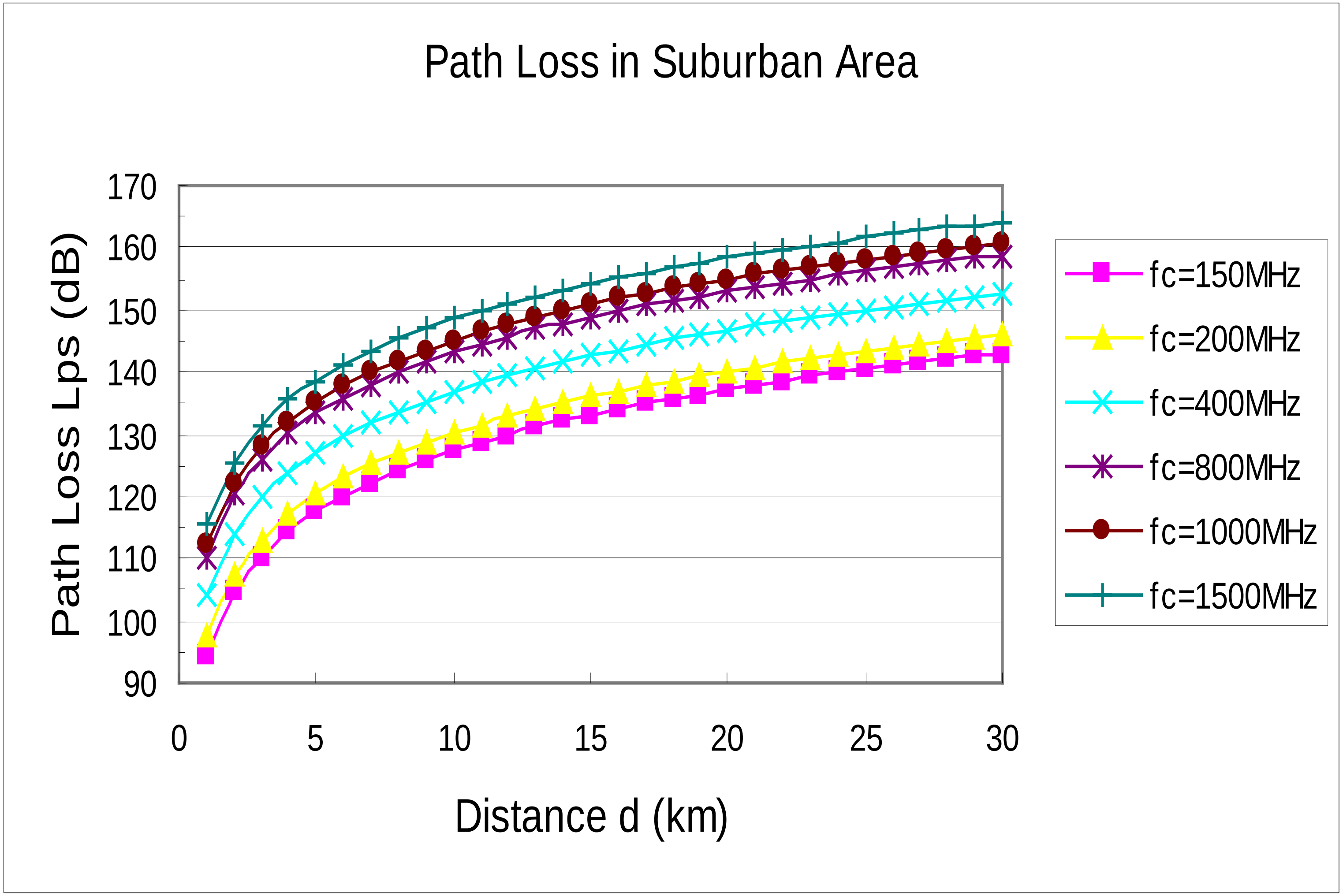
Path-Loss in Urban Area: Large City



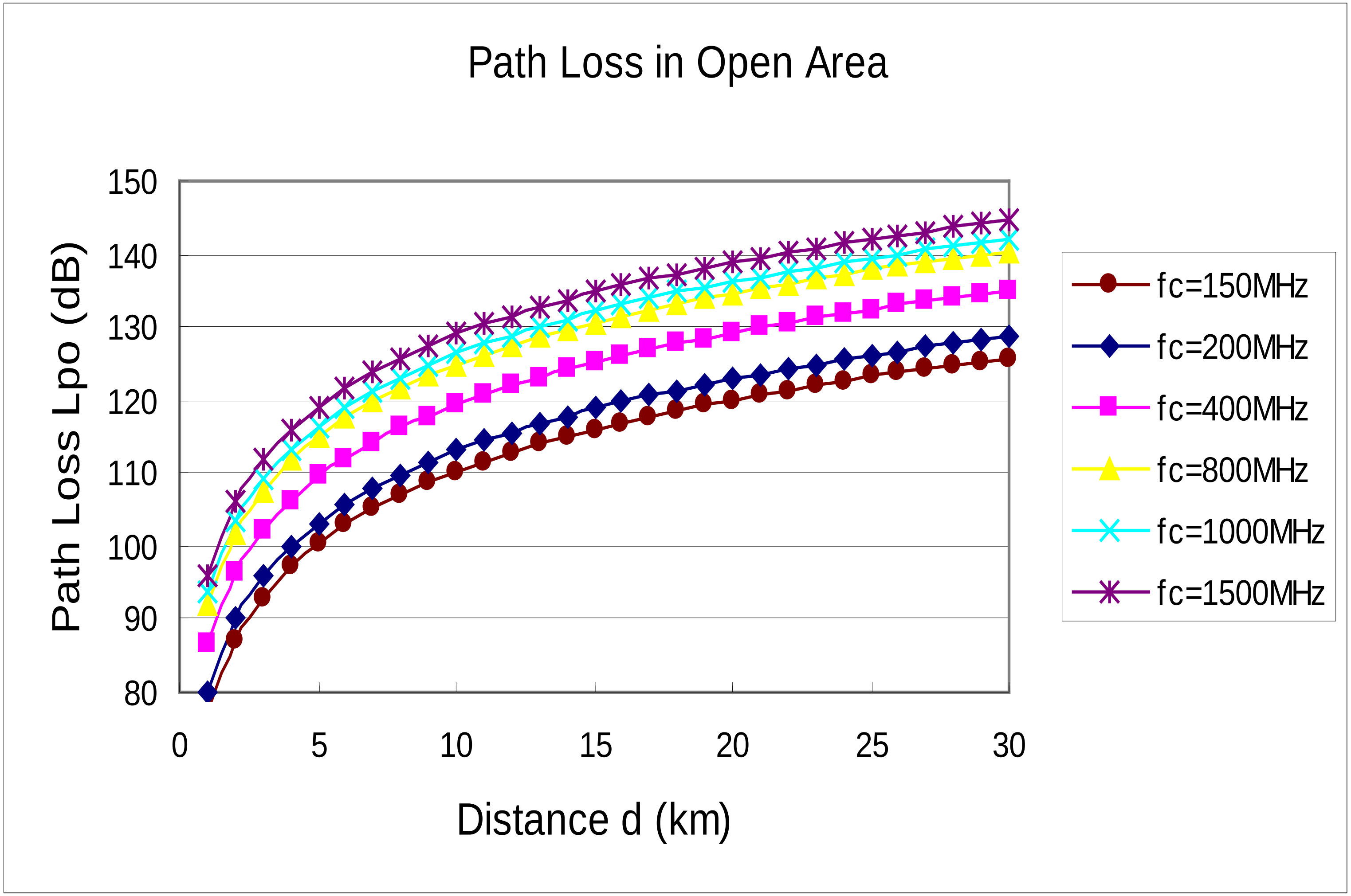
Path-Loss in Urban Area: Small and Medium Cities



Path Loss in Suburban Area



Path Loss in Open Area



Radio Propagation and Antenna

- Wired and wireless medium
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  - Shadow fading
  - Different environments
- Effect of Multipath and Doppler
  - Multipath fading
  - Doppler spectrum

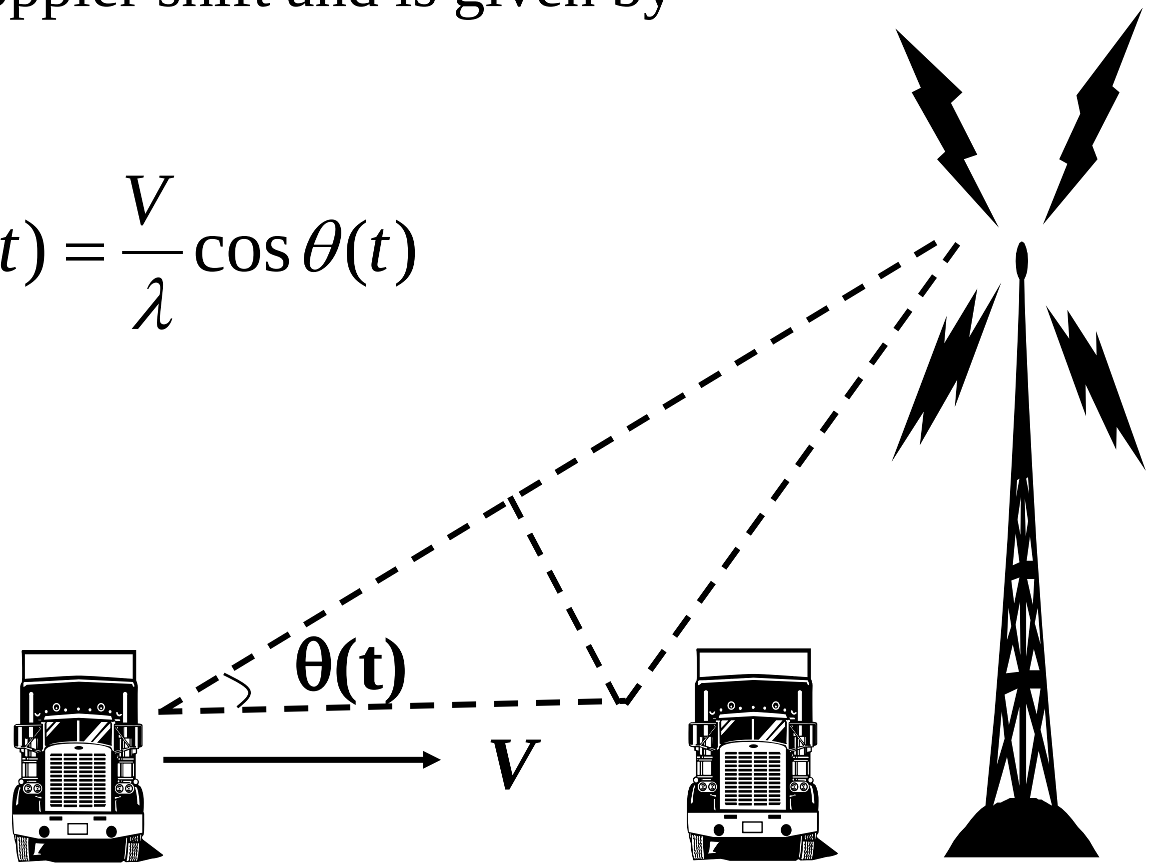
Effect of Multipath and Doppler

- *Small-scale fading*: The received signal is rapidly fluctuating due to the mobility of the terminal causing changes in multiple signal components arriving via different paths.
- There are two effects which contribute to the rapid fluctuation of the signal amplitude.
  - *Multipath fading:* caused by the addition of signals arriving via different paths.
  - *Doppler*: caused by the movement of the mobile terminal toward or away from the base station transmitter.
- Small-scale fading results in very high bit error rates. It is not possible to simply increase the transmit power to overcome the problem
  - Error control coding, diversity schemes, directional antennas.

Effect of Multipath and Doppler

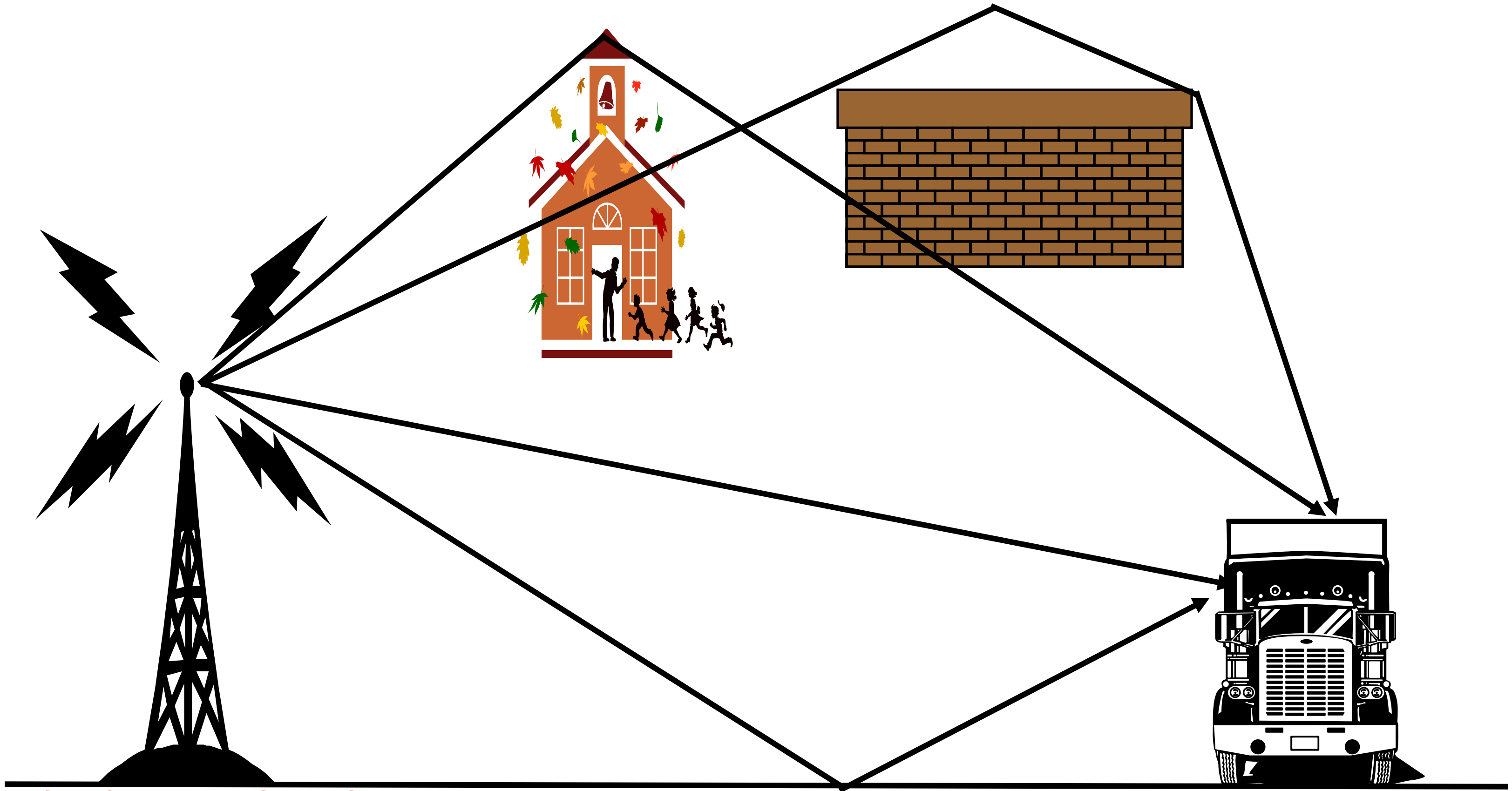
- BS transmits a single frequency f, the received signal at the MT at time t has a frequency of $f+v(t)$.
- $v(t)$ is the Doppler shift and is given by

$$v(t) = \frac{V}{\lambda}\cos\theta(t)$$



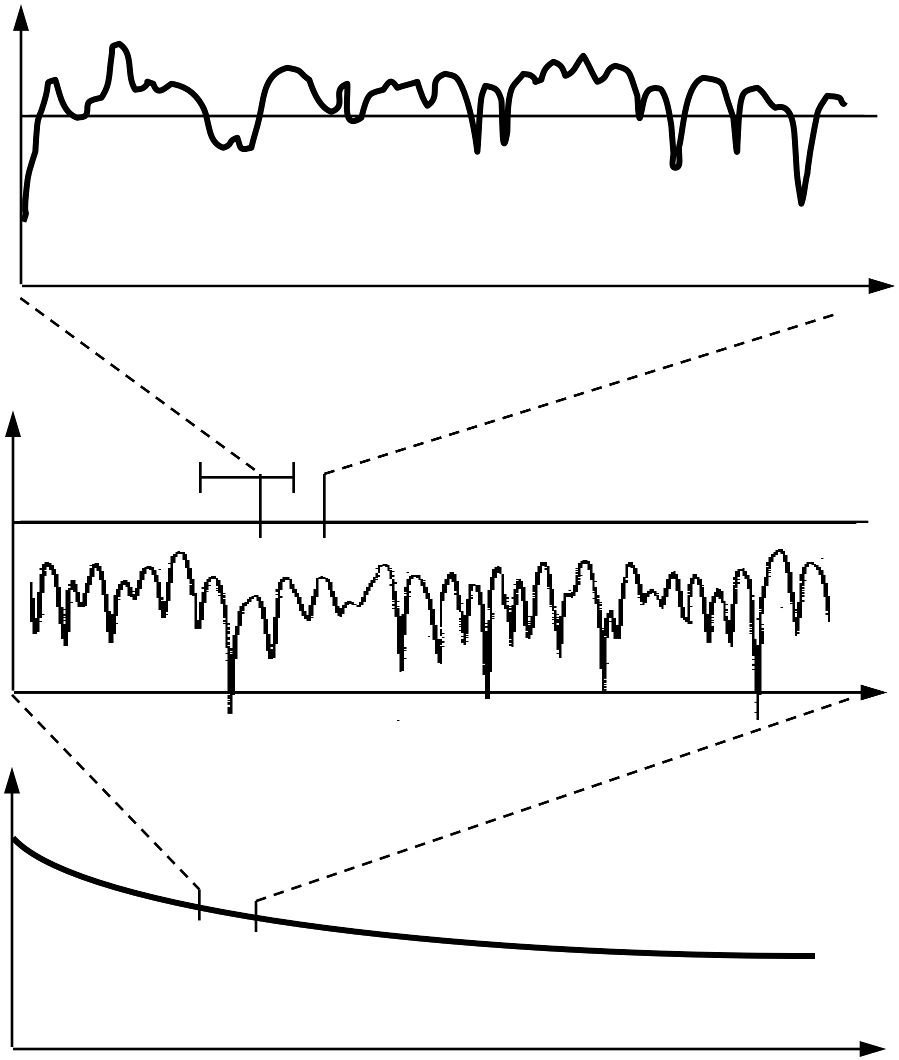
Modeling of Multipath Fading

- Results in fluctuations of the signal amplitude because of the addition of signals arriving with different *phases*.
- This phase difference is caused due to the fact that signals have traveled different distances by traveling along different *paths*.



Modeling of Multipath Fading (Narrowband)

- **Narrowband fading**
  - Autocorrelation
  - Power spectral density
- Because of the phases of the arriving paths are changing rapidly, the received signal amplitude undergoes rapid fluctuation that is often modeled as a <u>*random variable*</u>.

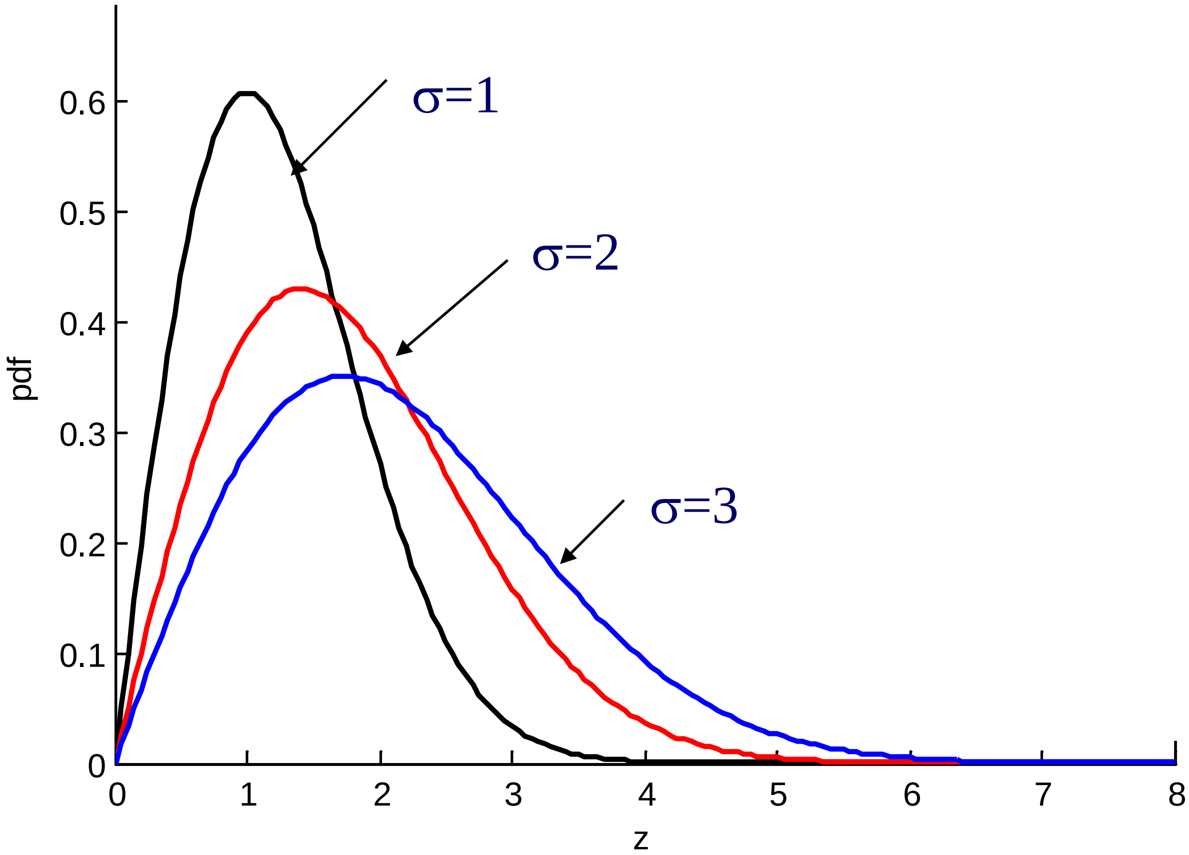


- Rayleigh distribution (NLOS)
  - Most commonly used distribution for multipath fading (the envelope distribution of received signal) is Rayleigh distribution with pdf

$$f_{ray}(z) = \frac{2z}{P_r}\exp\left(-\frac{z^2}{P_r}\right) = \frac{z}{\sigma^2}\exp\left(-\frac{z^2}{2\sigma^2}\right), z \geq 0$$

  - Assume that all signals suffer nearly the same attenuation, but arrive with different phases.
  - σ^2 is the variance.
  - Middle value r_m of envelope signal within sample range to be satisfied by $P(r \leq r_m) = 0.5$. We have $r_m = 1.777\,\sigma$.

Raleigh Distribution



The pdf of the envelope variation

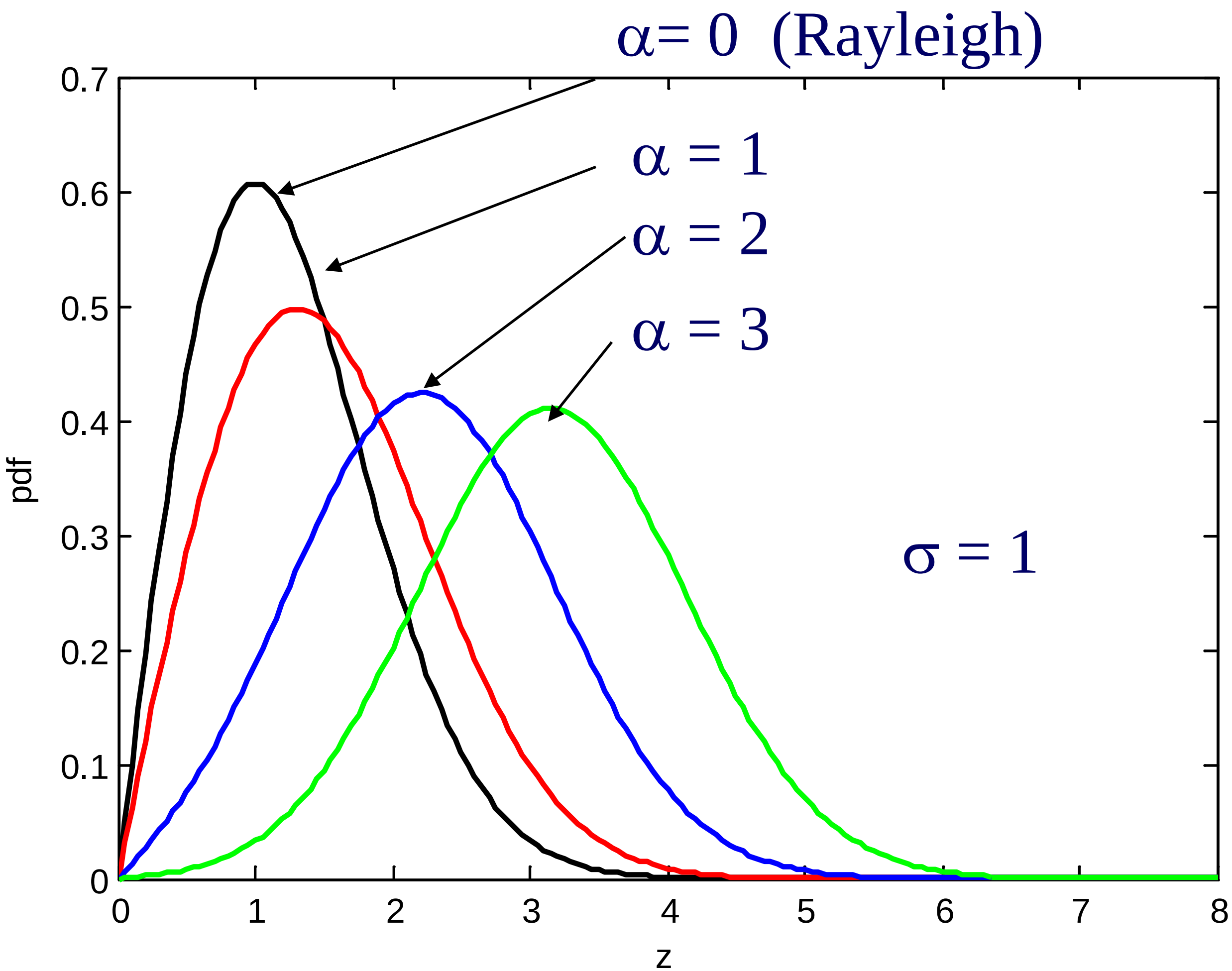
- Ricean distribution (LOS – transmitter is close)
  - When a strong LOS signal component also exists, the pdf is given by

$$f_{ric}(z) = \frac{z}{\sigma^2}\exp\left(\frac{-(z^2+\alpha^2)}{2\sigma^2}\right) I_0\left(\frac{\alpha z}{\sigma^2}\right), z \ge 0, \alpha \ge 0$$

  - α is a factor that determines how strong the LOS component is relative to the rest of the multipath signals. If α=0, then it becomes Rayleigh distribution.
  - $I_0(x)$ is the zero-order modified Bessel function of the first kind.
  - $K = \frac{\alpha^2}{2\sigma^2}$, $K = 0 \Rightarrow Rayleigh, K = \infty \Rightarrow nofading$

$$f_{ric}(z) = \frac{2z(K+1)}{P_r}\exp\left(-K - \frac{(K+1)z^2}{P_r}\right) I_0\left(2z\sqrt{\frac{K(K+1)}{P_r}}\right), z \ge 0$$

Rician Distribution



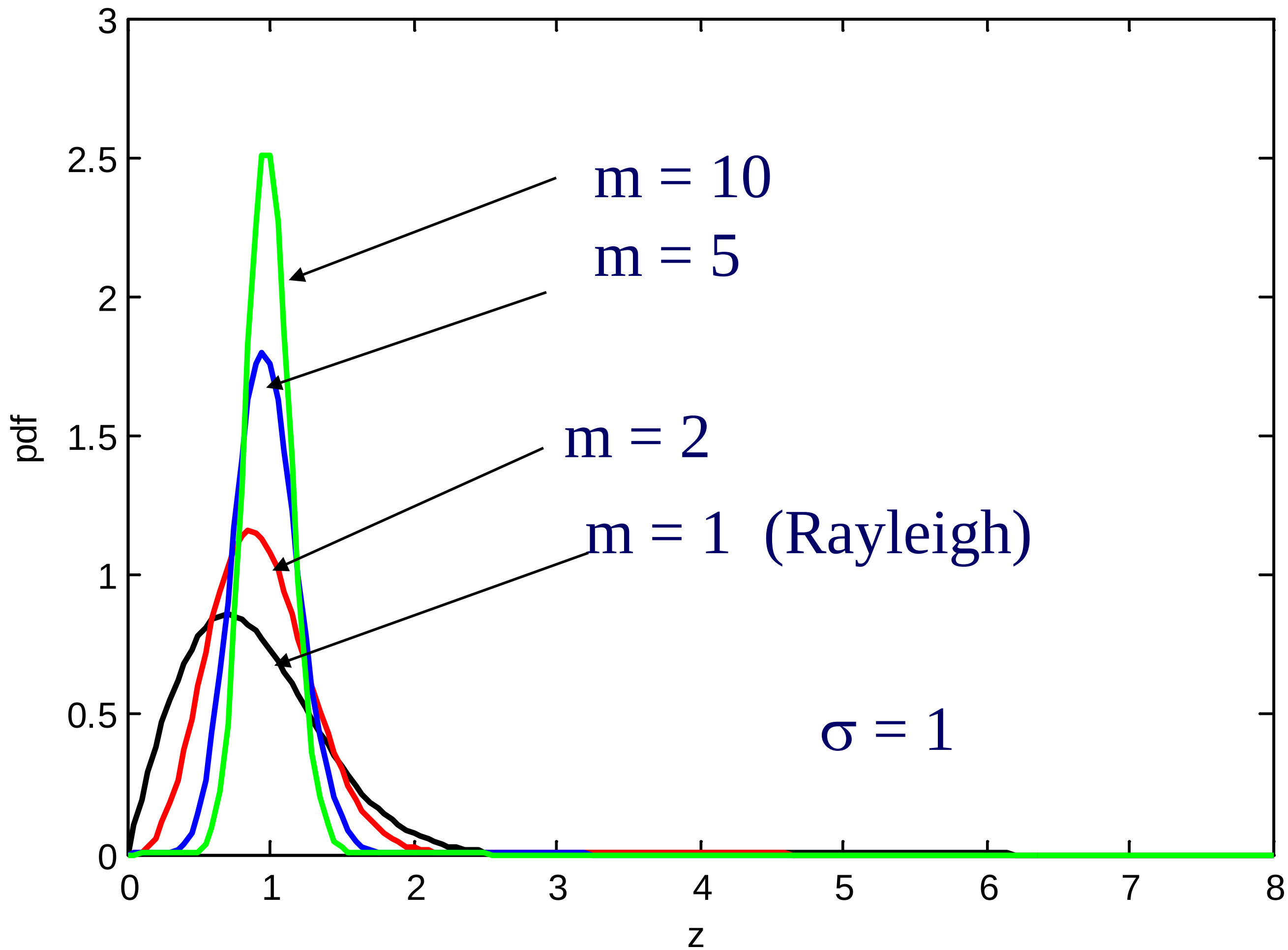
The pdf of the envelope variation

- Nakagami-m fading (general model)
  - Parameters adjusted to fit a variety of empirical measurements

$$f_{naka}(z) = \frac{2m^m z^{2m-1}}{\Gamma(m) P_r^m} \exp\left(\frac{-mz^2}{P_r}\right) I_0\left(\frac{\alpha z}{\sigma^2}\right), m \geq 0.5$$

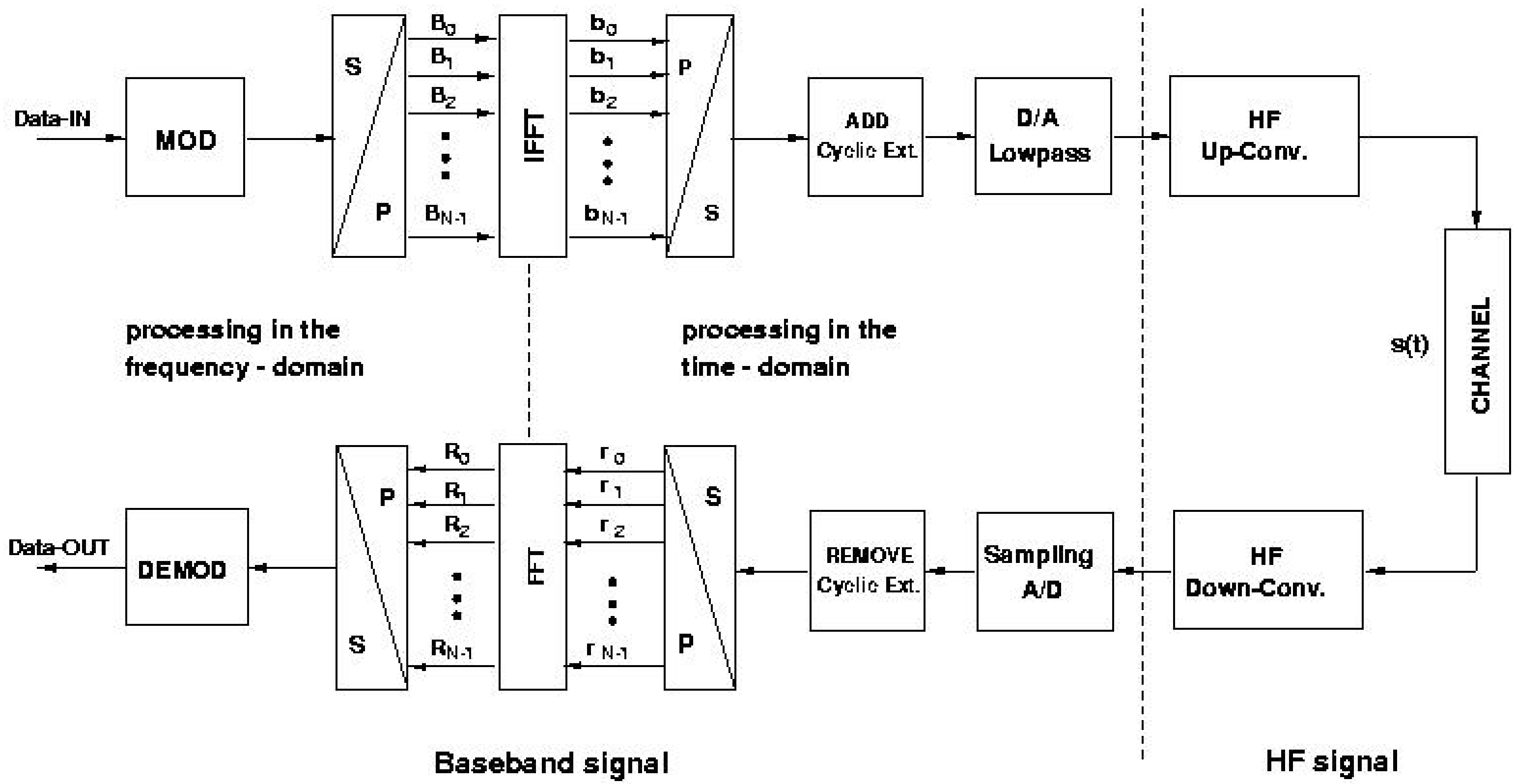
  - m=1, Rayleigh fading
  - $m = (K+1)^2/(2K+1)$, approximately Rician fading
  - $m = \infty$, no fading

Rician Distribution



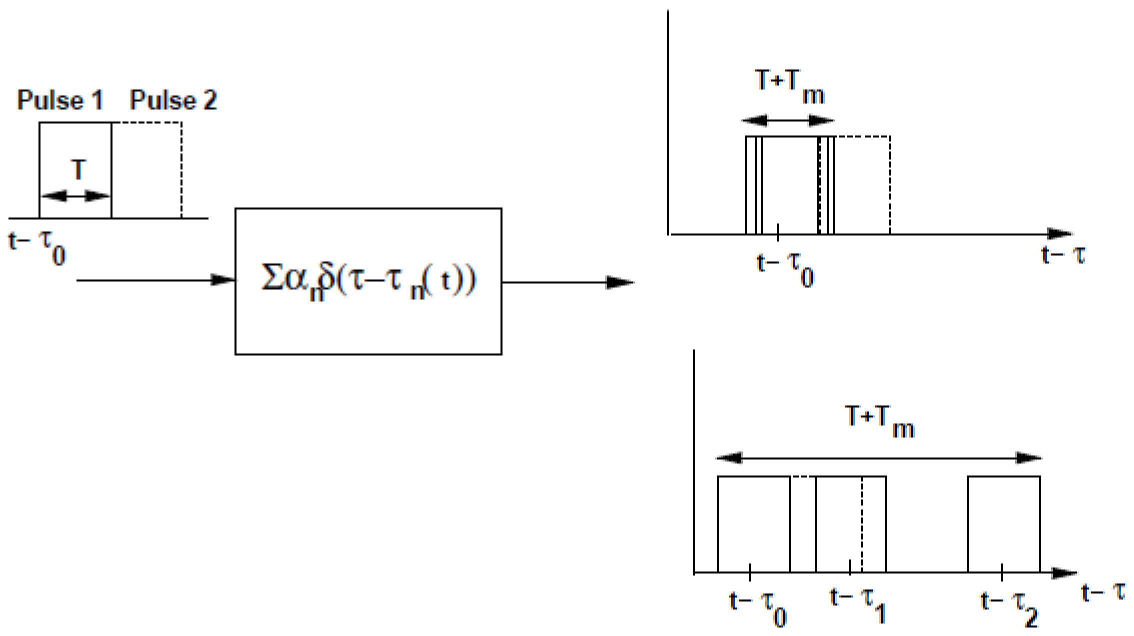
The pdf of the envelope variation

Practical System (Narrowband fading)



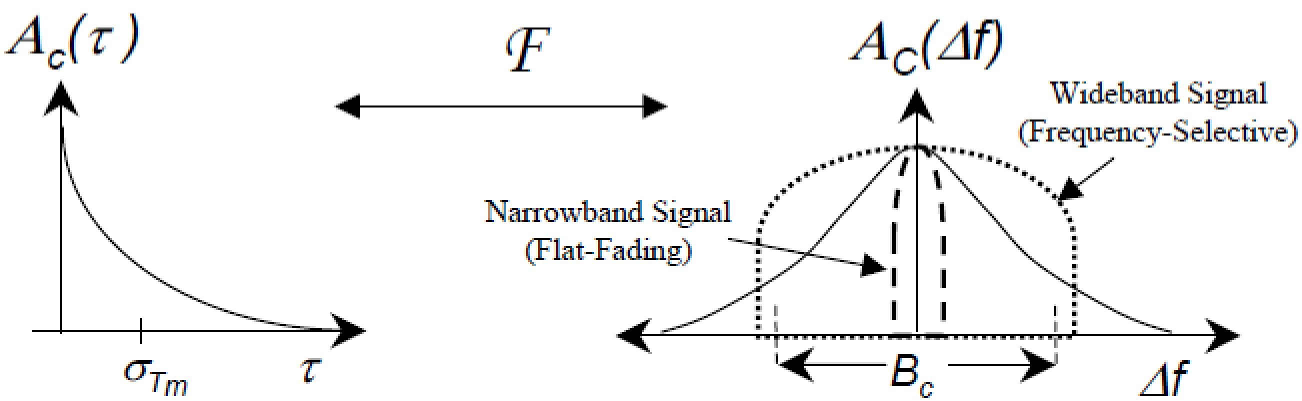
OFDM

- **Wideband fading**
  - Multipath delay spread

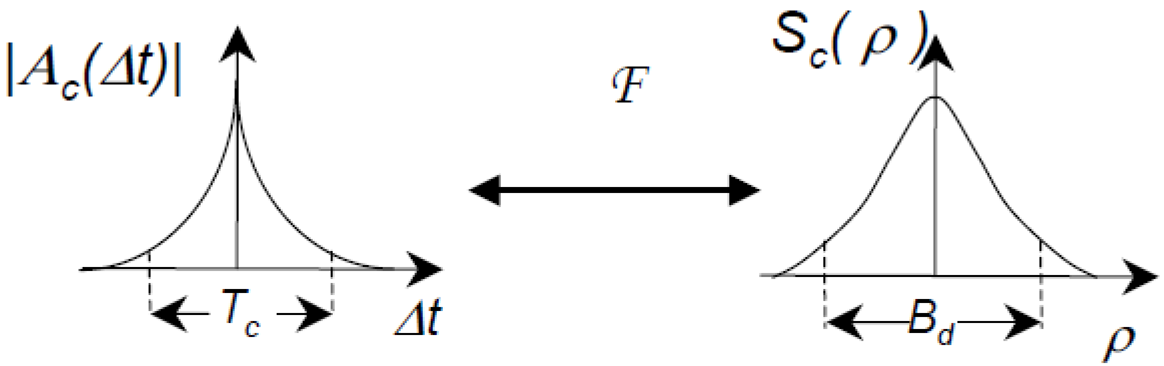


  - Intersymbol interference (ISI)
  - Equalization, Multicarrier modulation, spread spectrum

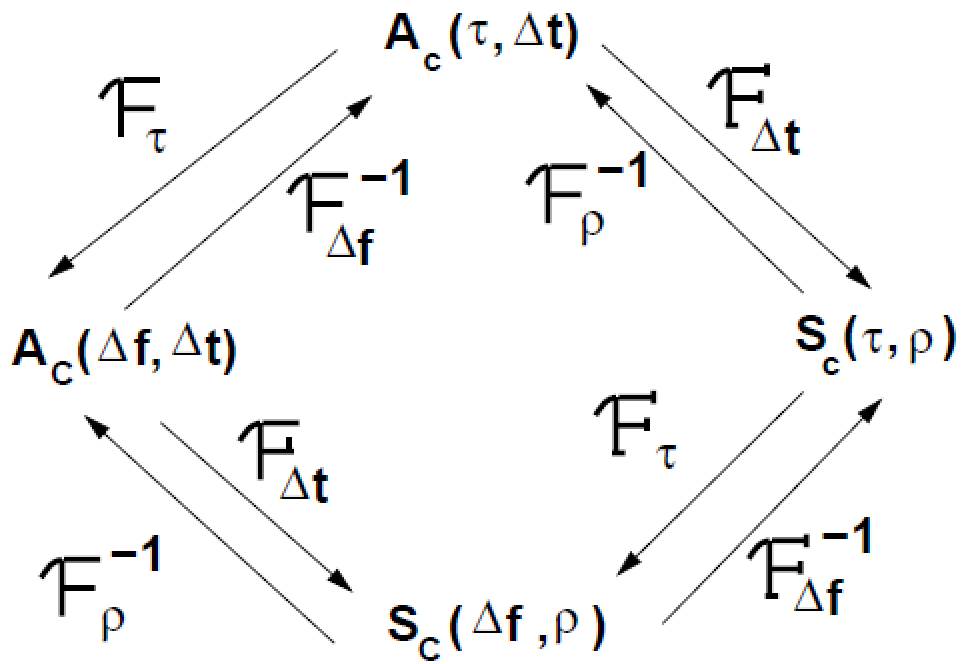
- **Wideband fading**
  - Deterministic: deterministic scattering function
  - Random: scattering function
  - Important characterizations
    - Power delay profile (PDP)
    - Coherence bandwidth



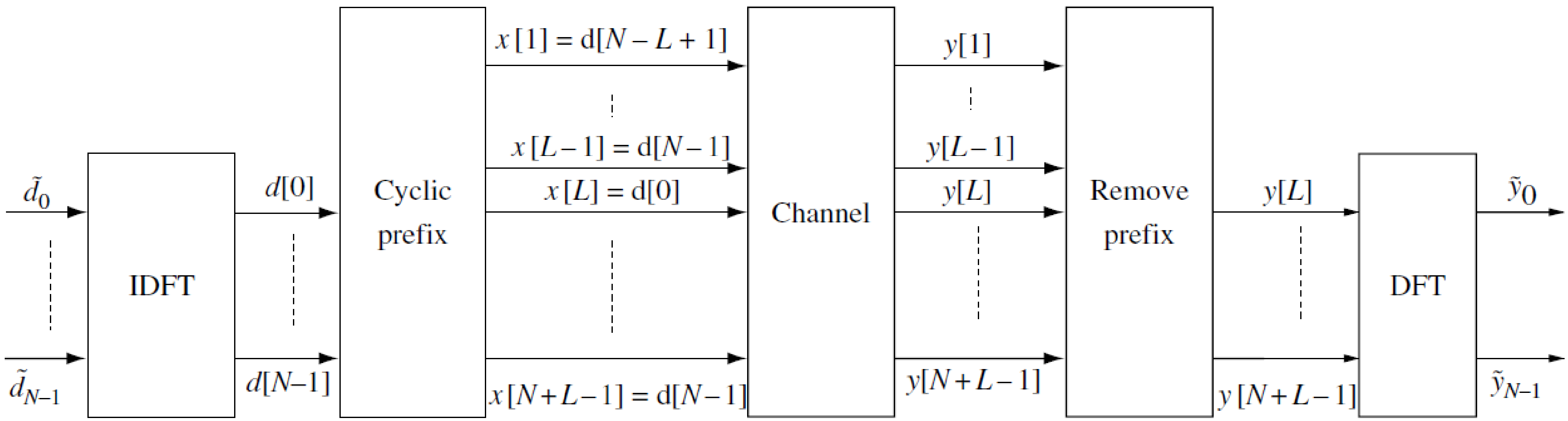
- **Wideband fading**
  - Important characterizations
    - Doppler power spectrum
    - Coherence time



- Fourier transform relationship



- General processing procedures

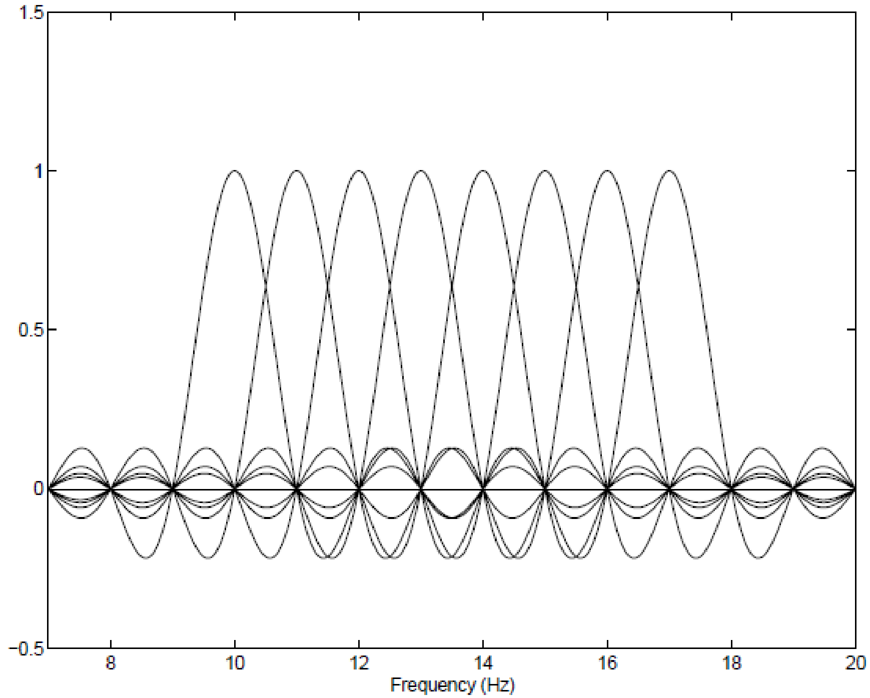


IDEA: If channel is LTI, sinusoids are eigenfunctions.

- Challenges
  - PAPR (Peak to Average Power Ratio)

$$\frac{\max_i |x[i]|^2}{E\{|x[i]|^2\}}$$

  - Frequency/time offset



Summary: Wireless Communications

- Comparison of wired and wireless medium
- Frequency bands and licenses
- Radio propagation mechanism
  - Reflection and transmission
  - Diffraction
  - Scattering
- Indoor and outdoor radio propagation
- Distance-power relationship
- Shadow effect
- Path-loss model
  - Free space propagation
  - Marocell environments
  - Microcell environments
- Multipath and Doppler spectrum
  - Narrowband fading
  - Wideband fading
  - Wideband system (OFDM)

Capacity

AWGN

- Consider a discrete-time AWGN channel with input and output relationship

$$y[i] = x[i] + n[i]$$

- Denote SNR=P/N_0B.
- The capacity is given by (Shannon capacity)

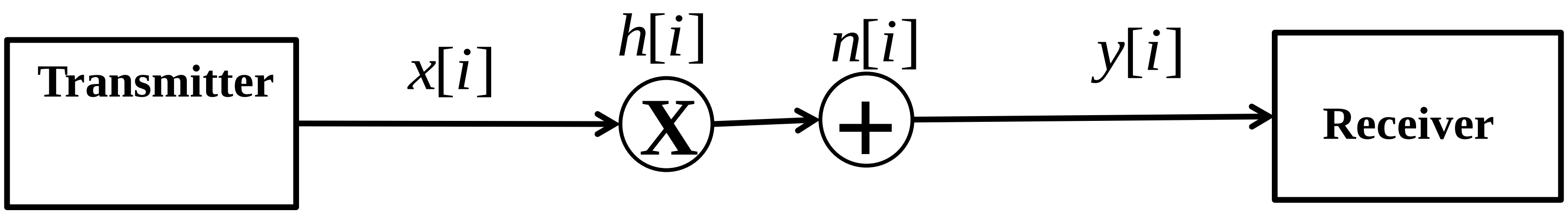
$$C = B\log_2(1+\mathrm{SNR})$$

Also interesting works on finite-blocklength code

Flat Fading Channels

- Consider a discrete-time channel with stationary and ergodic time-varying gain $h[i]$ and AWGN $n[i]$, the input and output relationship is given by

$$y[i] = h[i]x[i] + n[i]$$



- Depending on the availability of channel state/side information (CSI), we have different channel capacity:
  - Channel distribution information (CDI)
  - CSI at the receiver
  - CSI at the transmitter and the receiver

CDI

- The distribution of the channel gain $z[i]=|h[i]|^2$ is known to the transmitter and receiver
  - Generally unknown
  - Known for IID Rayleigh fading and finite-state Markov channels

- IID Rayleigh fading
  - Optimal input: a finite number of points, one of which is located at zero
  - Still no closed form expression

- FSMC channel
  - Capacity expressions for IID and general inputs
  - Depends on all past inputs and outputs (Complicated)

CSI at the receiver

- Assume $z[i]=|h[i]|^2$ is known at the receiver. Also assume that both the transmitter and receiver know the distribution of $z[i]$.
  - Ergodic capacity (Shannon capacity)
  - Capacity with outage

- Ergodic capacity

$$C=\int_0^{\infty} B\log_2(1+\mathrm{SNR}z)\,p(z)dz$$

  - Sufficiently long codeword to average over the distribution of $z[i]$.
  - Advanced codes: rateless code.

    U. Erez, M. D. Trott, and G. W. Wornell, "Rateless coding for Gaussian channels," *IEEE Trans. Inform. Theory*, vol. 58, no. 2, pp. 530 - 547, Feb. 2012.

$$C=\mathrm{E}\{B\log_2(1+\mathrm{SNR}z)\}=\int_0^{\infty} B\log_2(1+\mathrm{SNR}z)\,p(z)dz\le B\log_2(1+\mathrm{SNR}\mathrm{E}\{z\})$$

  - Imperfect CSI significantly decreases the capacity.

CSI at the receiver

- Capacity with outage
  - CSI stays constant over a long period (slowly varying channel)
  - Since the transmitter has no CSI, it sends information at a FIXED transmission rate.

$$R = B\log_2(1+\mathrm{SNR}z_{\min})$$

  - If $z[i] \geq z_{\min}$, the receiver can successfully decode the received signal; otherwise, ourage occurs.
  - The probability of outage

$$P_{\mathrm{out}} = \Pr\left\{z \leq z_{\min}\right\}$$

  - The average rate of the system is then given by

$$C_{\mathrm{out}} = \left(1 - P_{\mathrm{out}}\right)R = \left(1 - P_{\mathrm{out}}\right)B\log_2(1+\mathrm{SNR}z_{\min})$$

CSI at the transmitter and the receiver

- The transmitter can adapt its transmission strategy with respect to the CSI
  - Shannon capacity (optimal power and rate adaptation)
  - Zero-outage capacity
  - Outage capacity

- Shannon capacity
  - SNR= P/N_0B. With power adaptation, we define

$$\mu[i] = \frac{P[i]}{N_0 B}$$

  - $\mu(z)$: the specific value at a given channel state, we have

$$\mathrm{E}\{\mu(z)\} = \int_0^\infty \mu(z) p(z) dz \le \mathrm{SNR}$$

  - The capacity with given $\mu(z)$ can be expressed as

$$C = \mathrm{E}\{B \log_2(1+\mu(z)z)\} = \int_0^\infty B \log_2(1+\mu(z)z) p(z) dz$$

  - The capacity of fading channels with average power constraint is defined as

$$C = \max_{\int_0^\infty \mu(z) p(z) dz \le \mathrm{SNR}} \int_0^\infty B \log_2(1+\mu(z)z) p(z) dz$$

CSI at the transmitter and the receiver

- Shannon capacity
  - Optimal power control policy is "water-filling"

$$\mu(z) = \begin{cases} \frac{1}{\alpha} - \frac{1}{z} & z \geq \alpha \\ 0 & \text{else} \end{cases}$$

with α satisfying

$$\mathrm{E}\{\mu(z)\} = \int_{\alpha}^{\infty} \left(\frac{1}{\alpha} - \frac{1}{z} \right) p(z)\,dz = \mathrm{SNR}$$

CSI at the transmitter and the receiver

- Zero-outage capacity
  - Maximum data rate that can be maintained in all non-outage channel states multiplied by the probability of non-outage.
  - "Truncated channel inversion" power control policy

$$\mu(z) = \frac{\sigma}{z} \ , \qquad \sigma = 1/\mathrm{E}\{1/z\}$$

  - Capacity is given by

$$C = B\log_2(1+\sigma) = B\log_2(1+1/\mathrm{E}\{1/z\})$$

CSI at the transmitter and the receiver

- Outage capacity
  - Constant received SNR at the receiver.
  - "Channel inversion" power control policy

$$\mu(z) = \begin{cases} \dfrac{\sigma}{z} & z \geq \alpha \\ 0 & \text{else} \end{cases}, \quad \sigma = 1/\mathrm{E}_\alpha\{1/z\}$$

  - Outage probability $P_{\text{out}} = \Pr\{z \leq \alpha\}$
  - Outage capacity associated with P_{out} is given by

$$C = B\log_2(1 + 1/\mathrm{E}_\alpha\{1/z\})\Pr\{z \geq \alpha\}$$

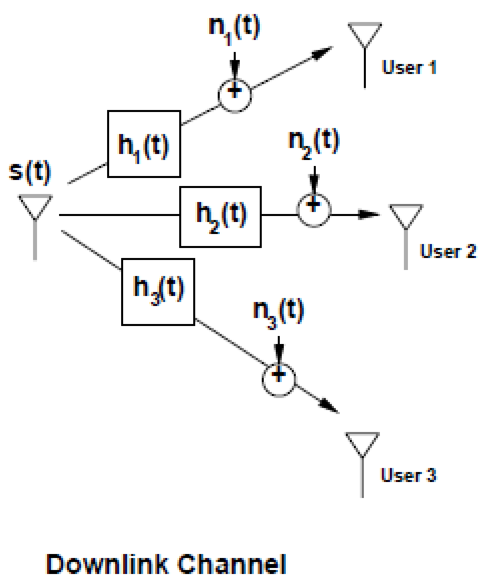
  - Maximum outage capacity of the system is

$$C = \max_\alpha B\log_2(1 + 1/\mathrm{E}_\alpha\{1/z\})\Pr\{z \geq \alpha\}$$

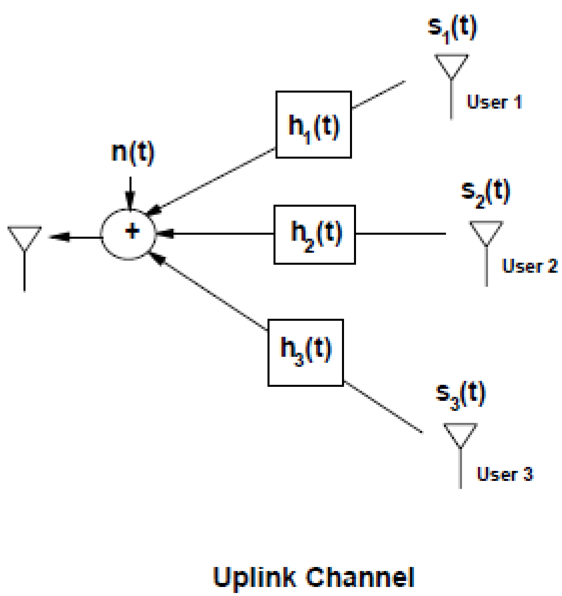
Multiuser Systems

Multiuser Channels

- The multiuser channel models

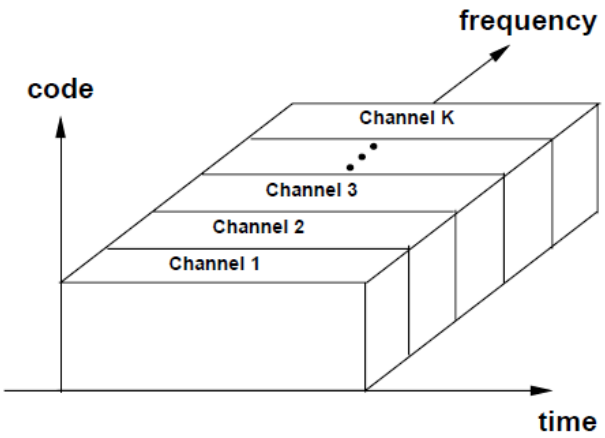


Broadcast Channel (BC)

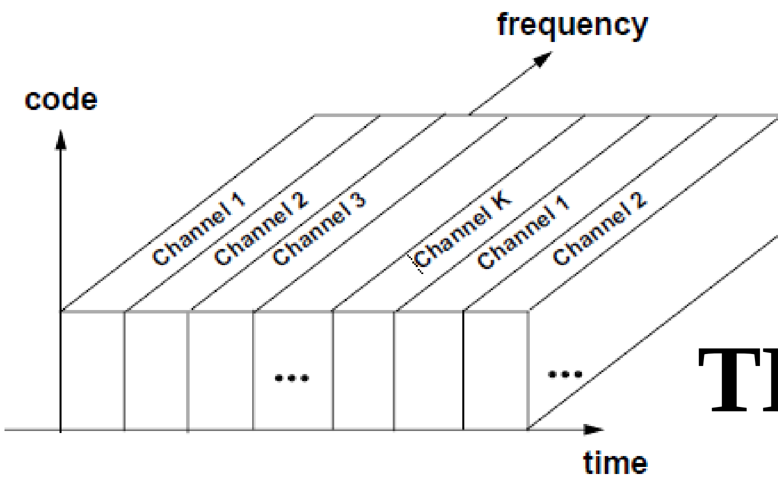


Multi-Access Channel (MAC)

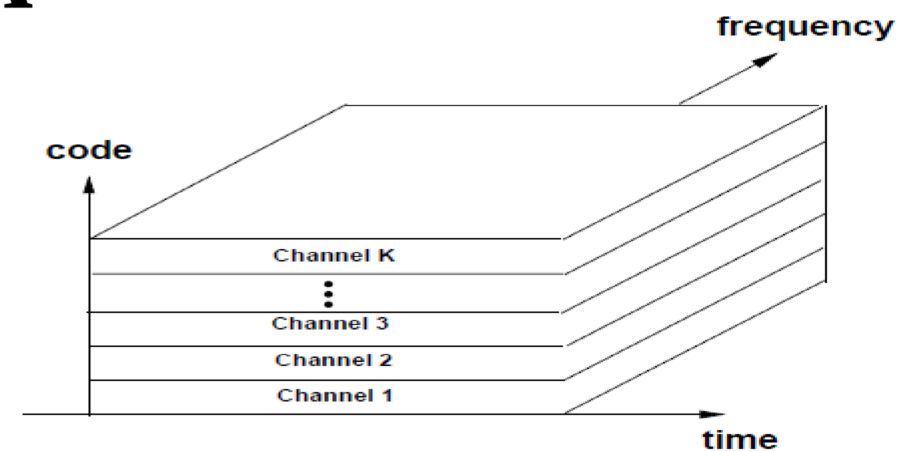
- Access Methods



FDMA



TDMA



CDMA

Multiuser Channels

- Random Access
  - Packets of N bits, transmission rate of R bits/s

$$\tau = N / R$$

  - Packet error rate: probability of decoding error

$$\Pr\{C < R\}$$

  - Generally assume Poisson process of arrival packets.

$$p(X(t) = k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

  - Traffic load:

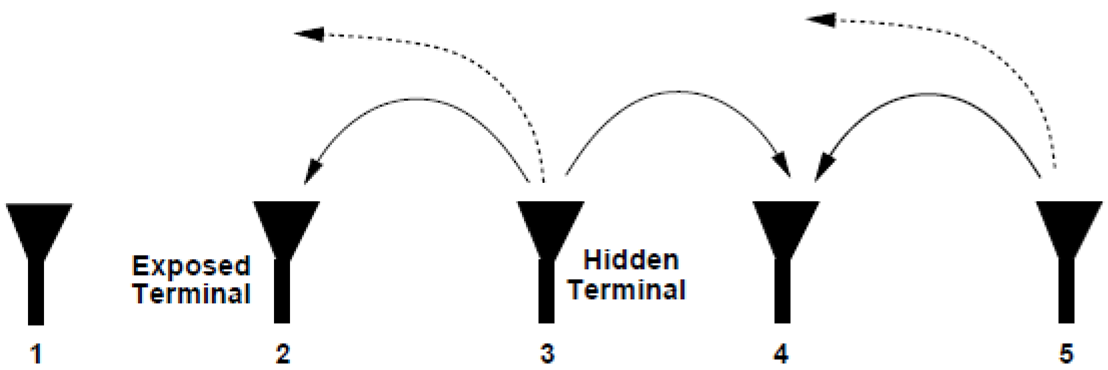
$$L = \lambda \tau = \lambda \bullet 1 / R_p$$

$L > 1$: on average more packets arrive in the system, so unstable

  - Throughput $T = L(1 - \Pr\{C < R\})$
  - Effective data rate RT

Multiuser Channels

- Carrier Sense Multiple Access (CSMA)
  - Sense the channel and delay the transmission if the channel is in use



  - Each node can hear its immediate neighbor but no other nodes. Node 3 and 5 each wish to transmit to node 4. If node 5 starts transmission, since node 3 cannot detect this transmission, it assumes that the channel is idle, and begins its transmission, causing collision with node 5's transmission**. Node 3 is "hidden" from node 5 because it cannot detect node 5's transmission.**

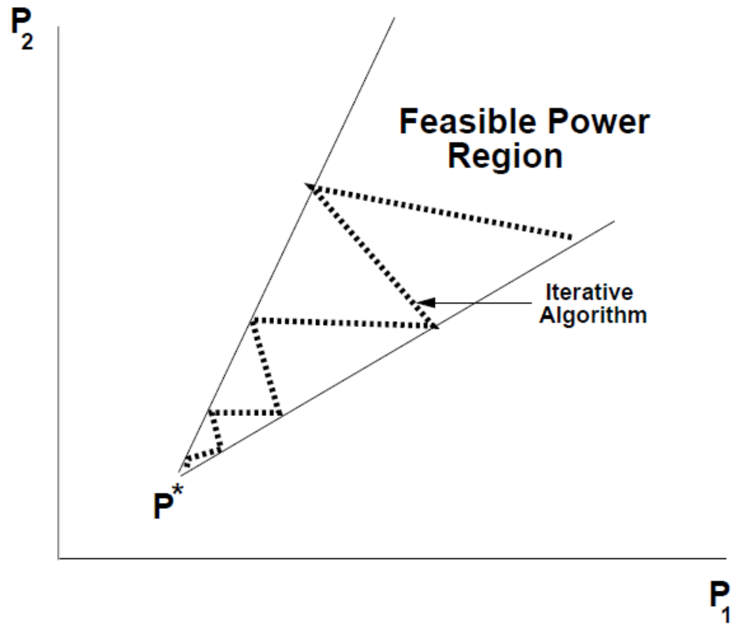
RTS-CTS protocol

- Power Control
  - Downlink is easy
  - Uplink

    SINR of user k:

$$\gamma_k = \frac{g_k P_k}{n + \rho \sum_{j \neq k} g_j P_j}, \quad k = 1, \ldots, K,$$

Certain minimum SINR requirement is enforced. Then, there will be the associated algorithm.



Multiuser Channels

- Broadcast Channel
  - We consider a broadcast channel where the transmitter sends different data to different receivers.
  - Capacity region (rate region).
  - Two-user AWGN channel. Consider the discrete input-output relationship given by

$$y_k[i] = x[i] + n_k[i], k = 1,2.$$

  - TD

$$\mathcal{C}_{TD} = \bigcup_{\{\tau:\, 0\le\tau\le1\}} \left(R_1 = \tau B \log_2\left(1 + \frac{P}{n_1 B}\right), R_2 = (1-\tau) B \log_2\left(1 + \frac{P}{n_2 B}\right)\right)$$

$$\mathcal{C}_{TD,VP} = \bigcup_{\{\tau, P_1, P_2:\, 0\le\tau\le1;\ \tau P_1 + (1-\tau)P_2 = P\}} \left(R_1 = \tau B \log_2\left(1 + \frac{P_1}{n_1 B}\right), R_2 = (1-\tau) B \log_2\left(1 + \frac{P_2}{n_2 B}\right)\right)$$

- Broadcast Channel
  - FD

$$\mathcal{C}_{FFD} = \bigcup_{\{P_1, P_2:\, P_1+P_2=P\}} \left(R_1 = B_1 \log_2\left(1 + \frac{P_1}{n_1 B_1}\right), R_2 = B_2 \log_2\left(1 + \frac{P_2}{n_2 B_2}\right) \right)$$

$$\mathcal{C}_{FD} = \bigcup_{\{P_1, P_2, B_1, B_2:\, P_1+P_2=P;\, B_1+B_2=B\}} \left(R_1 = B_1 \log_2\left(1 + \frac{P_1}{n_1 B_1}\right), R_2 = B_2 \log_2\left(1 + \frac{P_2}{n_2 B_2}\right) \right)$$

  - CD with successive interference cancellation (SIC)

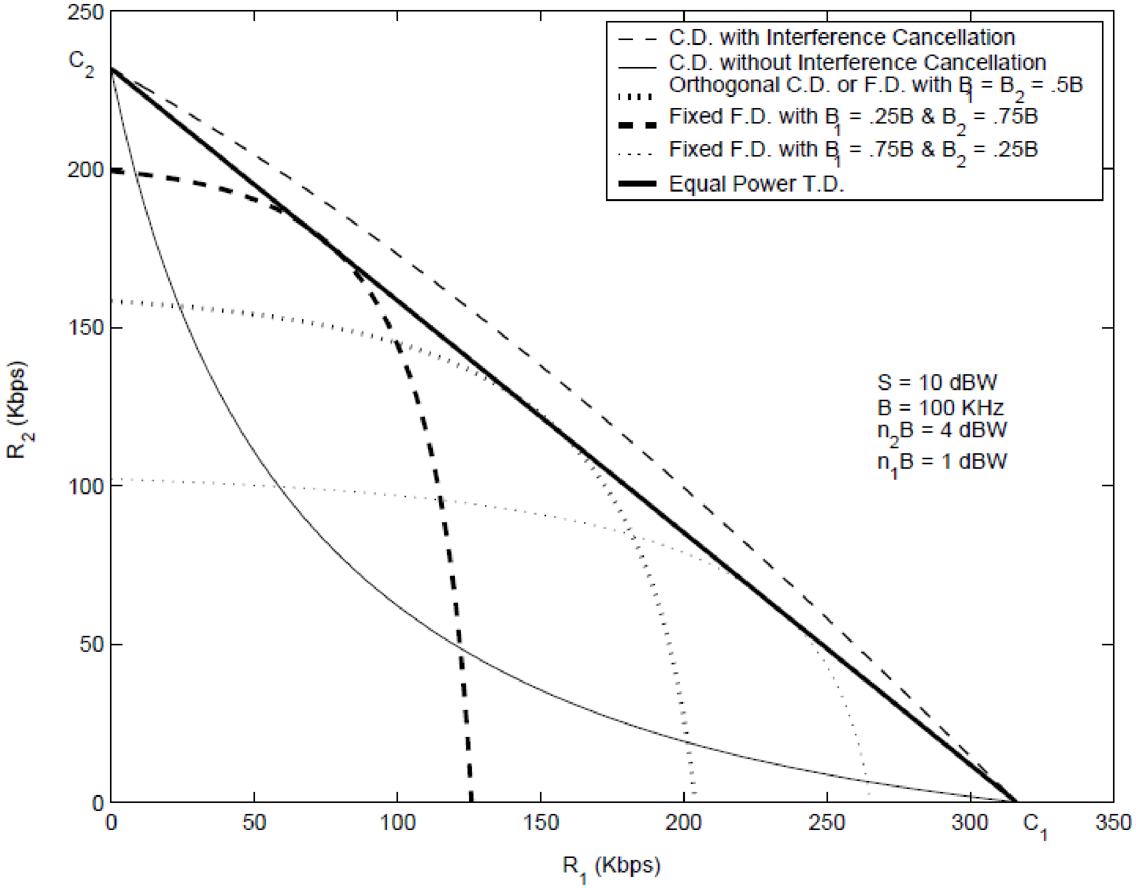
$$n_1 < n_2$$

$$\mathcal{C}_{BC} = \bigcup_{\{P_1, P_2:\, P_1+P_2=P\}} \left(R_1 = B \log_2\left(1 + \frac{P_1}{n_1 B}\right), R_2 = B \log_2\left(1 + \frac{P_2}{n_2 B + P_1}\right) \right)$$

$$\mathcal{C}_{BC} = \bigcup_{\{P_k:\, \sum_{k=1}^{K} P_k=P\}} \left\{ (R_1, \ldots, R_K) : R_k = B \log_2\left(1 + \frac{P_k}{n_k B + \sum_{j=1}^{K} P_j \mathbf{1}[n_k > n_j]}\right) \right\}$$

Multiuser Channels

- Broadcast Channel



Multiuser Channels

- Broadcast Channel
  - Fading channel. Consider the discrete input-output relationship given by

$$y_k[i] = h_k[i]x[i] + w_k[i].$$

  - $E\{|w_k[i]|^2\} = N_0$. Denote $n_k[i] = N_0 / |h_k[i]|^2$ as the effective time-varying noise with noise vector

$$\mathbf{n}[i] = (n_1[i], \ldots, n_K[i]).$$

  - The set of all power control policies satisfying the average power constraint

$$\mathcal{F}_{BC} \equiv \left\{ \mathcal{P} : \mathbf{E}_{\mathbf{n}} \left[\sum_{k=1}^{K} P_k(\mathbf{n}) \right] \leq \overline{P} \right\}$$

  - The instantaneous rate region given power control policy and fading states

$$\mathcal{C}_{BC}(\mathbf{P}(\mathbf{n})) = \left\{ (R_1(\mathbf{P}(\mathbf{n}), \ldots, R_K(\mathbf{P}(\mathbf{n})) : R_k(\mathbf{P}(\mathbf{n})) = B \log_2 \left(1 + \frac{P_k(\mathbf{n})}{n_k B + \sum_{j=1}^{K} P_j(\mathbf{n}) \mathbf{1}[n_k > n_j]} \right) \right\}$$

  - The average achievable rate region with power control over fading states

$$\mathcal{C}_{BC}(\mathcal{P}) = \{ R_k : R_k \leq \mathbf{E}_{\mathbf{n}} [R_k(\mathbf{P}(\mathbf{n}))], \quad k = 1, 2, \ldots, K \}$$

  - Then, the capacity region

$$\mathcal{C}_{BC}(\overline{P}) = \bigcup_{\mathcal{P} \in \mathcal{F}_{BC}} \mathcal{C}_{BC}(\mathcal{P})$$

Multiuser Channels

- Multi-Access Channel
  - We consider a multi-access channel where multiple transmitter sends data to a single receiver.
  - Capacity region (rate region).
  - Two-user AWGN channel. Consider the discrete input-output relationship given by

$$y[i] = h_1[i]x_1[i] + h_2[i]x_2[i] + n[i],$$

  - The two-user MAC capacity region is the closed convex hull of vectors (R_1,R_2) satisfying

$$R_k \le B\log_2\left(1+\frac{g_k P_k}{N_0 B}\right), k = 1, 2$$

$$R_1 + R_2 \le B\log_2\left(1+\frac{g_1 P_1 + g_2 P_2}{N_0 B}\right)$$

- Multi-Access Channel
  - K-user AWGN channel. The MAC capacity region is the closed convex hull

$$\mathcal{C}_{MAC} = \left\{ (R_1, \ldots, R_K) : \sum_{k \in S} R_k \leq B \log_2 \left(1 + \frac{\sum_{k \in S} g_k P_k}{N_0 B} \right), \forall S \subset \{1, 2, \ldots, K\} \right\}$$

  - FD

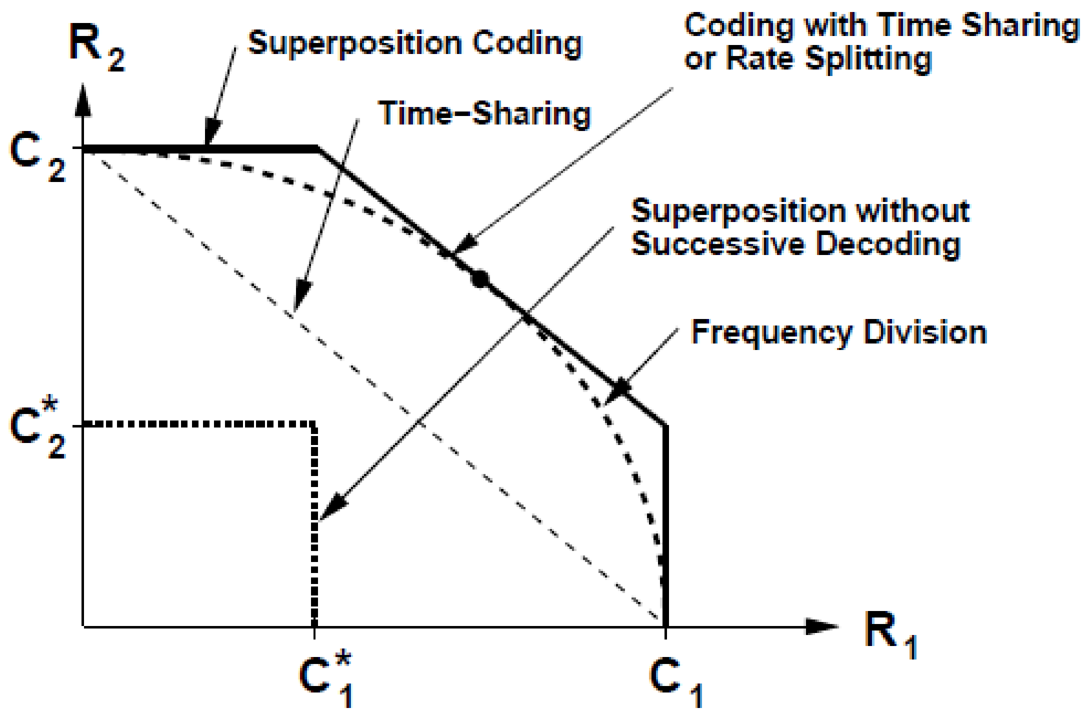
$$\mathcal{C}_{FD} = \bigcup_{\{B_1, B_2 : B_1 + B_2 = B\}} \left(R_1 = B_1 \log_2 \left(1 + \frac{g_1 P_1}{N_0 B_1} \right), R_2 = B_2 \log_2 \left(1 + \frac{g_2 P_2}{N_0 B_2} \right) \right)$$

  - TD

$$\mathcal{C}_{TD,VP} = \bigcup_{\{\tau_1, \tau_2 : \tau_1 + \tau_2 = 1\}} \left(R_1 = \tau_1 B \log_2 \left(1 + \frac{g_1 P_1}{N_0 \tau_1 B} \right), R_2 = \tau_2 B \log_2 \left(1 + \frac{g_2 P_2}{N_0 \tau_2 B} \right) \right)$$

- Multi-Access Channel
  - The sum-rate capacity is the maximum sum of rates of the users, where the maximum is taken over all rate vectors in the capacity region.

$$\mathcal{C}_{MACSR} = B \log_2 \left(1 + \frac{\sum_{k=1}^{K} g_k P_k}{N_0 B} \right)$$



Multiuser Channels

- Multi-Access Channel
  - Multi-access fading channel.
  - The set of all power control policies

$$\mathcal{F}_{MAC} \equiv \{\mathcal{P}: \; \mathbf{E}_{\mathbf{g}}[P_k(\mathbf{g})] \leq \overline{P}_k, k = 1, \ldots, K\}$$

  - The instantaneous MAC capacity region given power control policy and fading states

$$\mathcal{C}_{MAC}(P_1(\mathbf{g}), \ldots, P_K(\mathbf{g})) = \left\{ (R_1, \ldots, R_K) : \sum_{k \in S} R_k \leq B \log_2 \left(1 + \frac{\sum_{k \in S} g_k P_k(\mathbf{g})}{N_0 B} \right), \forall S \subset \{1, 2, \ldots, K\} \right\}$$

  - The average achievable rate over fading states

$$\mathcal{C}_{MAC}(\mathcal{P}) = \left\{ (R_1, \ldots, R_K) : \sum_{k \in S} R_k \leq \mathbf{E}_{\mathbf{g}} \left[B \log_2 \left(1 + \frac{\sum_{k \in S} g_k P_k(\mathbf{g})}{N_0 B} \right) \right], \forall S \subset \{1, 2, \ldots, K\} \right\}$$

  - The capacity region

$$\mathcal{C}_{MAC}(\overline{P}_1, \ldots, \overline{P}_K) = \bigcup_{\mathcal{P} \in \mathcal{F}_{MAC}} \mathcal{C}_{MAC}(\mathcal{P}).$$

  - The sum-rate capacitv

$$\mathcal{C}_{MACSR} = \max_{\mathcal{P} \in \mathcal{F}_{MAC}} \mathbf{E}_{\mathbf{g}} \left[B \log_2 \left(1 + \frac{\sum_{k=1}^{K} g_k P_k(\mathbf{g})}{N_0 B} \right) \right]$$

Multiuser Channels

- Multi-Access Channel
  - The instantaneous MAC capacity region given power control policy and fading states

$$\mathcal{C}_{MAC}(P_1(\mathrm{g}),\ldots,P_K(\mathrm{g})) = \left\{(R_1,\ldots,R_K) : \sum_{k\in S} R_k \le B\log_2\left(1+\frac{\sum_{k\in S} g_k P_k(\mathrm{g})}{N_0 B}\right), \forall S \subset \{1,2,\ldots,K\}\right\}$$

  - The set of rates that can be maintained in all fading states

$$\mathcal{C}^0_{MAC}(\mathcal{P}) = \bigcap_{\mathrm{g}} \mathcal{C}_{MAC}(\mathcal{P}).$$

  - The zero-outage capacity region

$$\mathcal{C}^0_{MAC}(\overline{P}_1,\ldots,\overline{P}_K) = \bigcup_{\mathcal{P}\in\mathcal{F}_{MAC}} \bigcap_{\mathrm{g}} \mathcal{C}_{MAC}(\mathcal{P}).$$

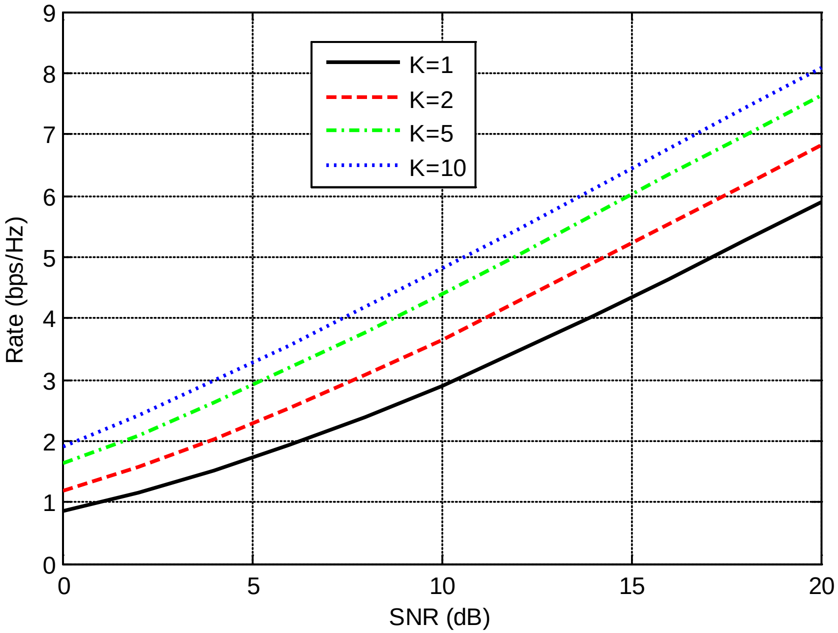
Multiuser Channels

- Multiuser Diversity
  - A system serving many users with independent fading.
  - User selection with best channel condition.

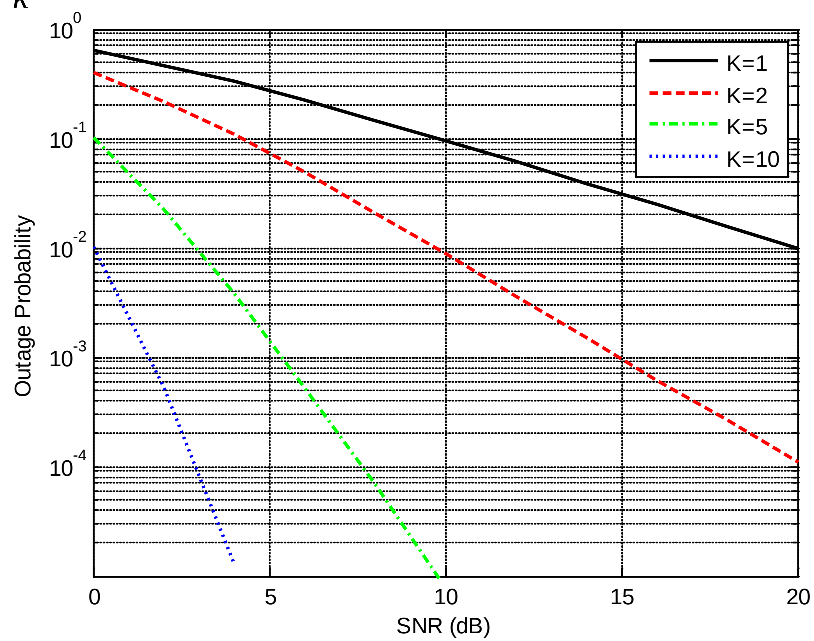
one user: $$C = \log_2(1 + \mathrm{SNR}z)$$

K users: $$C = \log_2(1 + \mathrm{SNR}\max_k z_k)$$

opportunistic



Capacity

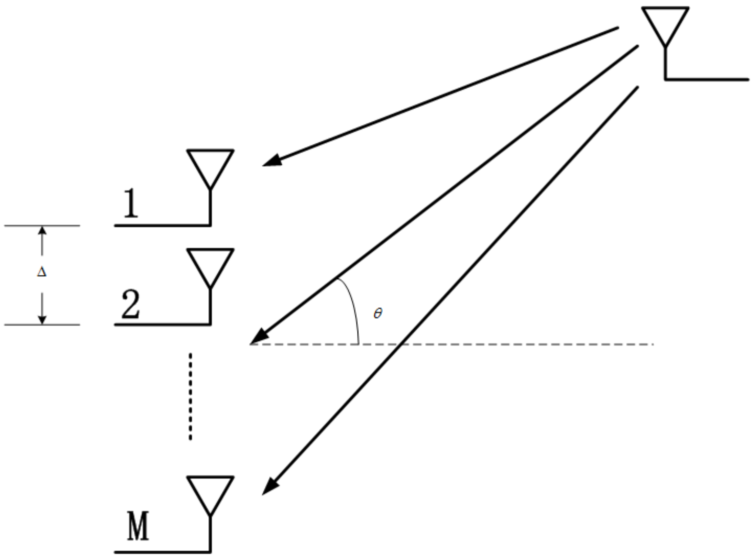


Outage probability

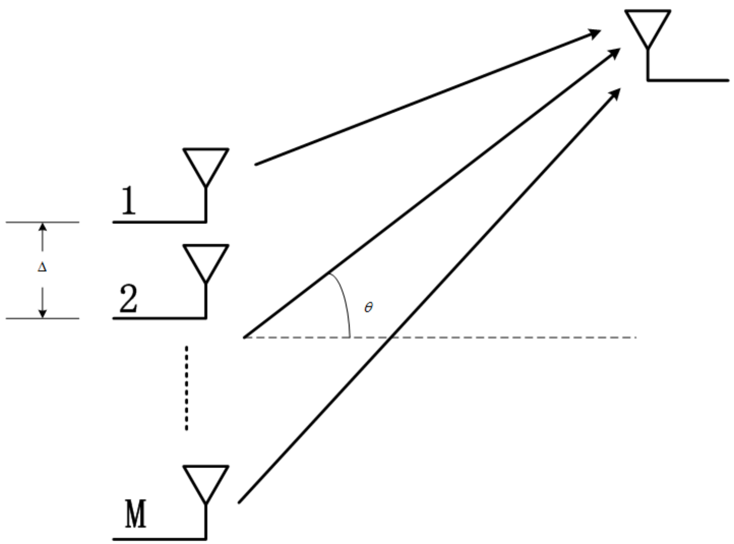
MIMO

Channel Modeling

- Line-of-sight
  - SIMO/MISO.



SIMO



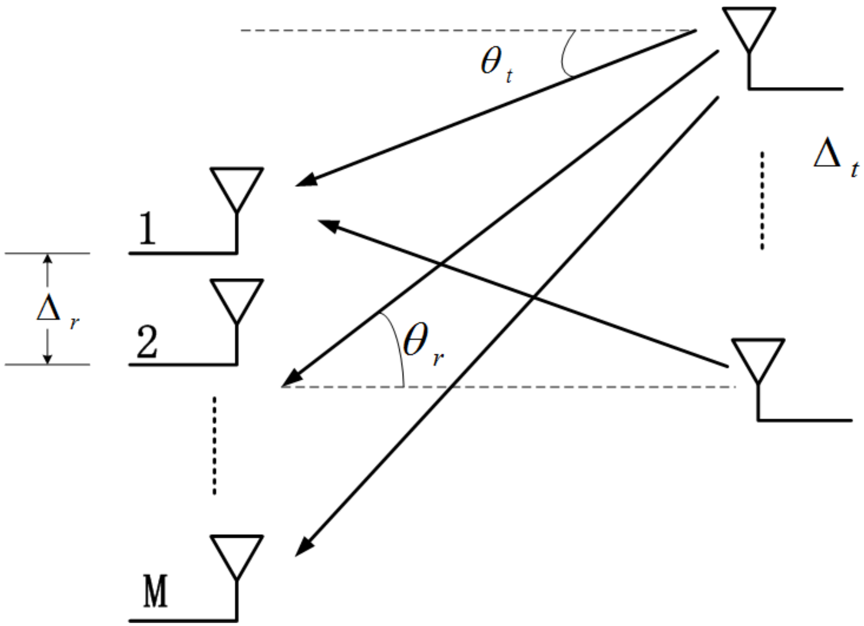
MISO

$$\mathbf{y} = \mathbf{h}x + \mathbf{w}$$

$$y = \mathbf{h}^H \mathbf{x} + w$$

Channel Modeling

- Line-of-sight
  - MIMO.

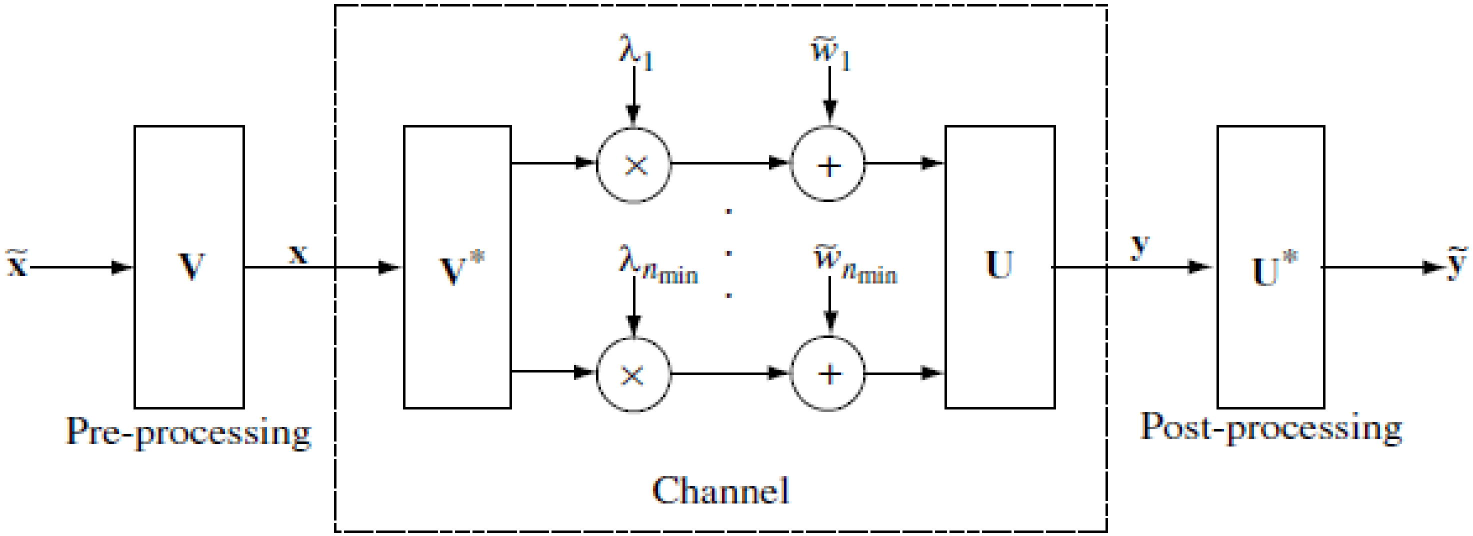


MIMO

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$$

Channel Modeling

- Line-of-sight
  - MIMO processing.

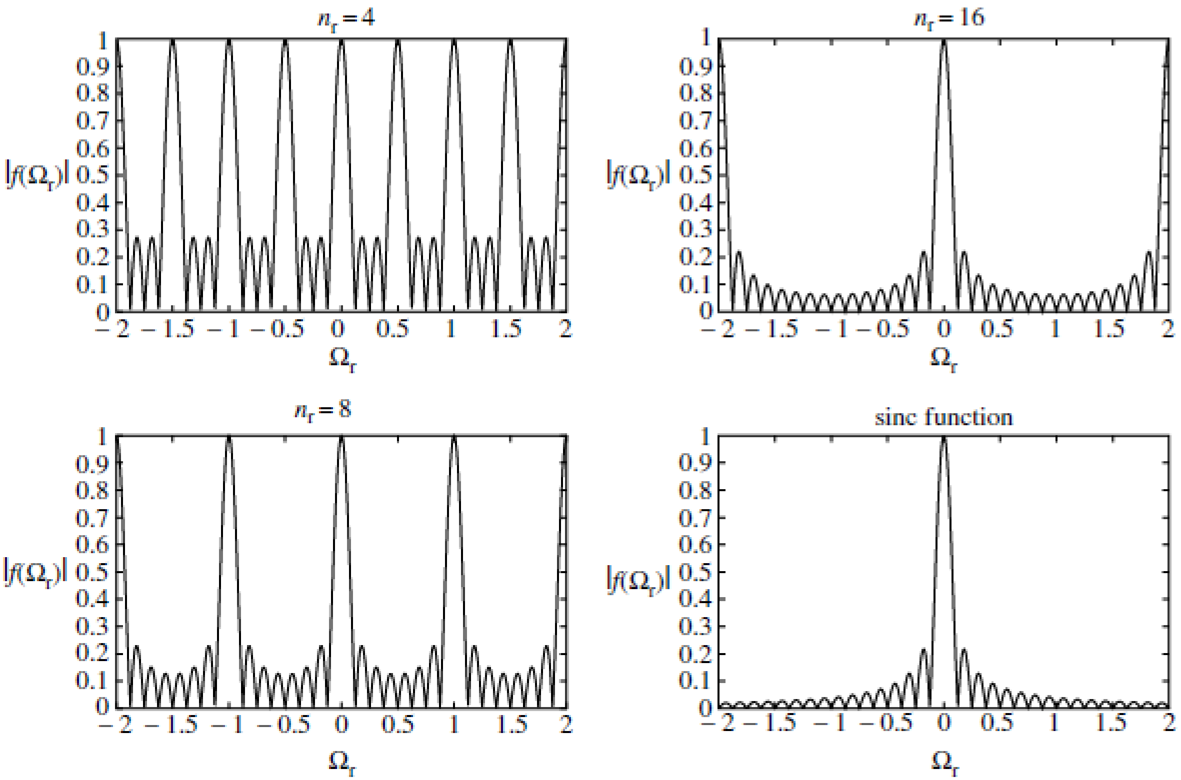


$$\tilde{\mathbf{y}} = \Lambda \tilde{\mathbf{x}} + \tilde{\mathbf{w}}$$

Channel Modeling

- MIMO Fading Channels
  – Spatial resolvability.

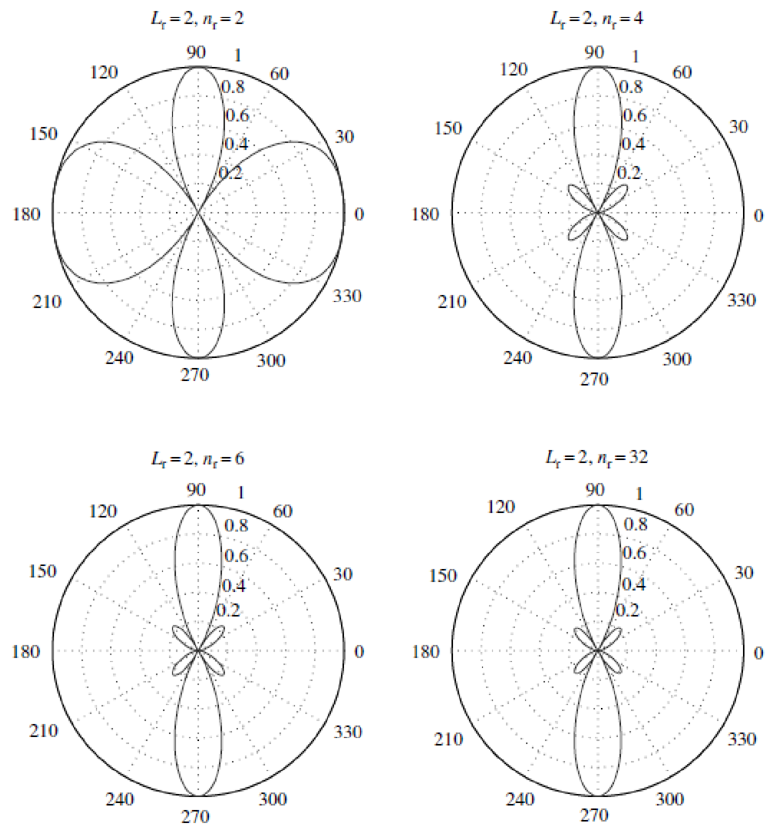
$$f(\Omega_r) = \mathbf{e}^H(\theta_1)\mathbf{e}(\theta_2)$$



Channel Modeling

- MIMO Fading Channels
  - Beam pattern.

$$f(\Omega_r) = \mathbf{e}^H(\theta_0)\mathbf{e}(\theta)$$



Channel Modeling

- MIMO Fading Channels
  - Resolvable paths.
  - Angular domain representation

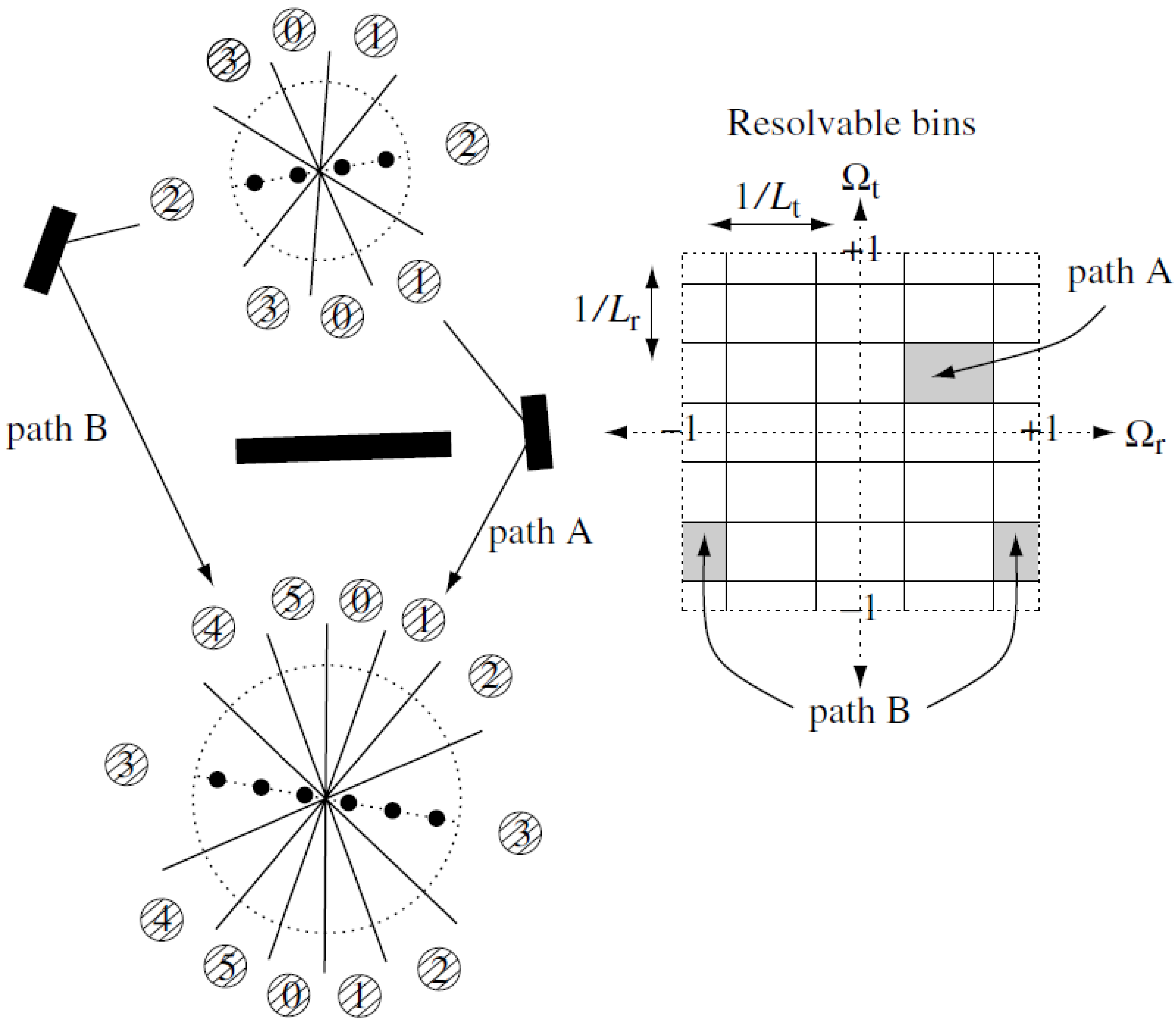
$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$$

$$\mathbf{H} = \sum_i a_i \sqrt{n_r n_t}\, e^{-j2\pi \frac{d^{(i)}}{\lambda}} \mathbf{e}_r(\theta_r)\mathbf{e}_t^H(\theta_t)$$

$$\mathbf{y}^{\mathbf{a}} = \mathbf{H}^{\mathbf{a}}\mathbf{x}^{\mathbf{a}} + \mathbf{w}^{\mathbf{a}}$$

$$\mathbf{H}^{\mathbf{a}} = \mathbf{U}_r^H \mathbf{H}\mathbf{U}_t$$

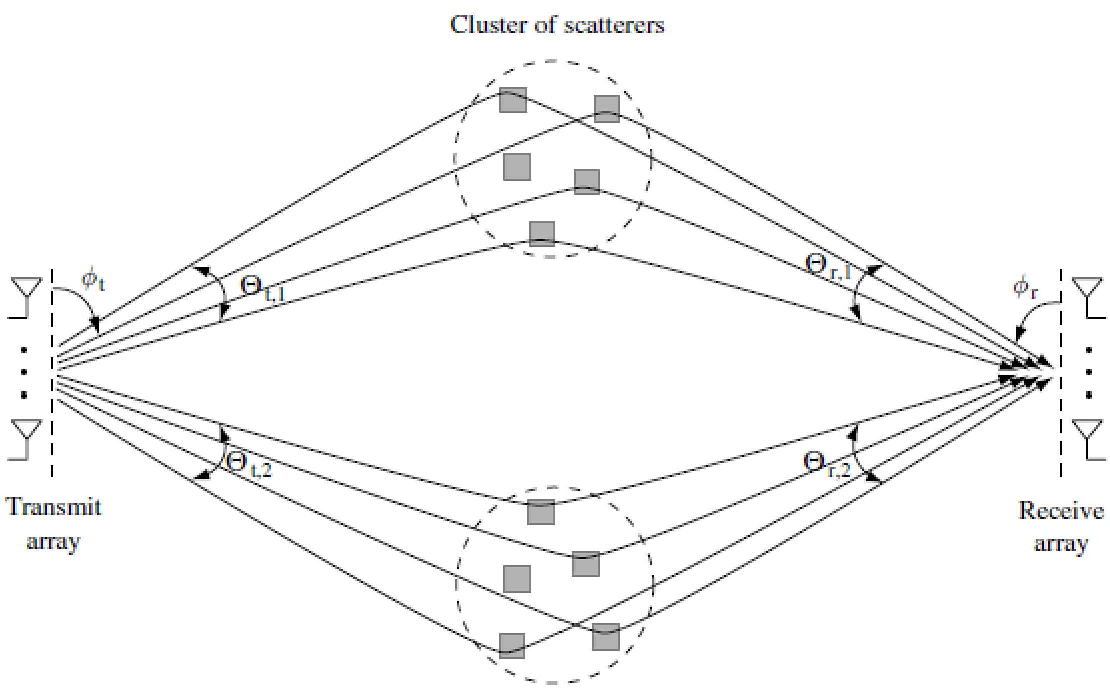
  - IID Rayleigh fading model (entries are IID CSCG).



0
1
3
2
2
1
3
0
path B
path A
4
5
0
1
2
3
3
4
5
0
1
2
Resolvable bins
Ω_t
$1/L_t$
+1
$1/L_r$
path A
−1
+1
Ω_r
−1
path B

Channel Modeling

- MIMO Fading Channels
  - Clustered response models.



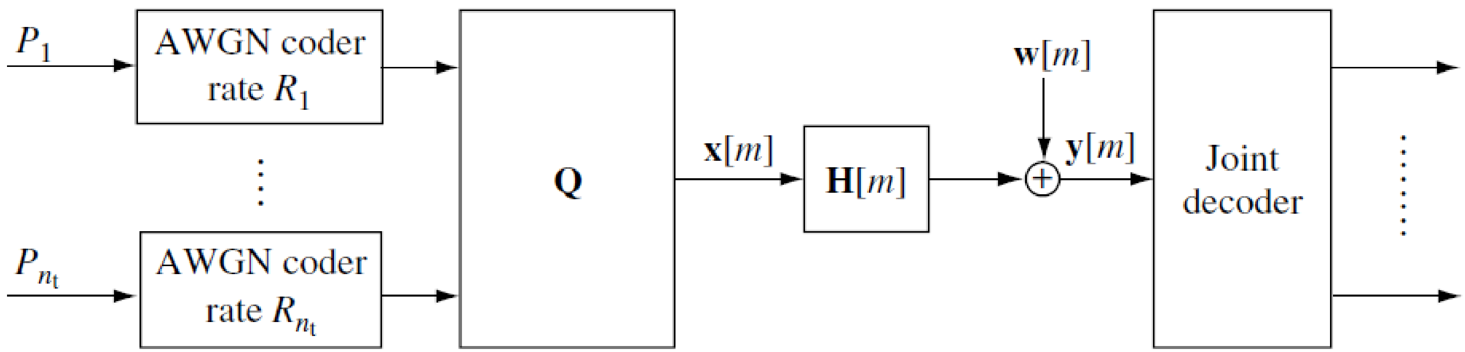
DOF: $$\min\left\{\sum_k \lceil L_t|\Theta_{tk}|\rceil, \sum_k \lceil L_r|\Theta_{tk}|\rceil\right\}$$

Capacity-CSIR

- Narrow-band MIMO
- Slow-fading channel

$$\mathbf{y}[m] = \mathbf{H}\mathbf{x}[m] + \mathbf{w}[m]$$

– V-BLAST



– Capacity: $R < \log\det\left(\mathbf{I}_{n_r} + \frac{1}{N_0}\mathbf{H}\mathbf{K}_x\mathbf{H}^*\right)$ bits/s/Hz

where $\mathbf{K}_x := \mathbf{Q}\,\mathrm{diag}\{P_1, \ldots, P_{n_t}\}\mathbf{Q}^*$

Capacity-CSIR

- Narrow-band MIMO

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$$

- Fast fading

$$\mathbf{y}[m] = \mathbf{H}[m]\mathbf{x}[m] + \mathbf{w}[m]$$

  – CSI at the receiver

$$C = \max_{\mathbf{K}_x:\mathrm{Tr}[\mathbf{K}_x]\le P} \mathbb{E}\left[\log\det\left(\mathbf{I}_{n_r} + \frac{1}{N_0}\mathbf{H}\mathbf{K}_x\mathbf{H}^*\right)\right]$$

Angular domain

$$\mathbf{K}_x = \mathbf{U}_t \Lambda \mathbf{U}_t^H$$

$$C = \max_{\Lambda:\mathrm{Tr}\{\Lambda\}\le P} \mathbb{E}\left[\log\det\left(\mathbf{I}_{n_r} + \frac{1}{N_0}\mathbf{U}_r\mathbf{H}^a\boldsymbol{\Lambda}\mathbf{H}^{a*}\mathbf{U}_r^*\right)\right]$$

$$= \max_{\Lambda:\mathrm{Tr}[\Lambda]\le P} \mathbb{E}\left[\log\det\left(\mathbf{I}_{n_r} + \frac{1}{N_0}\mathbf{H}^a\boldsymbol{\Lambda}\mathbf{H}^{a*}\right)\right].$$

IID Rayleigh

$$\mathbf{K}_x = \frac{P}{n_t}\mathbf{I}_{n_t}$$

$$C = \mathbb{E}\left[\log\det\left(\mathbf{I}_{n_r} + \frac{\mathsf{SNR}}{n_t}\mathbf{H}\mathbf{H}^*\right)\right]$$

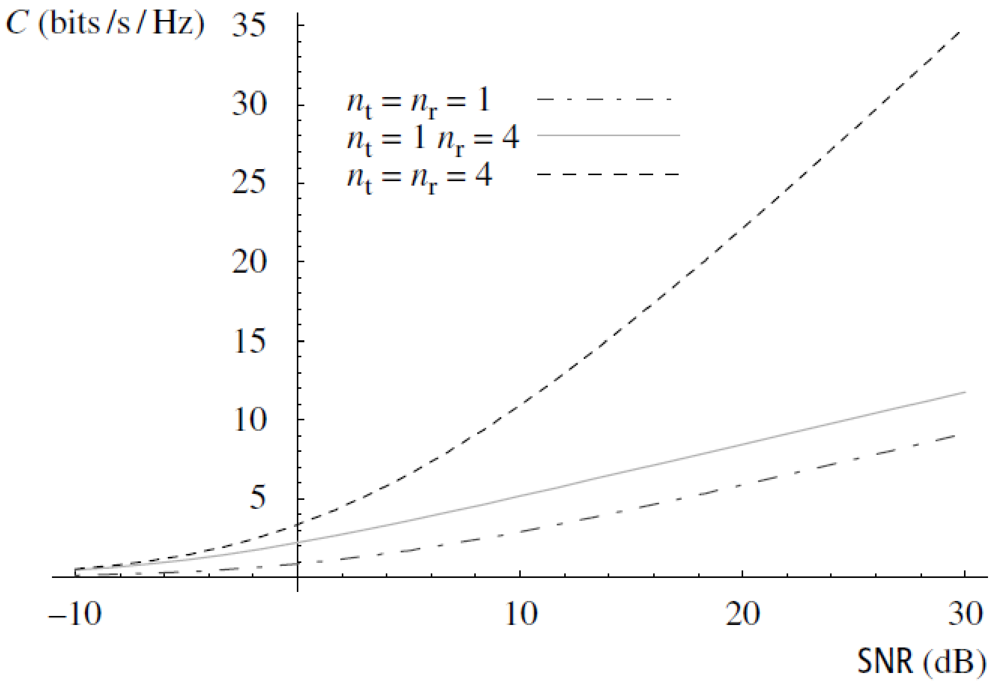
$$C = \mathbb{E}\left[\sum_{i=1}^{n_{\min}} \log\left(1 + \frac{\mathsf{SNR}}{n_t}\lambda_i^2\right)\right]$$

$$= \sum_{i=1}^{n_{\min}} \mathbb{E}\left[\log\left(1 + \frac{\mathsf{SNR}}{n_t}\lambda_i^2\right)\right]$$

Capacity-CSIR

- Rayleigh fading
  - High-SNR regime

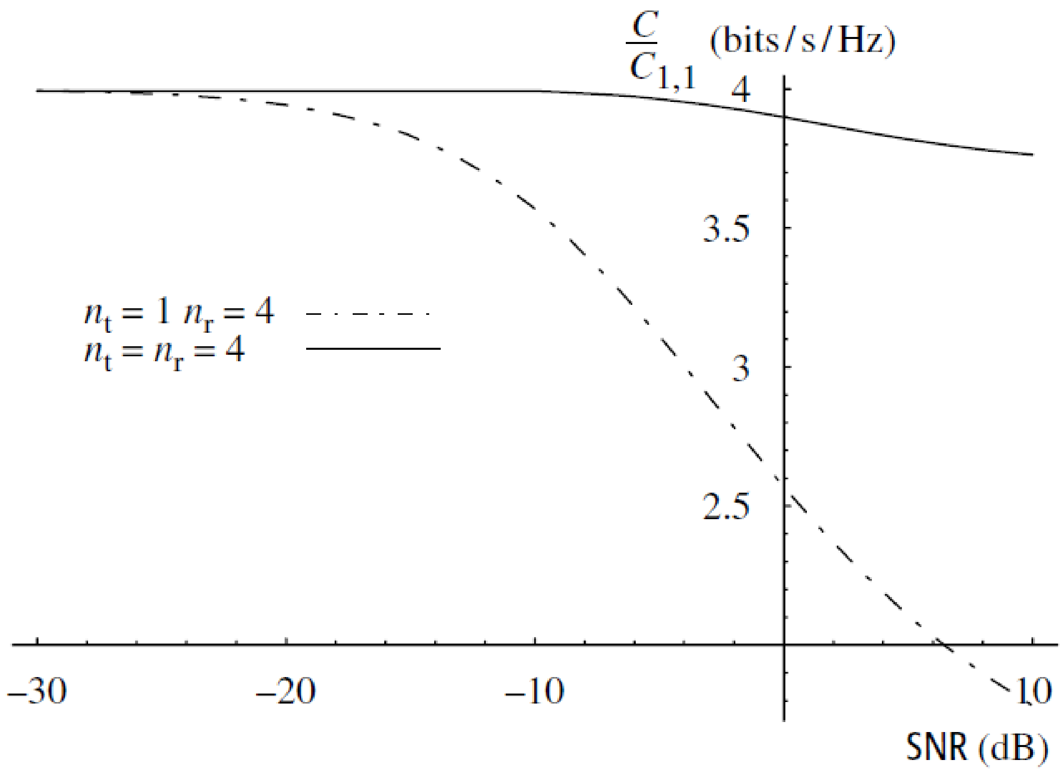
$$C \approx n_{\min} \log \frac{\mathsf{SNR}}{n_t} + \sum_{i=1}^{n_{\min}} \mathbb{E}[\log \lambda_i^2]$$



Capacity-CSIR

- Rayleigh fading
  - Low-SNR regime

$$
\begin{aligned}
C &= \sum_{i=1}^{n_{\min}} \mathbb{E}\left[\log\left(1+\frac{\mathsf{SNR}}{n_t}\lambda_i^2\right)\right] \\
&\approx \sum_{i=1}^{n_{\min}} \frac{\mathsf{SNR}}{n_t}\mathbb{E}\left[\lambda_i^2\right]\log_2 e \\
&= \frac{\mathsf{SNR}}{n_t}\mathbb{E}[\mathrm{Tr}[\mathbf{H}\mathbf{H}^*]]\log_2 e \\
&= \frac{\mathsf{SNR}}{n_t}\mathbb{E}\left[\sum_{i,j}|h_{ij}|^2\right]\log_2 e \\
&= n_r \mathsf{SNR}\log_2 e \text{ bits/s/Hz.}
\end{aligned}
$$



Capacity-CSIR

- Rayleigh fading
  - Large antenna-array regime

$$C_{nn}(\mathsf{SNR}) = \mathbb{E}\left[\sum_{i=1}^{n} \log\left(1 + \mathsf{SNR}\frac{\lambda_i^2}{n}\right)\right]$$

The limiting density of squared singular values

$$f^*(x) = \begin{cases} \frac{1}{\pi}\sqrt{\frac{1}{x} - \frac{1}{4}} & 0 \le x \le 4, \\ 0 & \text{else.} \end{cases}$$

Then,

$$\frac{1}{n}\sum_{i=1}^{n} \log\left(1 + \mathsf{SNR}\frac{\lambda_i^2}{n}\right) \to \int_0^4 \log(1 + \mathsf{SNR}x) f^*(x)\mathrm{d}x.$$

So

$$c^*(\mathsf{SNR}) := \int_0^4 \log(1 + \mathsf{SNR}x) f^*(x)\mathrm{d}x$$

$$= 2\log\left(1 + \mathsf{SNR} - \frac{1}{4}F(\mathsf{SNR})\right) - \frac{\log e}{4\mathsf{SNR}}F(\mathsf{SNR}).$$

where

$$F(\mathsf{SNR}) := \left(\sqrt{4\mathsf{SNR} + 1} - 1\right)^2$$

$$\lim_{n\to\infty} \frac{C_{nn}(\mathsf{SNR})}{n} = c^*(\mathsf{SNR})$$

Capacity-CSIR

- Asymptotic regime
  - MISO with a large transmit antenna array

$$C_{n1} = \mathbb{E}\left[\log\left(1 + \frac{\mathsf{SNR}}{n}\sum_{i=1}^{n}|h_i|^2\right)\right] \text{bits/s/Hz.}$$

$$C_{n1} \to \log(1 + \mathsf{SNR}) = C_{\text{awgn}}.$$

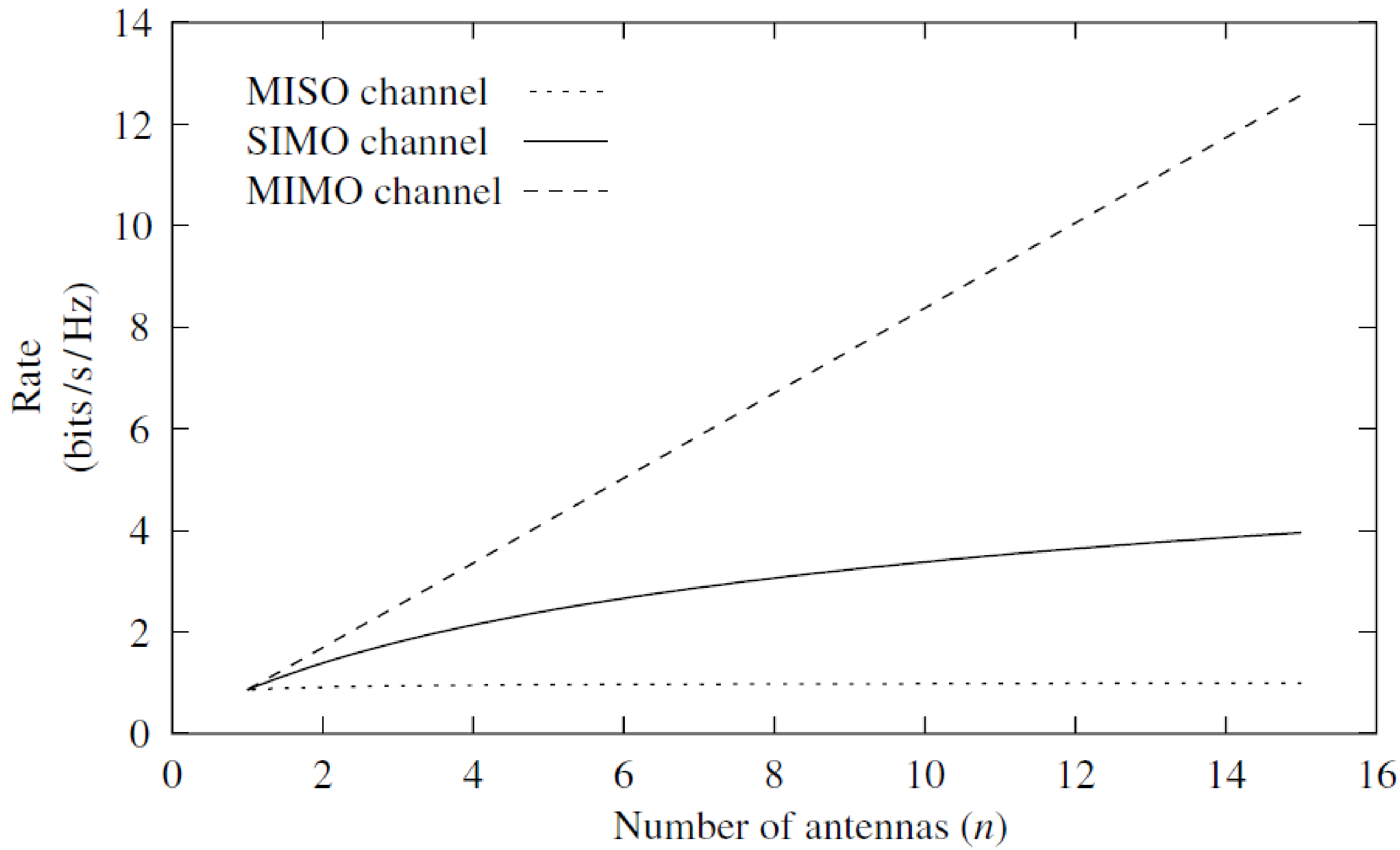
  - SIMO with a large receive antenna array

$$C_{1n} = \mathbb{E}\left[\log\left(1 + \mathsf{SNR}\sum_{i=1}^{n}|h_i|^2\right)\right]$$

$$C_{1n} \approx \log(n\mathsf{SNR}) = \log n + \log \mathsf{SNR},$$

Capacity-CSIR

- Asymptotic regime



Capacity-CSIR and CSIT

- Parallel channels

$$C = \sum_{i=1}^{n_{\min}} \mathbb{E}\left[\log\left(1 + \frac{P^*(\lambda_i)\lambda_i^2}{N_0}\right)\right]$$

$$P^*(\lambda) = \left(\mu - \frac{N_0}{\lambda^2}\right)^+,$$

$$\sum_{i=1}^{n_{\min}} \mathbb{E}\left[\left(\mu - \frac{N_0}{\lambda_i^2}\right)^+\right] = P.$$

Capacity-CSIR and CSIT

- Receiver algorithms
  - Zero-forcing (ZF) (*linear nulling*)

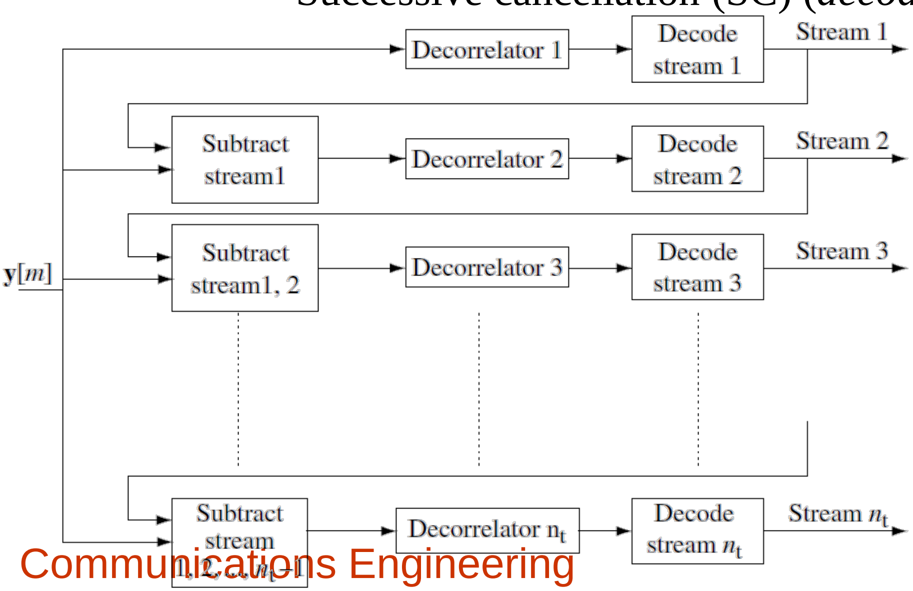
$$\mathbf{H}^{\dagger} := (\mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*.$$

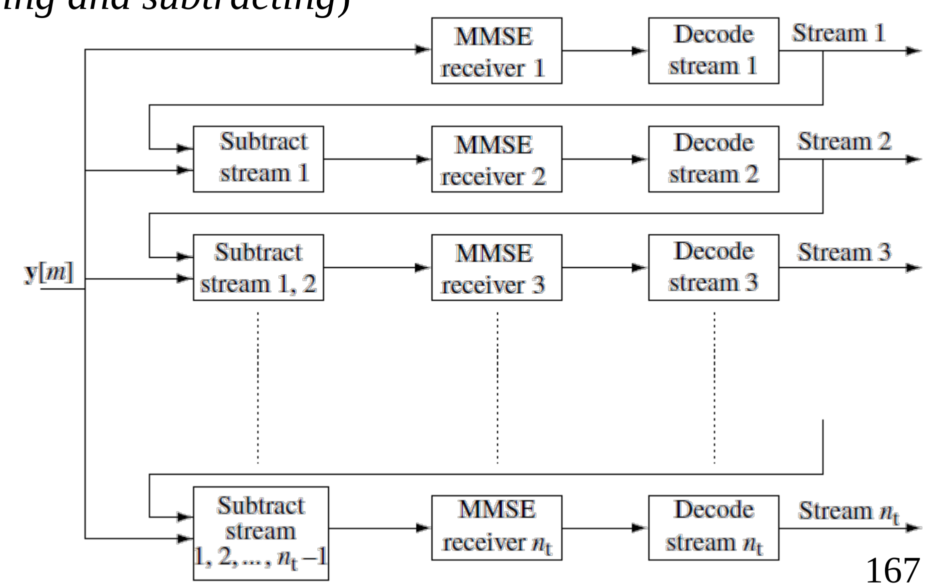
  - Minimum mean squared error (MMSE) (*linear nulling*)

$$\mathbf{v}_{\mathrm{mmse}} := \mathbf{K}_z^{-1}\mathbf{h},$$

$$\mathbf{K}_{z_k} := N_0\mathbf{I}_{n_r} + \sum_{i\neq k}^{n_t} P_i\mathbf{h}_i\mathbf{h}_i^*,$$

  - Successive cancellation (SC) (*decoding and subtracting*)



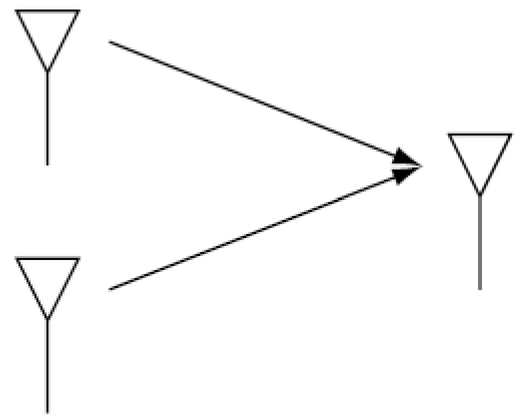


Diversity-Multiplexing-Tradeoff (DMT)

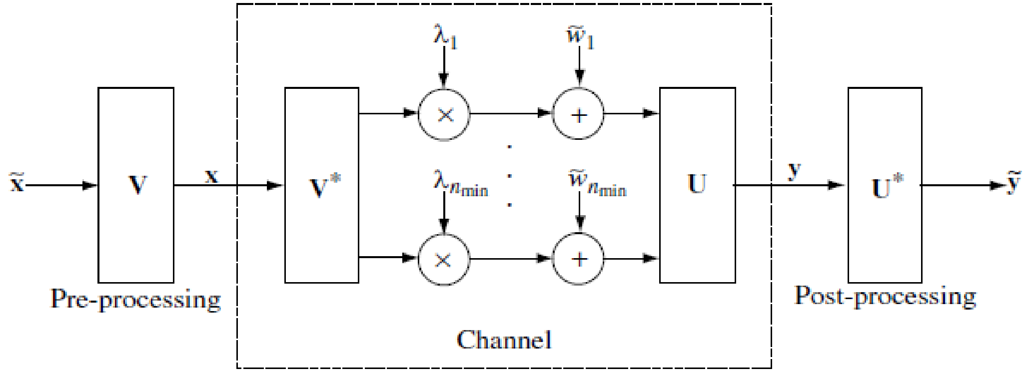
- Slow fading

$$\mathbf{y}[m] = \mathbf{H}\mathbf{x}[m] + \mathbf{w}[m]$$

- Diversity
  - Deep fading leads to errors.
  - Information symbols passing through multiple signal paths with independent fading
  - **Time**, **frequency**, **space**
  - **Alamouti Scheme**



- Multiplexing
  - Number of independent streams



Diversity-Multiplexing-Tradeoff (DMT)

- IID Rayleigh fading
  - Maximum diversity gain: $n_t n_r$
  - Maximum spatial multiplexing (fast fading): $n_{\min} \log SNR$
- Fixed rate R
  - Random variable for capacity.
  - High-SNR regime: R is a fraction of the fast fading capacity.

$$R = r \log SNR$$

A diversity gain d*(r) is achieved at the multiplexing gain r, if

$$r = \lim_{SNR\to\infty} \frac{R}{\log SNR}, \text{ and } d^*(r) = \lim_{SNR\to\infty} \frac{-\log p_{out}(R)}{\log SNR}.$$

- The curve d*(r) is the diversity-multiplexing tradeoff of the slow fading channel.

Diversity-Multiplexing-Tradeoff (DMT)

- Scalar Rayleigh fading

$$y[m] = hx[m] + w[m]$$

  - PAM/QAM: error probability determined by the minimum distance between the constellation points.
  - PAM: With 2^R constellation points, the minimum distance

$$D_{\min} \approx \frac{\sqrt{SNR}}{2^R}$$

  - High-SNR error probability

$$p_e \approx Q\left(\frac{D_{\min}}{2}\right) \approx \frac{1}{2}\left(1 - \sqrt{\frac{D_{\min}^2}{4 + D_{\min}^2}}\right) \approx \frac{1}{D_{\min}^2} \approx \frac{2^{2R}}{SNR}$$

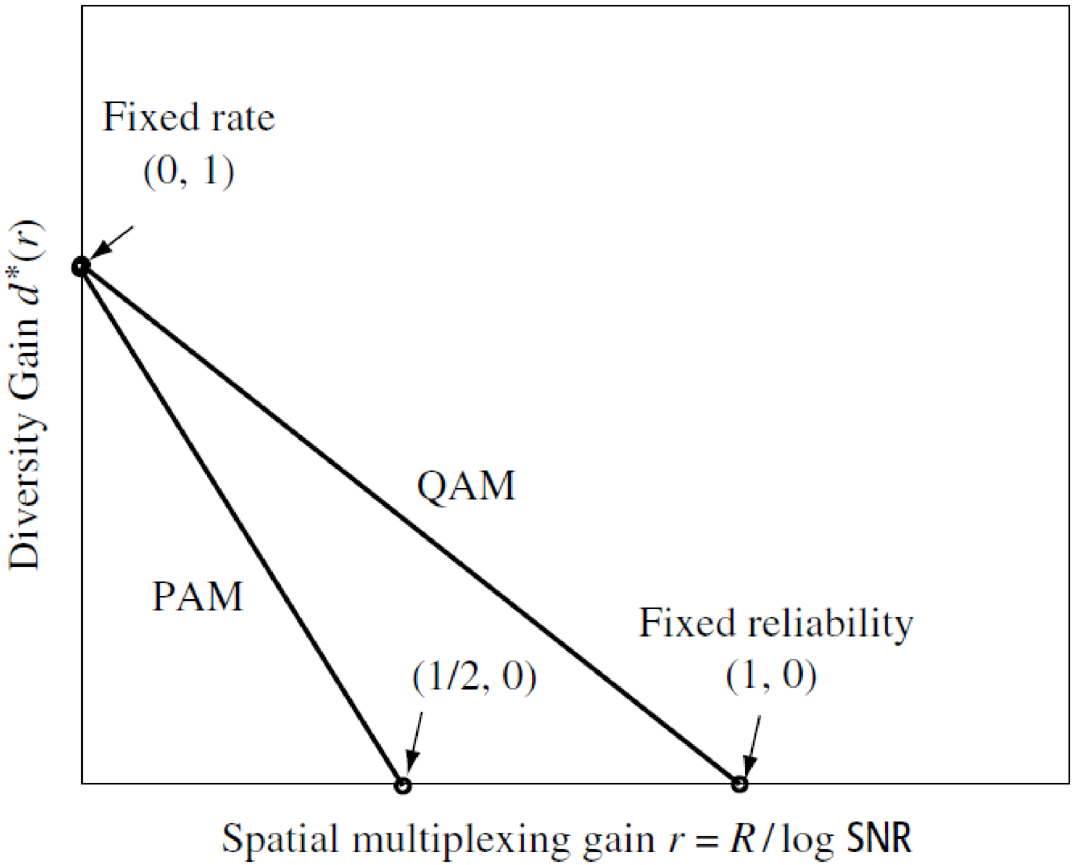
  - Let $R = r \log SNR$. Then, we have

$$p_e \approx \frac{1}{SNR^{1\text{-}2r}}$$

  - So d(r)=1-2r, r $\in$[0,1/2].
  - d(r)=1-r, r $\in$[0,1] for QAM.

Diversity-Multiplexing-Tradeoff (DMT)

- Scalar Rayleigh fading



Diversity-Multiplexing-Tradeoff (DMT)

- Parallel Rayleigh fading

$$y_l[m] = h_l x_l[m] + w_l[m], l = 1, \cdots, L$$

  - Target rate for each sub-channel $R = r \log SNR$.
  - Outage is

$$p_{\text{out}} = \mathbb{P}\left\{ \sum_{\ell=1}^{L} \log(1 + |h_\ell|^2 \mathsf{SNR}) < Lr \log \mathsf{SNR} \right\}$$

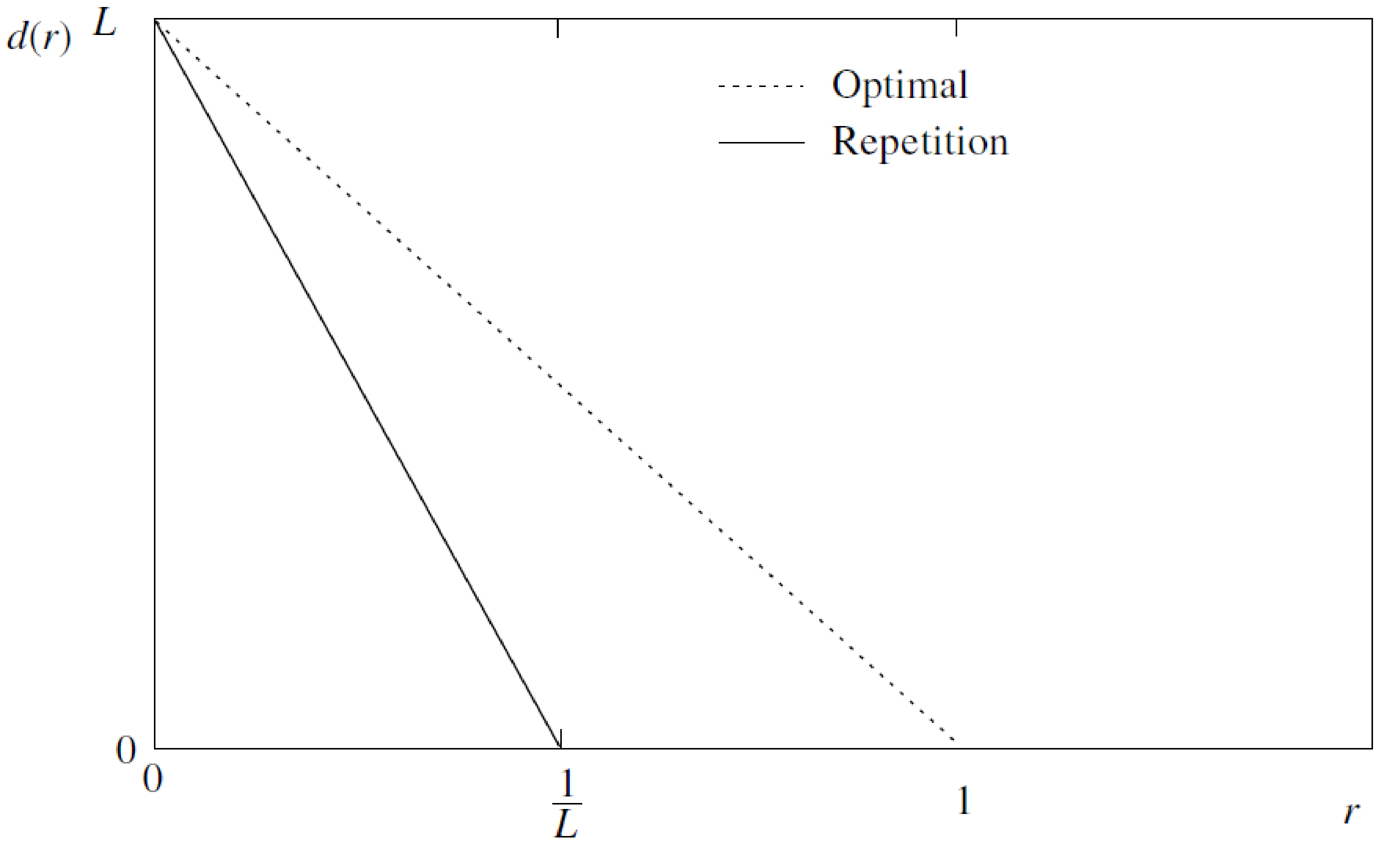
$$p_{\text{out}} \approx (\mathbb{P}\{\log(1 + |h_1|^2 \mathsf{SNR}) < r \log \mathsf{SNR}\})^L \approx \frac{1}{\mathsf{SNR}^{L(1-r)}}$$

  - d(r)=L(1-r), r $\in$[0,1].
  - Sending the same symbol over L sub-channels (repetition diversity), and combines the received symbols coherently at the subchannels:

d(r)=L(1-Lr), r $\in$[0,1/L].

Diversity-Multiplexing-Tradeoff (DMT)

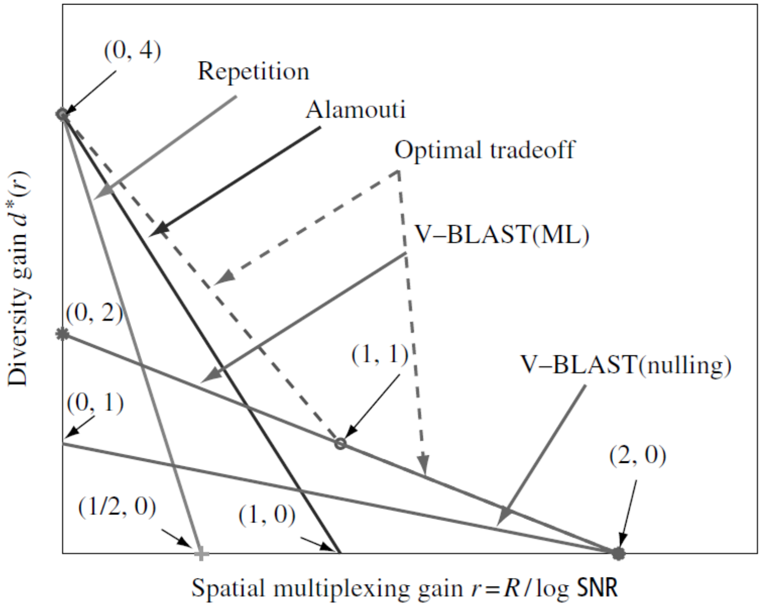
- Parallel Rayleigh fading



Diversity-Multiplexing-Tradeoff (DMT)

- 2x2 MIMO Rayleigh fading

	Classical diversity gain	Degrees of freedom utilized	D–M tradeoff
Repetition	4	1/2	$4-8r,\ r\in[0,1/2]$
Alamouti	4	1	$4-4r,\ r\in[0,1]$
V-BLAST (ML)	2	2	$2-r,\ r\in[0,2]$
V-BLAST (nulling)	1	2	$1-r/2,\ r\in[0,2]$
Channel itself	4	2	$4-3r,\ r\in[0,1]$ $2-r,\ r\in[1,2]$



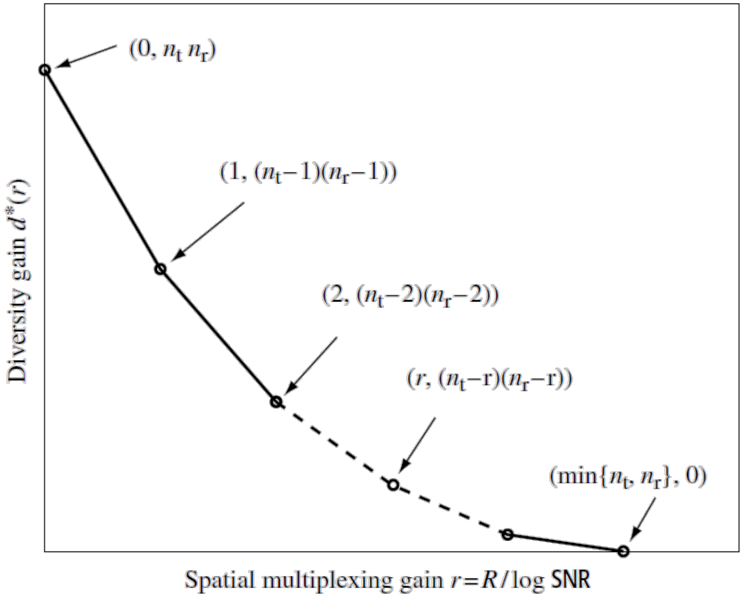
Diversity-Multiplexing-Tradeoff (DMT)

- n_txn_r MIMO Rayleigh fading
  - Outage

$$p_{\text{out}}^{\text{iid}}(r \log \mathsf{SNR}) = \mathbb{P}\left\{\log\det\left(\mathbf{I}_{n_r} + \frac{\mathsf{SNR}}{n_t}\mathbf{H}\mathbf{H}^*\right) < r\log\mathsf{SNR}\right\}$$

  - DMT: piecewise linear curve with joining points

$$(k, (n_t - k)(n_r - k)), k = 0, \ldots, n_{\min}$$



Multi-user

- Uplink
  - Channel input-output relationship (single TX antenna)

$$\mathbf{y}[m] = \sum_{k=1}^{K} \mathbf{h}_k x_k[m] + \mathbf{w}[m]$$

  - SDMA: spatial signatures of the users
  - Capacity region

$$R_1 < \log\left(1 + \frac{\|\mathbf{h}_1\|^2 P_1}{N_0}\right),$$

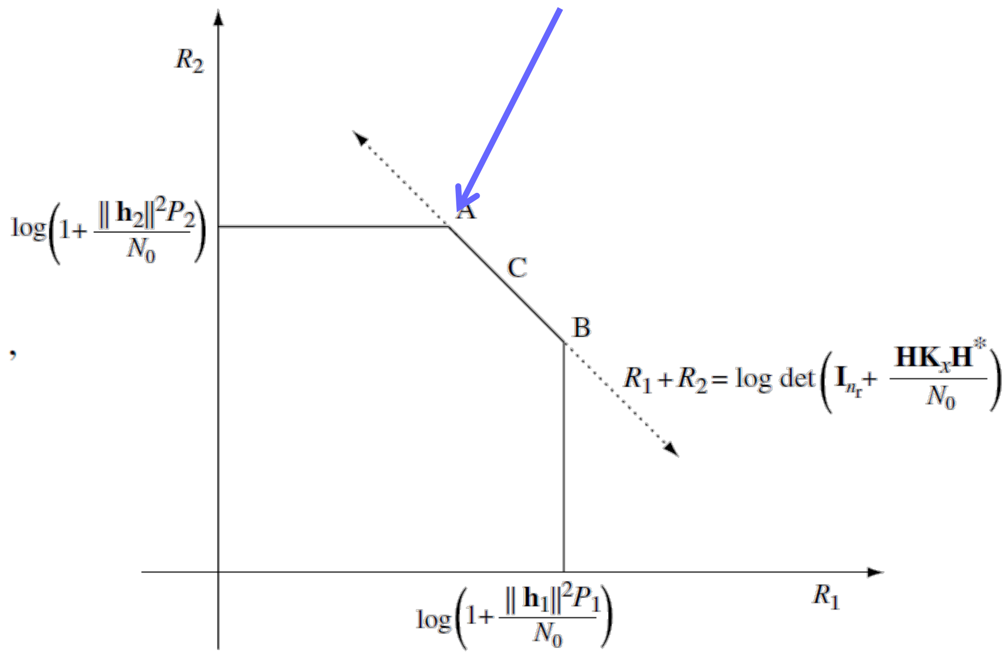
$$R_2 < \log\left(1 + \frac{\|\mathbf{h}_2\|^2 P_2}{N_0}\right),$$

$$R_1 + R_2 < \log\det\left(\mathbf{I}_{n_r} + \frac{1}{N_0}\mathbf{H}\mathbf{K}_x\mathbf{H}^*\right),$$

$$\mathbf{K}_x = \mathrm{diag}(P_1, P_2).$$

$$R_2 = \log\left(1 + \frac{P_2\|\mathbf{h}_2\|^2}{N_0}\right),$$

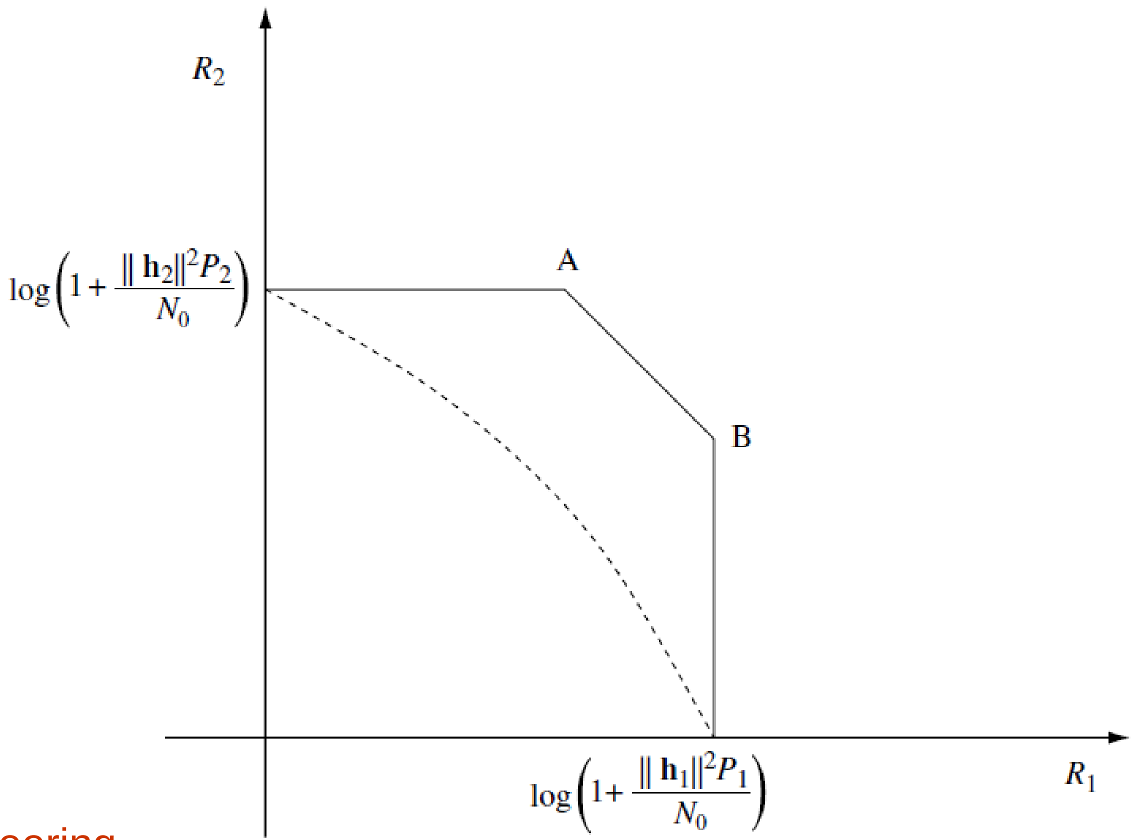
$$R_1 = \log(1 + P_1\mathbf{h}_1^*(N_0\mathbf{I}_{n_r} + P_2\mathbf{h}_2\mathbf{h}_2^*)^{-1}\mathbf{h}_1).$$



Multi-user

- Uplink
  - Orthogonal transmission

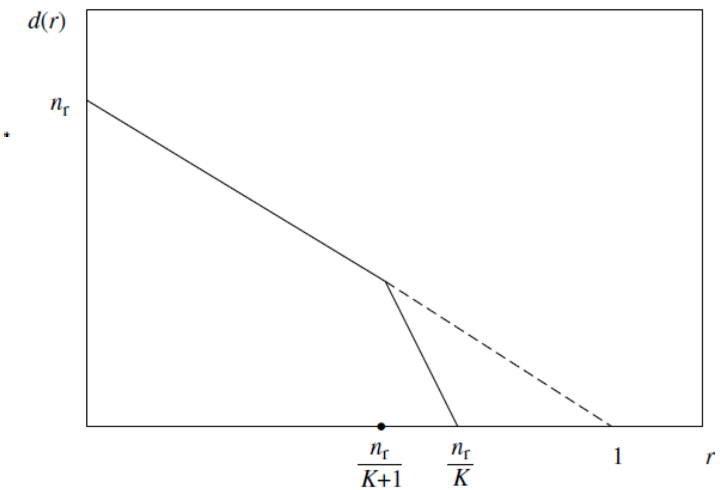
$$\left(\alpha \log\left(1+\frac{P_1\|\mathbf{h}_1\|^2}{\alpha N_0}\right), (1-\alpha)\log\left(1+\frac{P_2\|\mathbf{h}_2\|^2}{(1-\alpha)N_0}\right)\right)$$



Multi-user

- Uplink
  - Slow fading: DMT

$$(0, n_r), \qquad \left(\frac{n_r}{K+1}, \frac{n_r(K-n_r+1)}{K+1}\right), \qquad \text{and} \left(\frac{n_r}{K}, 0\right).$$



  - Fast fading: Capacity region

CSIR

$$R_k \le \mathbb{E}\left[\log\left(1+\frac{\|\mathbf{h}_k\|^2 P_k}{N_0}\right)\right], \qquad k=1,2.$$

$$R_1+R_2 \le \mathbb{E}\left[\log\det\left(\mathbf{I}_{n_r}+\frac{1}{N_0}\mathbf{H}\mathbf{K}_x\mathbf{H}^*\right)\right].$$

CSIR +CSIT

$$R_k \le \mathbb{E}\left[\log\left(1+\frac{\|\mathbf{h}_k\|^2 P_k(\mathbf{h}_1,\mathbf{h}_2)}{N_0}\right)\right], \qquad k=1,2,$$

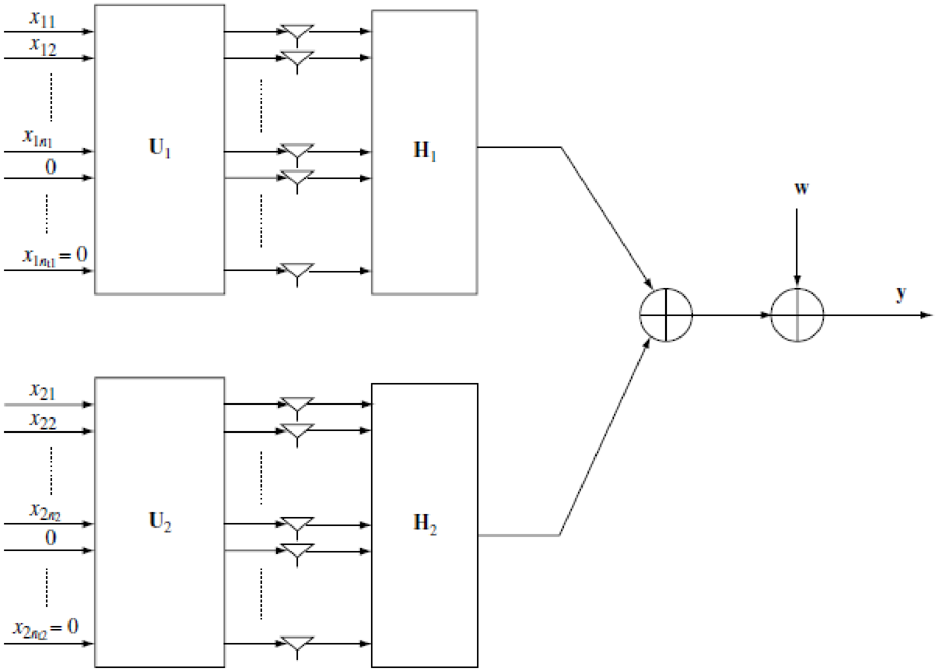
$$R_1+R_2 \le \mathbb{E}\left[\log\det\left(\mathbf{I}_{n_r}+\frac{1}{N_0}\mathbf{H}\mathbf{K}_x\mathbf{H}^*\right)\right].$$

Multi-user

- MIMO Uplink
  – Channel input-output relationship

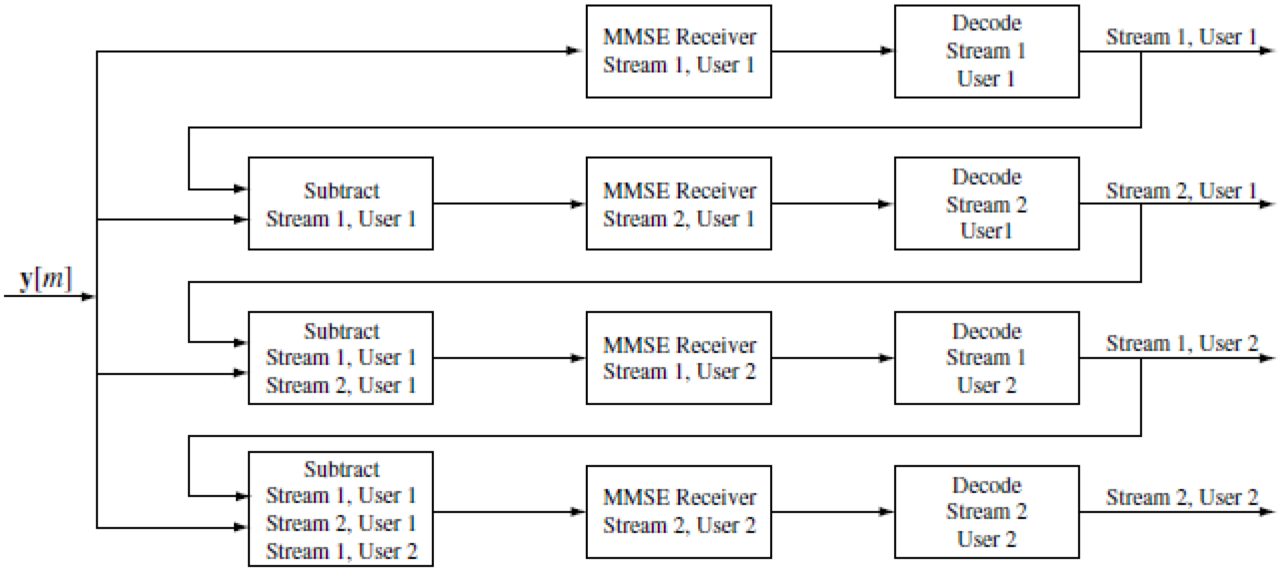
$$\mathbf{y}[m] = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{x}_k[m] + \mathbf{w}[m],$$

  – Transmitter



Multi-user

- MIMO Uplink
  - Receiver

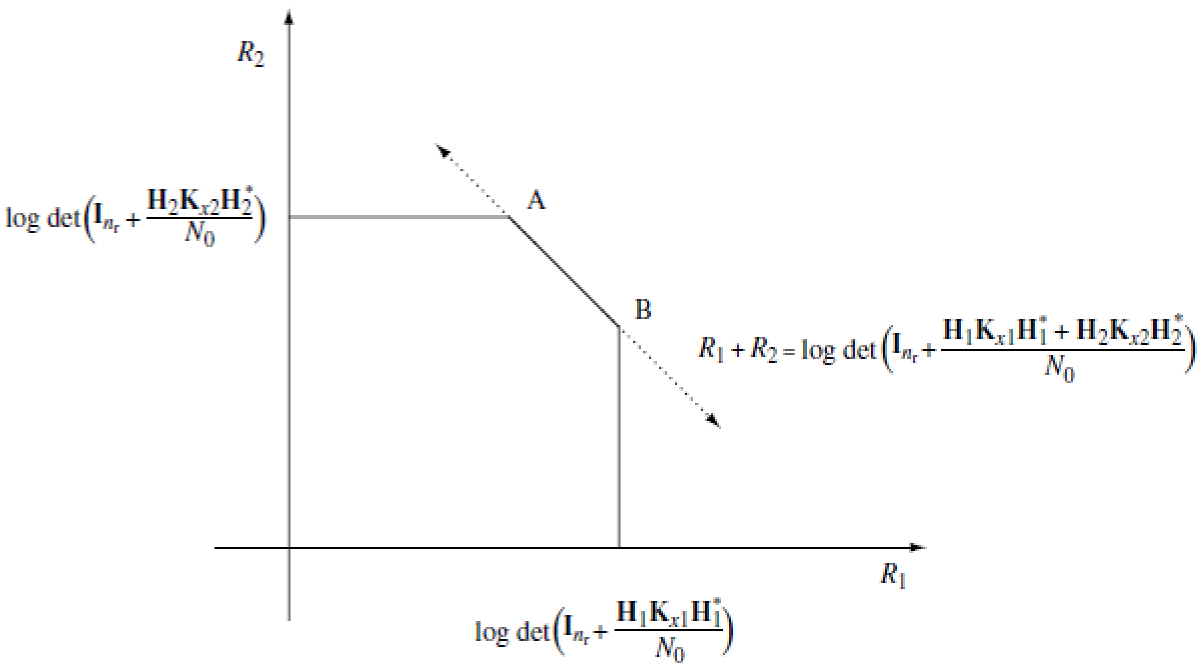


Multi-user

- MIMO Uplink
  - Capacity region

$$R_k \le \log\det\left(\mathbf{I}_{n_r} + \frac{1}{N_0}\mathbf{H}_k\mathbf{K}_{xk}\mathbf{H}_k^*\right), \qquad k = 1, 2,$$

$$R_1 + R_2 \le \log\det\left(\mathbf{I}_{n_r} + \frac{1}{N_0}\sum_{k=1}^{2}\mathbf{H}_k\mathbf{K}_{xk}\mathbf{H}_k^*\right).$$



Multi-user

- MISO Downlink
  - Channel input-output relationship

$$y_k[m] = \mathbf{h}_k^* \mathbf{x}[m] + w_k[m], \qquad k = 1, 2, \ldots, K.$$

  - Precoding

$$\mathbf{x}[m] = \sum_{k=1}^{K} \tilde{x}_k[m] \mathbf{u}_k,$$

  - Received signal at user k

$$\begin{aligned} y_k[m] &= (\mathbf{h}_k^* \mathbf{u}_k)\tilde{x}_k[m] + \sum_{j \neq k} (\mathbf{h}_k^* \mathbf{u}_j)\tilde{x}_j[m] + w_k[m], \\ &= (\mathbf{h}_k^* \mathbf{u}_k)\tilde{x}_k[m] + \sum_{j<k} (\mathbf{h}_k^* \mathbf{u}_j)\tilde{x}_j[m] \\ &\quad + \sum_{j>k} (\mathbf{h}_k^* \mathbf{u}_j)\tilde{x}_j[m] + w_k[m]. \end{aligned}$$

  - Rate for user k

$$R_k = \log(1 + \mathsf{SINR}_k),$$

$$\mathsf{SINR}_k = \frac{P_k \,|\, \mathbf{u}_k^* \mathbf{h}_k \,|^2}{N_0 + \sum_{j>k} P_j \,|\, \mathbf{u}_j^* \mathbf{h}_k \,|^2}.$$

Multi-user

- SIMO Downlink
  - Channel input-output relationship

$$\mathbf{y}_k[m] = \mathbf{h}_k x[m] + \mathbf{w}_k[m], \qquad k = 1, 2,$$

  - Matched filter at receivers

$$\tilde{y}_k[m] := \frac{\mathbf{h}_k^* \mathbf{y}_k[m]}{\|\mathbf{h}_k\|} = \|\mathbf{h}_k\| x[m] + w_k[m], \qquad k = 1, 2.$$

  - Suppose $\|\mathbf{h}_1\| \le \|\mathbf{h}_2\|$, rate for the users are

$$R_1 = \log\left(1 + \frac{P_1\|\mathbf{h}_1\|^2}{P_2\|\mathbf{h}_1\|^2 + N_0}\right), \quad R_2 = \log\left(1 + \frac{P_2\|\mathbf{h}_2\|^2}{N_0}\right).$$

  - MIMO downlink: Transmitter precoding + receiver algorithms