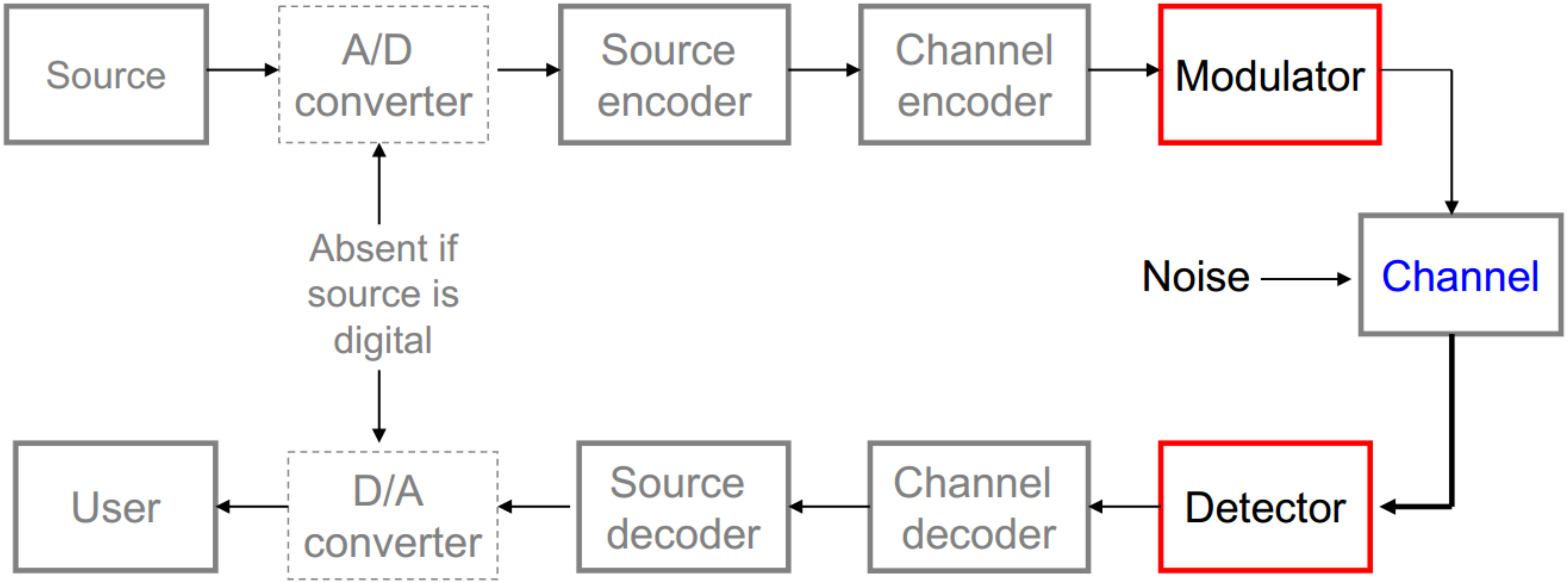
Outline

- Introduction
- Signal, random variable, random process and spectra
- Analog modulation
- Analog to digital conversion
- Digital transmission through baseband channels
- Signal space representation
- Optimal receivers
- **Digital modulation techniques**
- Channel coding
- Synchronization
- Information theory

Digital modulation techniques



- Binary digital modulation
- M-ary digital modulation
- Comparison study

Chapter 8.2，8.3.3，8.5-8.7，9.1-9.5，9.7

Digital modulation techniques

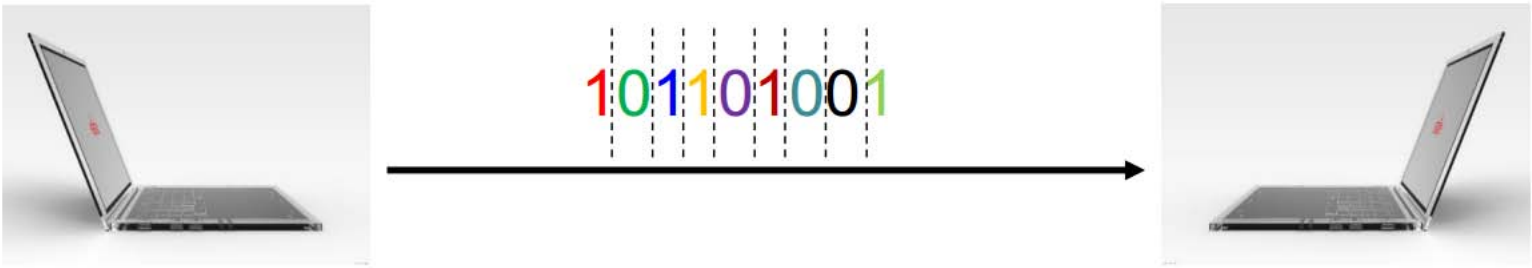
- In digital communications, the modulation process corresponds to **switching** or **keying** the amplitude, frequency, or phase of a **sinusoidal** carrier wave corresponding to incoming digital data
- Three basic digital modulation techniques

1. Amplitude-shift keying (ASK) - special case of AM
2. Frequency-shift keying (FSK) - special case of FM
3. Phase-shift keying (PSK) - special case of PM

- We use signal space approach in receiver design and performance analysis

Binary digital modulation

- In binary signaling, the modulator produces one of two distinct signals in response to one bit of source data at a time.

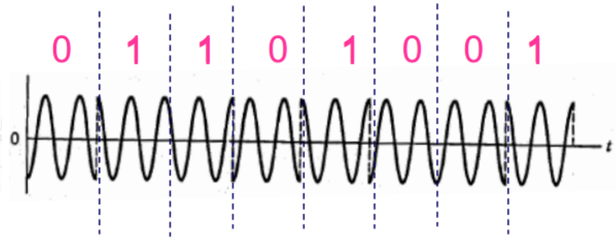


- Binary modulation type

Binary PSK | **Binary FSK** | **Binary ASK**

Binary digital modulation

- Binary Phase-Shift Keying (**BPSK**)
  - Modulation



"1" → $s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$

"0" → $s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$

  - $0 \le t < T_b$, T_b bit duration
  - f_c : carrier frequency, chosen to be n_c/T_b for some fixed integer n_c or $f_c >> 1/T_b$
  - E_b : transmitted signal energy per bit, i.e.,

$$\int_0^{T_b} s_1^2(t)dt = \int_0^{T_b} s_2^2(t)dt = E_b$$

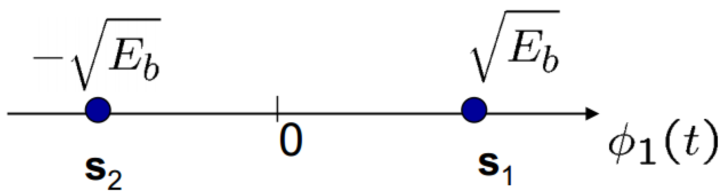
  - The pair of signals differ only in a 180-degree phase shift

Binary digital modulation

- Binary Phase-Shift Keying (**BPSK**)
  - Signal space representation:

$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t) \quad \text{with} \quad 0 \le t < T_b$$

  - So $s_1(t) = \sqrt{E_b}\phi_1(t)$ and $s_2(t) = -\sqrt{E_b}\phi_1(t)$
  - A binary PSK system is characterized by a signal space that is one-dimensional (N=1), and has two message points (M=2)



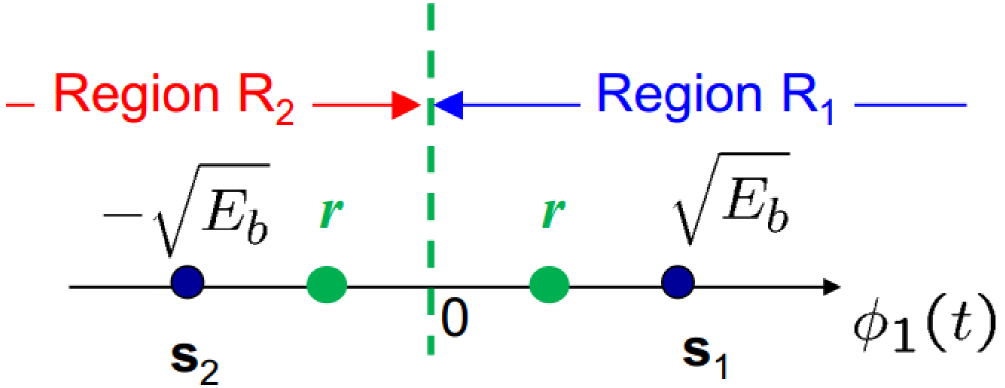
$$d_{12} = 2\sqrt{E_b}$$

  - Assume that the two signals are equally likely, i.e.,

$$P(s_1) = P(s_2) = 0.5$$

Binary digital modulation

- Binary Phase-Shift Keying (**BPSK**)
  - The optimal decision boundary is the midpoint of the line joining these two message points



  - Decision rule:

1. Guess signal $s_1(t)$ (or binary 1) was transmitted if the received signal point r falls in region R_1 $(r > 0)$
2. Guess signal $s_2(t)$ (or binary 0) was transmitted otherwise $(r \leq 0)$

Binary digital modulation

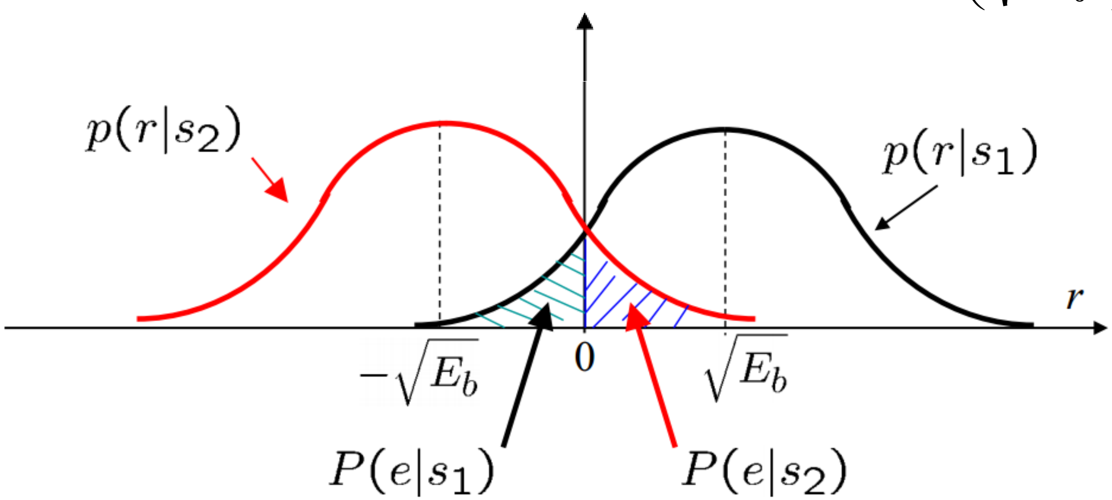
- Binary Phase-Shift Keying (**BPSK**)
  - Probability of error analysis.
  - The conditional probability of the receiver deciding in favor of $s_2(t)$ given that $s_1(t)$ was transmitted is

$$P(e|s_1) = P(r < 0|s_1)$$

$$= \int_{-\infty}^{0} \frac{1}{\sqrt{\pi N_0}} \exp\left\{-\frac{(r-\sqrt{E_b})^2}{N_0}\right\} dr = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

  - Due to symmetry

$$P(e|s_2) = P(r > 0|s_2) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$



Binary digital modulation

- Binary Phase-Shift Keying (**BPSK**)
  - Probability of error analysis.
  - Since the signals $s_1(t)$ and $s_2(t)$ are equally likely to be transmitted, the average probability of error is

$$P_e = 0.5P(e|\mathbf{s}_1) + 0.5P(e|\mathbf{s}_2) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

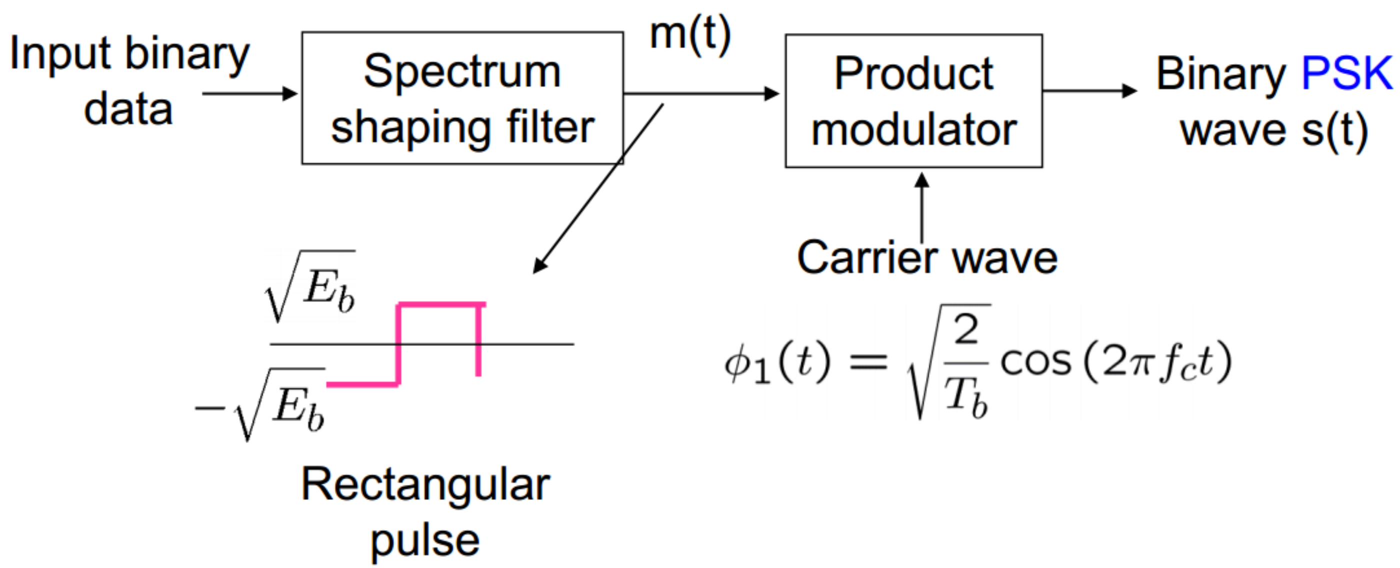
$$\Downarrow$$

P_e depends on ratio $\frac{E_b}{N_0}$

  - This ratio is normally called bit energy to noise density ratio (SNR/bit)

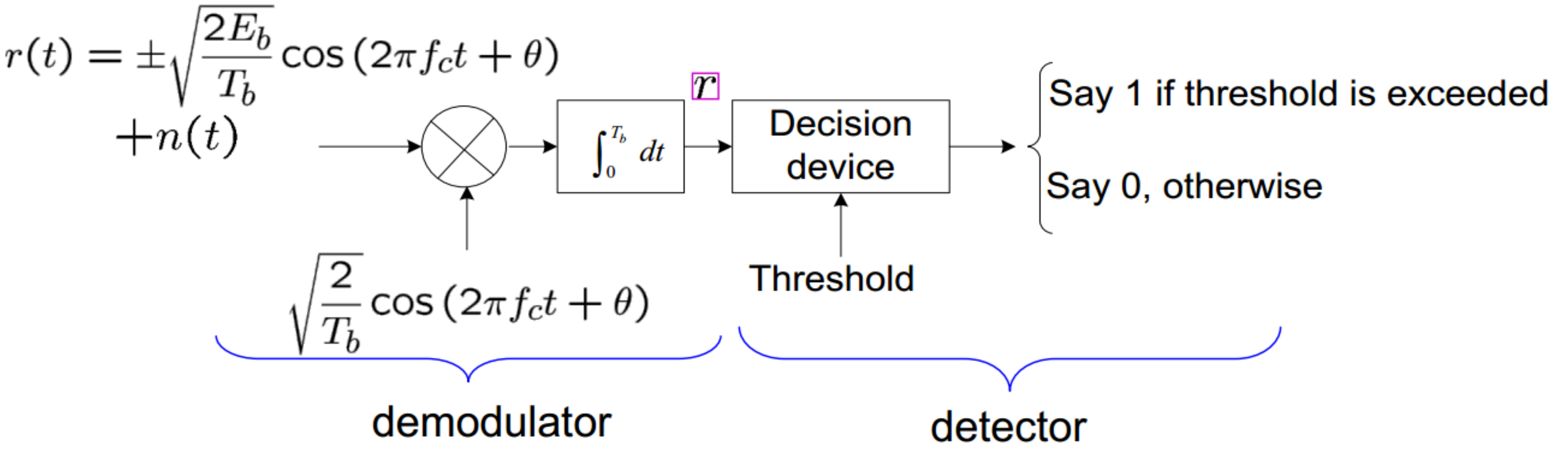
Binary digital modulation

- Binary Phase-Shift Keying (**BPSK**)
  - Transmitter.



Binary digital modulation

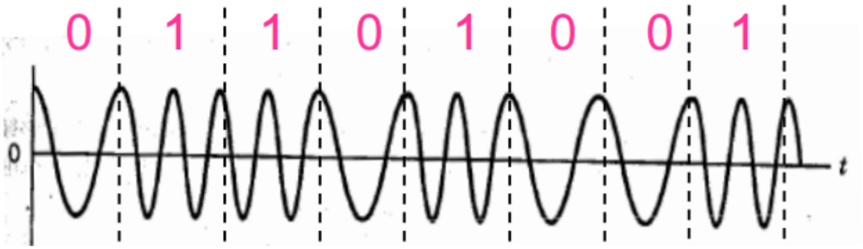
- Binary Phase-Shift Keying (**BPSK**)
  - Receiver.



  - θ is the carrier-phase offset, due to propagation delay or oscillators at the transmitter and receiver are not synchronous
  - The detection is coherent in the sense of phase synchronization and timing synchronization

Binary digital modulation

- Binary Frequency-Shift Keying (**BFSK**)
  - Modulation



"1" → $s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t)$

"0" → $s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t) \quad 0 \le t < T_b$

  - E_b : transmitted signal energy per bit

$$\int_0^{T_b} s_1^2(t)dt = \int_0^{T_b} s_2^2(t)dt = E_b$$

  - f_i : transmitted frequency with separation $\Delta f = f_1 - f_0$
  - Δf is selected so that $s_1(t)$ and $s_2(t)$ are orthogonal, i.e.,

$$\int_0^{T_b} s_1(t)s_2(t)dt = 0$$

(Example?)

Binary digital modulation

- Binary Frequency-Shift Keying (**BFSK**)
  - Signal space representation:

$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_1 t) \qquad 0 \le t < T_b$$

$$\phi_2(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_2 t) \qquad 0 \le t < T_b$$

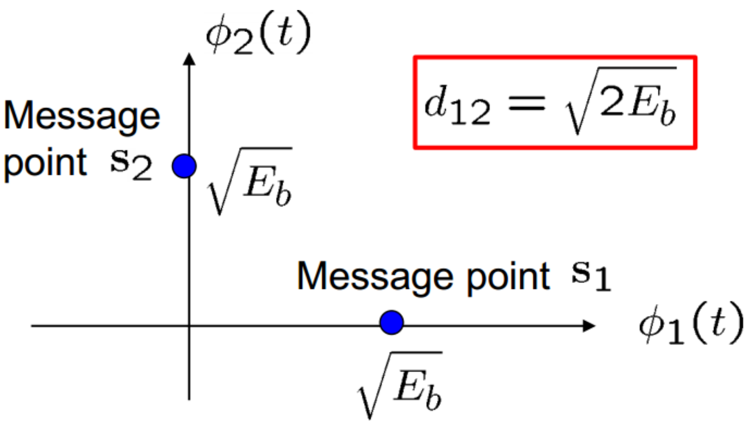
$\Rightarrow$

$$s_1(t) = \sqrt{E_b}\phi_1(t)$$

$$s_2(t) = \sqrt{E_b}\phi_2(t)$$

$$\mathbf{s}_1 = [\sqrt{E_b} \quad 0]$$

$$\mathbf{s}_2 = [0 \quad \sqrt{E_b}]$$



$$d_{12} = \sqrt{2E_b}$$

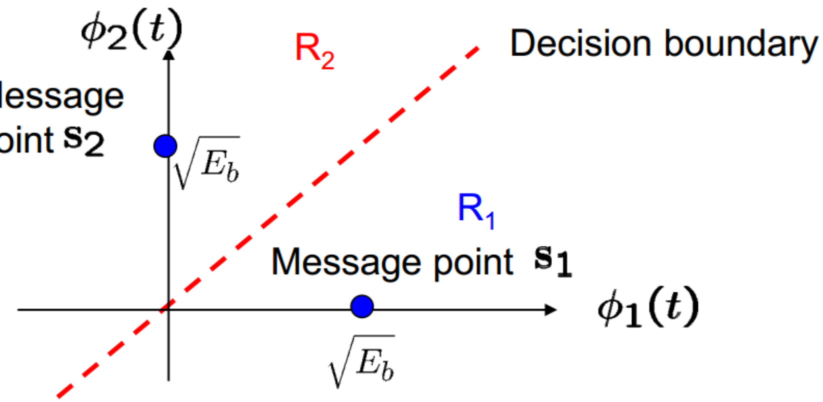
Binary digital modulation

- Binary Frequency-Shift Keying (**BFSK**)
  - Decision regions:

$$\vec{r} = [r_1 \quad r_2]$$

$$r_1 = \int_0^{T_b} r(t)\phi_1(t)dt$$

$$r_2 = \int_0^{T_b} r(t)\phi_2(t)dt$$



1. Guess signal $s_1(t)$ (or binary 1) was transmitted if the received signal point ***r*** falls in region R_1 ($r_2>r_1$)
2. Guess signal $s_2(t)$ (or binary 0) was transmitted otherwise ($r \leq 0$)

Binary digital modulation

- Binary Frequency-Shift Keying (**BFSK**)
  - Probability of error analysis.
  - Given that $\mathbf{s}_1$ is transmitted

$$r_1 = \sqrt{E_b} + n_1 \quad \text{and} \quad r_2 = n_2$$

  - Since the condition $r_2 > r_1$ corresponds to the receiver making a decision in favor of symbol $\mathbf{s}_2$, the conditional probability of error when $\mathbf{s}_1$ is transmitted is given by

$$P(e|s_1) = P(r_1 < r_2|s_1) = P(\sqrt{E_b} + n_1 < n_2)$$

  - n_1 and n_2 are i.i.d. Gaussian with $n_1, n_2 \in \mathcal{N}(0, N_0/2)$
  - Then $n = n_1 - n_2$ is Gaussian with $n \in \mathcal{N}(0, N_0)$

$$\Rightarrow \quad P(e|s_1) = P(n < -\sqrt{E_b}) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Binary digital modulation

- Binary Frequency-Shift Keying (**BFSK**)
  - Probability of error analysis.
  - By symmetry, we also have

$$P(e|s_2) = P(r_1 > r_2|s_2) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

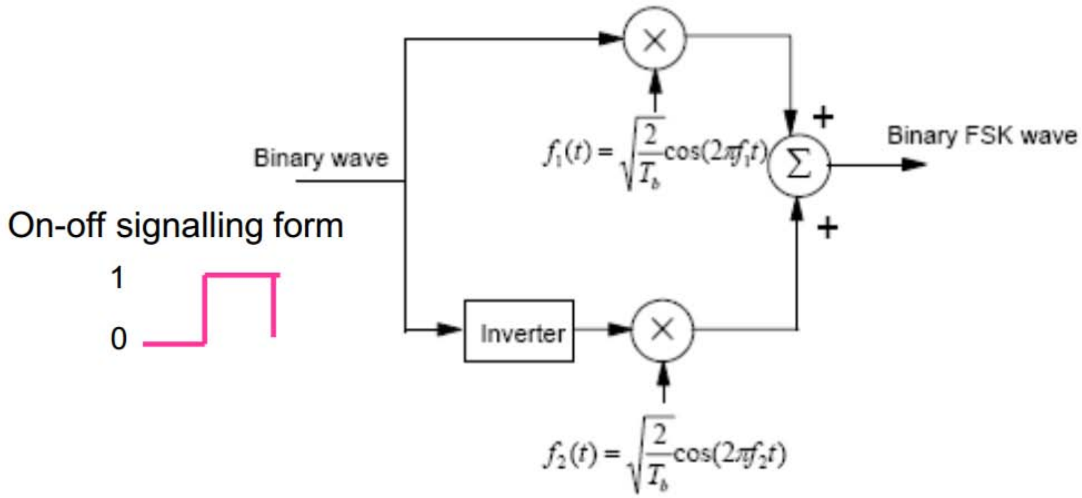
  - Since the two signals are equally likely to be transmitted, the average probability of error for coherent binary FSK is

$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \quad \Rightarrow \quad \textbf{3 dB} \text{ worse than BPSK}$$

To achieve the same P_e, BFSK needs 3dB more transmission power than BPSK

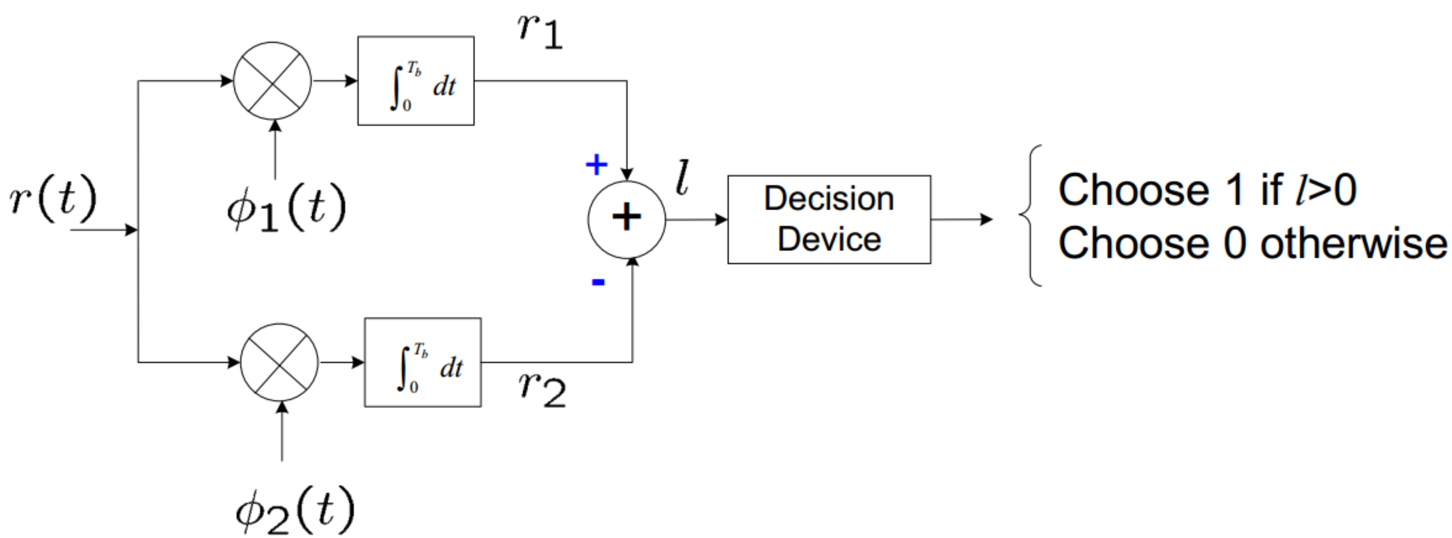
Binary digital modulation

- Binary Frequency-Shift Keying (**BFSK**)
  - Transmitter.



Binary digital modulation

- Binary Frequency-Shift Keying (**BFSK**)
  - Receiver.

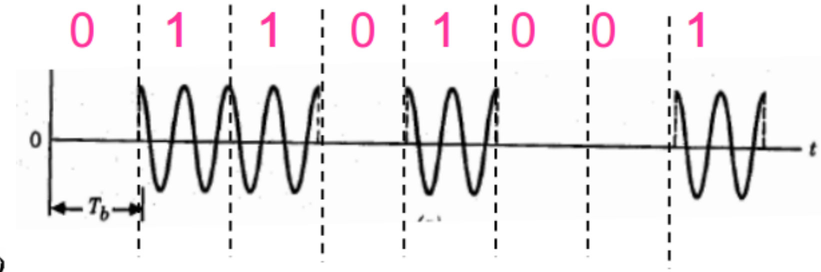


Binary digital modulation

- Binary Amplitude-Shift Keying (**BASK**)
  - Modulation.

"1" $\longrightarrow$ $s_1(t) = \sqrt{\dfrac{2E}{T_b}}\cos(2\pi f_c t)$

"0" $\longrightarrow$ $s_2(t) = 0 \qquad 0 \le t < T_b$



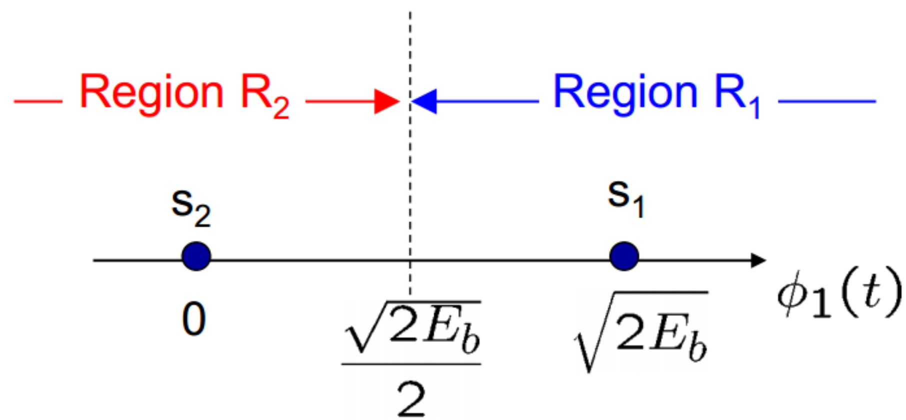
(On-off signaling)

  - Average energy per bit

$$E_b = \frac{E+0}{2} \qquad \text{i.e. } E = 2E_b$$

  - Decision region

$$d_{12} = \sqrt{2E_b}$$



Binary digital modulation

- Binary Amplitude-Shift Keying (**BASK**)
  - Probability of error analysis.
  - Average probability of error

Prove it!

$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Identical to that of coherent binary FSK

Binary digital modulation

- Comparison

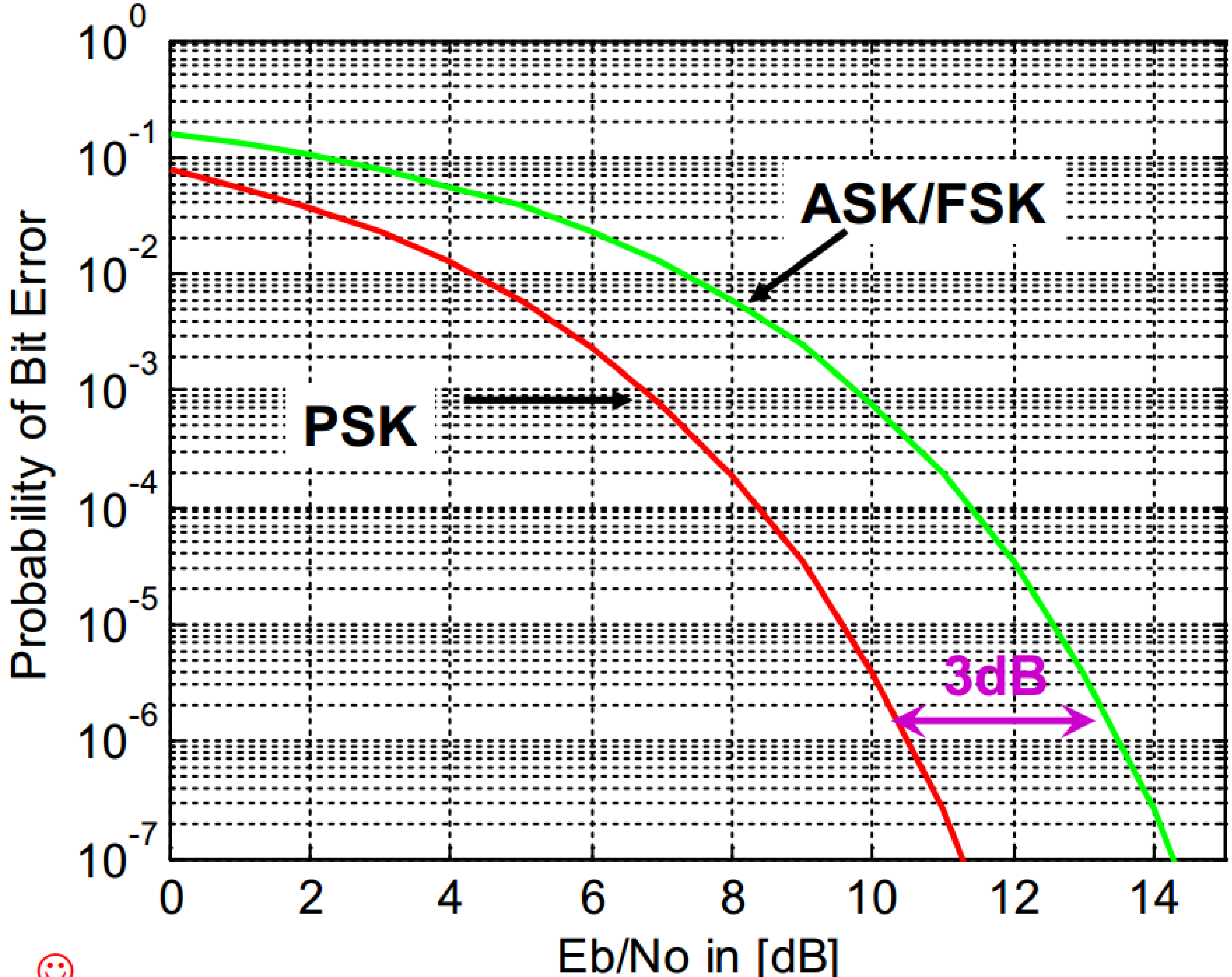
BPSK	BFSK	BASK
$d_{1,2} = 2\sqrt{E_b}$	$d_{1,2} = \sqrt{2E_b}$	$d_{1,2} = \sqrt{2E_b}$
$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$	$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$	$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$

- In general,

$$P_e = Q\left(\sqrt{\frac{d_{12}^2}{2N_0}}\right)$$

Binary digital modulation

- Comparison



Binary digital modulation

- Example
  - Binary data are transmitted over a microwave link at the rate of 10^6 bits/sec and the PSD of the noise at the receiver input is 10^{-10} watts/Hz.
  - Find the average carrier power required to maintain an average probability of error $P_e \leq 10^{-4}$ for coherent binary FSK.
  - What if noncoherent binary FSK?

Binary digital modulation

- Update
  - We have discussed coherent modulation schemes, e.g., BPSK, BFSK, BASK, which need coherent detection assuming that the receiver is able to detect and track the carrier wave's phase
  - In many practical situations, strict phase synchronization is not possible. In these situations, non-coherent reception is required.
  - We now consider non-coherent detection on binary FSK and differential phase-shift keying (DPSK)

Binary digital modulation

- Non-coherent scheme: BFSK
  - Consider a binary FSK system, the two signals are

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t + \theta_1)$$

$$0 \le t < T_b$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t + \theta_2)$$

  - θ_1, θ_2 : unknown random phases with uniform distribution

$$p_{\theta_1}(\theta) = p_{\theta_2}(\theta) = \begin{cases} 1/2\pi & \theta \in [0, 2\pi) \\ 0 & \text{else} \end{cases}$$

Binary digital modulation

- Non-coherent scheme: BFSK
  - Since

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_1 t + \theta_1) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_1 t)\cos(\theta_1) - \sqrt{\frac{2E_b}{T_b}}\sin(2\pi f_1 t)\sin(\theta_1)$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_2 t + \theta_2) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_2 t)\cos(\theta_2) - \sqrt{\frac{2E_b}{T_b}}\sin(2\pi f_2 t)\sin(\theta_2)$$

  - Choose four basis functions as

$$\phi_{1c}(t) = \sqrt{2/T_b}\cos(2\pi f_1 t) \quad \phi_{1s}(t) = -\sqrt{2/T_b}\sin(2\pi f_1 t)$$

$$\phi_{2c}(t) = \sqrt{2/T_b}\cos(2\pi f_2 t) \quad \phi_{2s}(t) = \sqrt{2/T_b}\sin(2\pi f_2 t)$$

  - Signal space representation

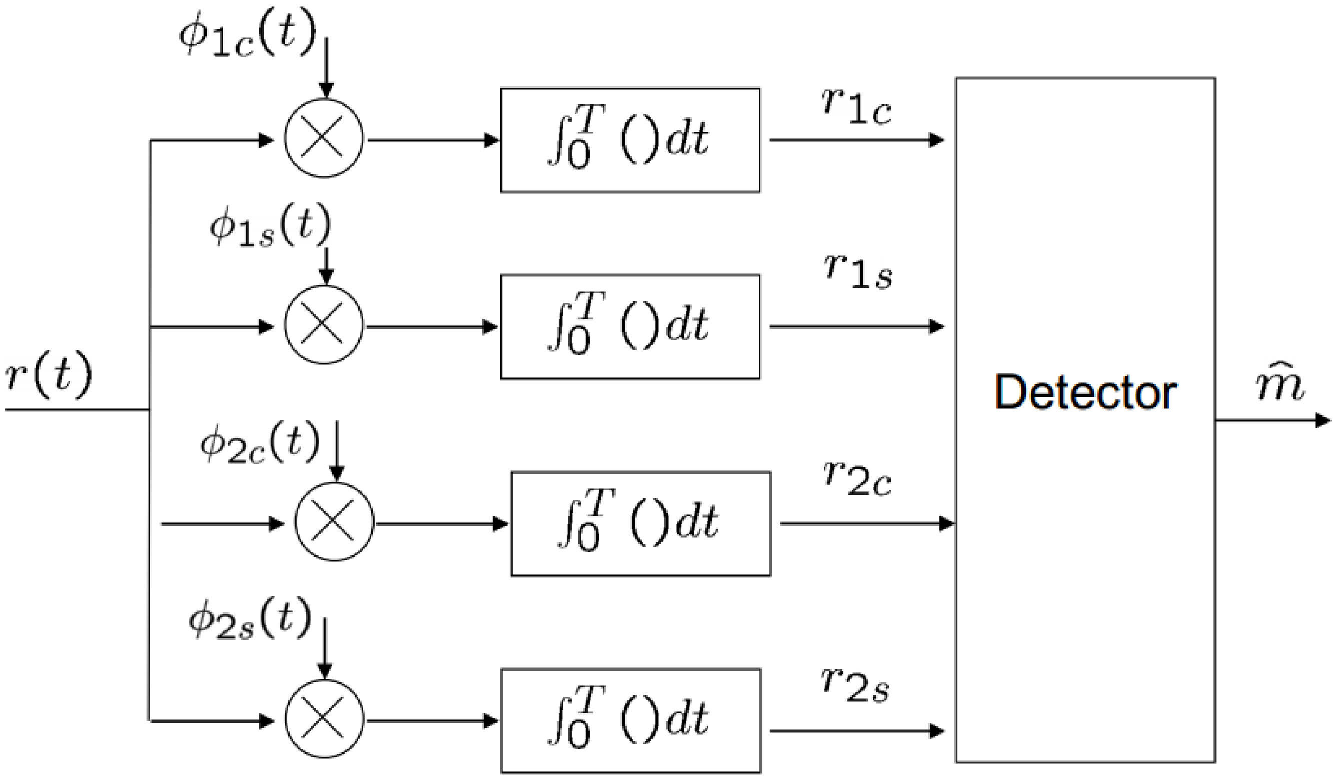
$$\vec{s}_1 = [\ \sqrt{E_b}\cos\theta_1 \quad \sqrt{E_b}\sin\theta_1 \quad 0 \quad 0\ ]$$

$$\vec{s}_2 = [\ 0 \quad 0 \quad \sqrt{E_b}\cos\theta_2 \quad \sqrt{E_b}\sin\theta_2\ ]$$

Binary digital modulation

- Non-coherent scheme: BFSK
  - The vector representation of the received signal

$$\vec{r} = [r_{1c} \quad r_{1s} \quad r_{2c} \quad r_{2s}]$$



Binary digital modulation

- Non-coherent scheme: BFSK
  - Decision rule:

$$f(\vec{r}|\vec{s}_1) \underset{\text{Choose } s_2}{\overset{\text{Choose } s_1}{\gtrless}} f(\vec{r}|\vec{s}_2)$$

ML

  - Conditional pdf

$$f(\vec{r}|\vec{s}_1,\theta_1) = \frac{1}{\pi N_0}\exp\left[-\frac{(r_{1c}-\sqrt{E_b}\cos\theta_1)^2+(r_{1s}-\sqrt{E_b}\sin\theta_1)^2}{N_0}\right] \times \frac{1}{\pi N_0}\exp\left[-\frac{r_{2c}^2+r_{2s}^2}{N_0}\right]$$

  - Similarly

$$f(\vec{r}|\vec{s}_2,\theta_2) = \frac{1}{\pi N_0}\exp\left[-\frac{r_{1c}^2+r_{1s}^2}{N_0}\right] \times \frac{1}{\pi N_0}\exp\left[-\frac{(r_{2c}-\sqrt{E_b}\cos\theta_2)^2+(r_{2s}-\sqrt{E_b}\sin\theta_2)^2}{N_0}\right]$$

Binary digital modulation

- Non-coherent scheme: BFSK
  - For ML decision, we need to evaluate

$$f(\vec{r}|\vec{s}_1) \geq f(\vec{r}|\vec{s}_2)$$

$$\frac{1}{2\pi}\int_0^{2\pi} f(\vec{r}|\vec{s}_1,\theta_1)d\theta_1 \geq \frac{1}{2\pi}\int_0^{2\pi} f(\vec{r}|\vec{s}_2,\theta_2)d\theta_2$$

  - Removing the constant terms

$$\left(\frac{1}{\pi N_0}\right)^2 \exp\left[-\frac{r_{1c}^{\ 2}+r_{1s}^{\ 2}+r_{2c}^{\ 2}+r_{2s}^{\ 2}+E}{N_0}\right]$$

  - We have

$$\frac{1}{2\pi}\int_0^{2\pi}\exp\left[\frac{2\sqrt{E}r_{1c}\cos(\phi_1)+2\sqrt{E}r_{1s}\sin(\phi_1)}{N_0}\right]d\phi_1$$

$$\geq \frac{1}{2\pi}\int_0^{2\pi}\exp\left[\frac{2\sqrt{E}r_{2c}\cos(\phi_1)+2\sqrt{E}r_{2s}\sin(\phi_1)}{N_0}\right]d\phi_2$$

Binary digital modulation

- Non-coherent scheme: BFSK
  - By definition

$$\frac{1}{2\pi}\int_0^{2\pi}\exp\left[\frac{2\sqrt{E}r_{1c}\cos(\phi_1)+2\sqrt{E}r_{1s}\sin(\phi_1)}{N_0}\right]d\phi_1 = I_0\left(\frac{2\sqrt{E(r_{1c}^2+r_{1s}^2)}}{N_0}\right)$$

where $I_0()$ is a modified Bessel function of the zero-th order

  - Thus, the decision rule becomes: choose $\mathbf{s}_1$ if

$$I_0\left(\frac{2\sqrt{E(r_{1c}^2+r_{1s}^2)}}{N_0}\right) \geq I_0\left(\frac{2\sqrt{E(r_{2c}^2+r_{2s}^2)}}{N_0}\right)$$

Binary digital modulation

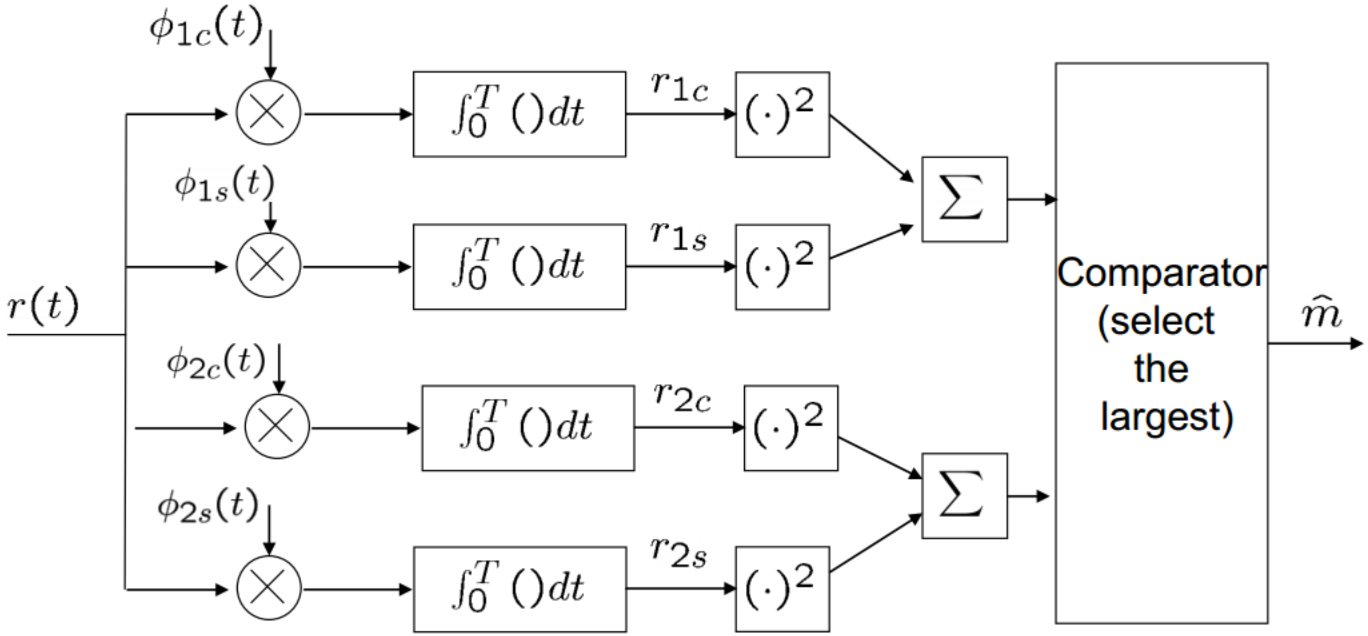
- Non-coherent scheme: BFSK
  - Note that this Bessel function is monotonically increasing. Therefore, we choose $\mathbf{s}_1$ if

$$\sqrt{r_{1c}^2 + r_{1s}^2} \geq \sqrt{r_{2c}^2 + r_{2s}^2}$$

1. Useful insight: we just compare the energy in the two frequencies and pick the larger (**envelope detector**)
2. Carrier phase is irrelevant in decision making

Binary digital modulation

- Non-coherent scheme: BFSK
  - Structure.

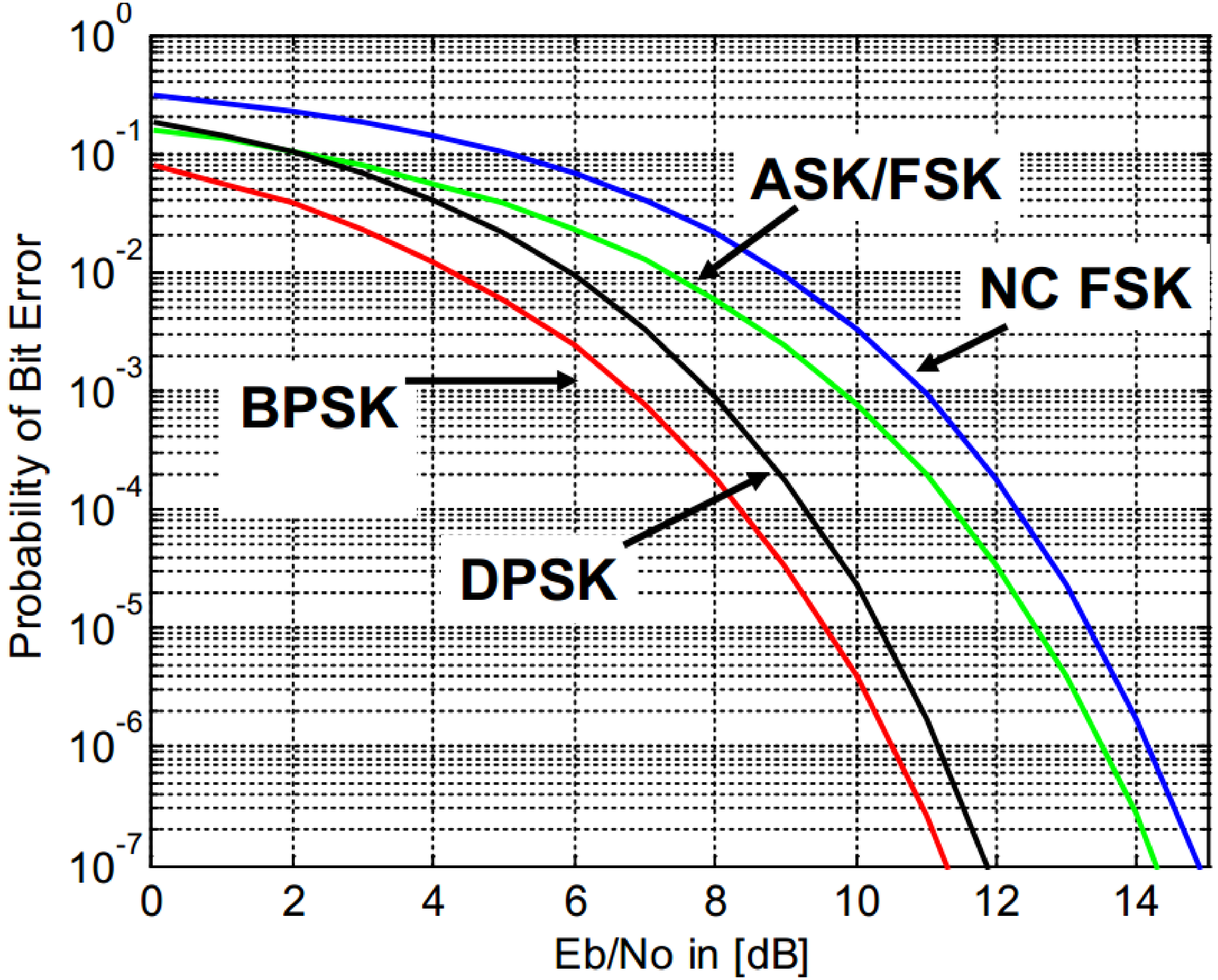


$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)$$

See Section 9.5.2

Binary digital modulation

- Comparison



Binary digital modulation

- Differential PSK (**DPSK**)
  - Non-coherent version of PSK
  - Phase synchronization is eliminated using differential encoding

1. Encode the information in phase difference between successive signal transmission.
2. Send "0" , advance the phase of the current signal by 180^{o}
3. Send "1", leave the phase unchanged

- Provided that the unknown phase θ contained in the received wave varies slowly (constant over two bit intervals), the phase difference between waveforms received in two successive bit intervals will be independent of θ

Binary digital modulation

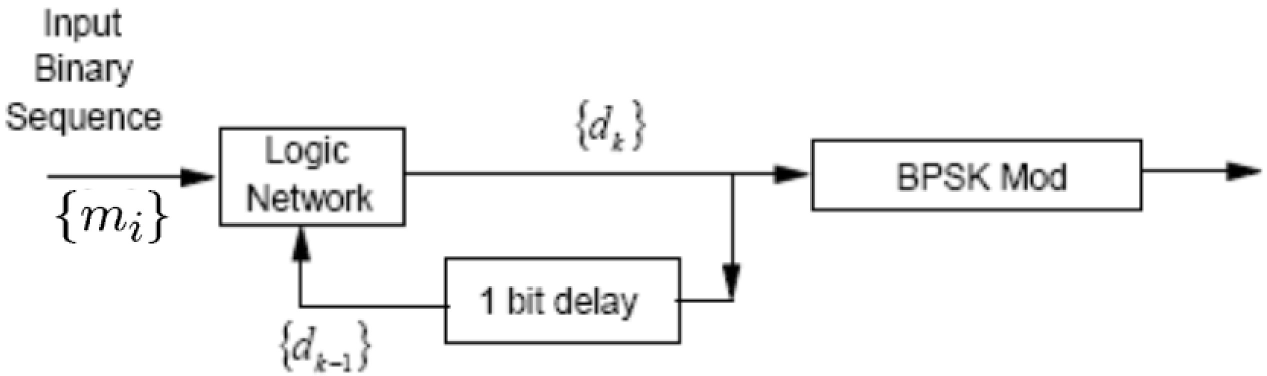
- Differential PSK (**DPSK**)
  - Generate DPSK signals in two steps
    1. Differential encoding of the information binary bits.
    2. Phase shift keying
  - Differential encoding starts with an arbitrary reference bit

Information sequence		1	0	0	1	0	0	1	1	$\{m_i\}$
Differentially encoded sequence	1 (Initial bit)	1	0	1	1	0	1	1	1	$\{d_i\}$
Transmitted Phase	0	0	π	0	0	π	0	0	0	

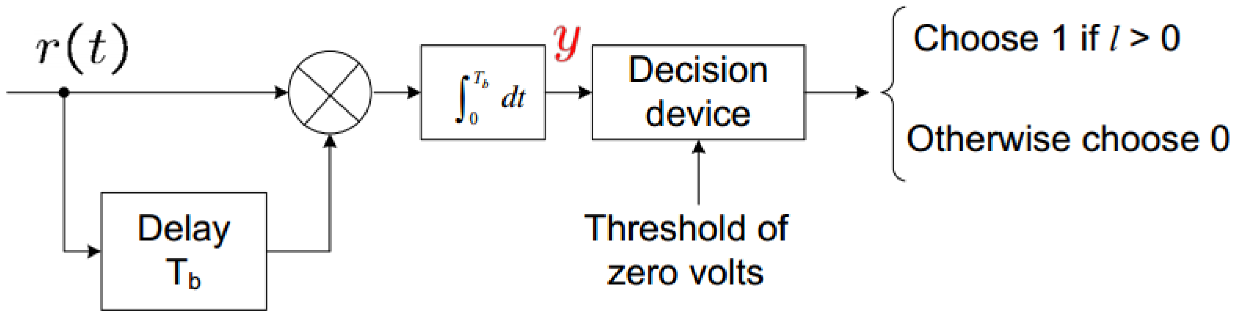
$$d_i = \overline{d_{i-1} \oplus m_i}$$

Binary digital modulation

- Differential PSK (**DPSK**)
  - Structure.



- Differential PSK (**DPSK**)
  - Differential detection.



  - Output of integrator (assume noise free)

$$y = \int_0^{T_b} r(t)r(t-T_b)dt = \int_0^{T_b} \cos(w_c t + \psi_k + \theta)\cos(w_c t + \psi_{k-1} + \theta)dt$$

$$\propto \cos(\psi_k - \psi_{k-1})$$

  - The unknown phase θ becomes irrelevant. The decision becomes: if $\psi_k - \psi_{k-1} = 0$ (bit 1), then y>0; if $\psi_k - \psi_{k-1} = \pi$ (bit 0), then y<0

$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$$

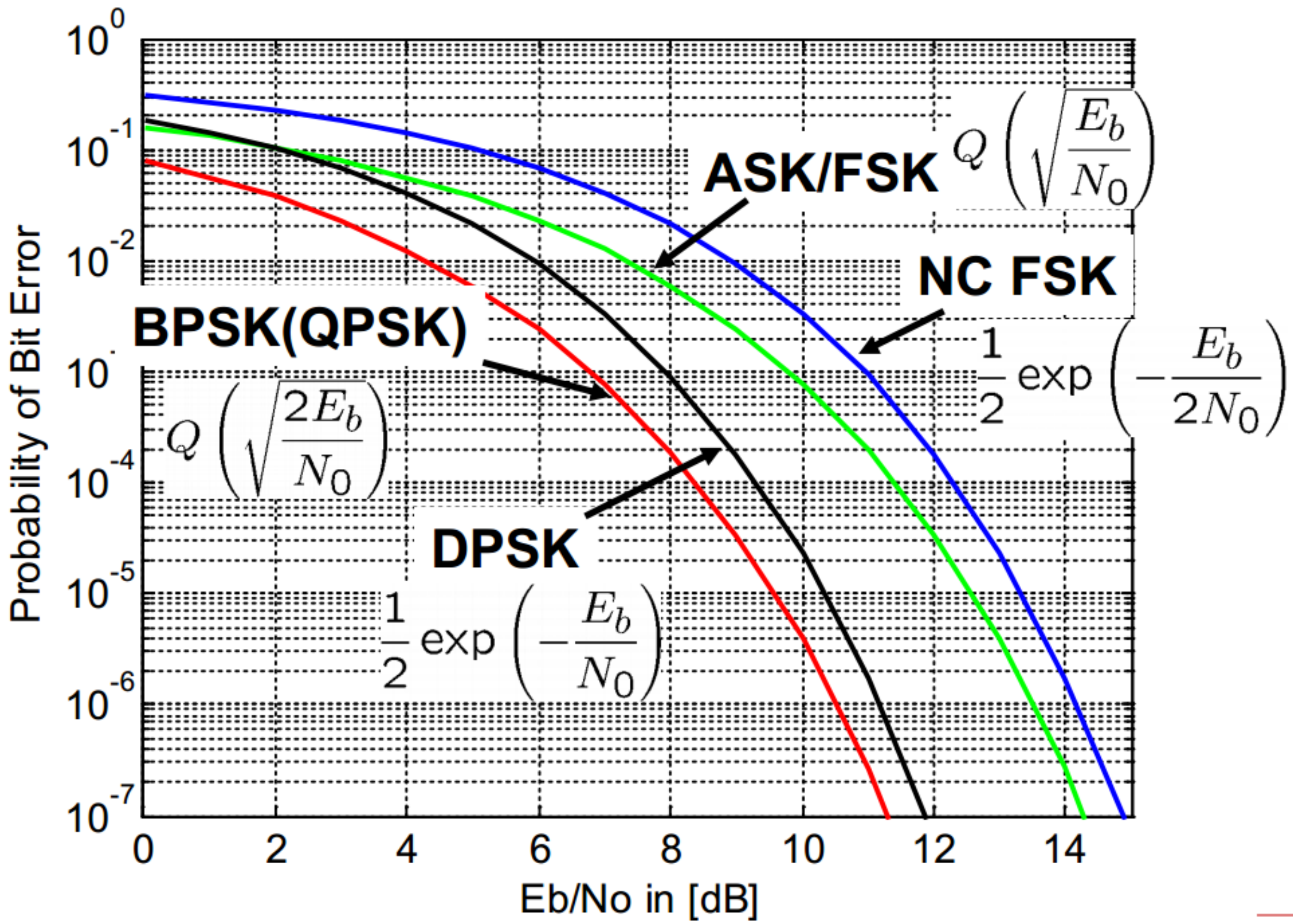
Binary digital modulation

- Comparison

Coherent PSK	$Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$
Coherent ASK	$Q\left(\sqrt{\frac{E_b}{N_0}}\right)$
Coherent FSK	$Q\left(\sqrt{\frac{E_b}{N_0}}\right)$
Non-Coherent FSK	$\frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)$
DPSK	$\frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$

Binary digital modulation

- Comparison



M-ary digital modulation

- Why?

M-ary digital modulation

- In binary data transmission, send only one of two possible signals during each bit interval T_b
- In M-ary data transmission, send one of M possible signals during each signaling interval T
- In almost all applications, $M=2^n$ and $T=nT_b$, where n is an integer
- Each of the M signals is called a **symbol**
- These signals are generated by changing the amplitude, phase, frequency, or combined forms of a carrier in M discrete steps.
- Thus, we have MASK, MPSK, MFSK, and **MQAM**

M-ary digital modulation

- M-ary Phase-shift Keying (**MPSK**)
  - Modulation: The phase of the carrier takes on M possible values

$$\theta_m = 2\pi(m-1)/M, \;\; m = 1, \ldots, M$$

  - Signal set

$$s_m(t) = \sqrt{\frac{2E_s}{T}} \cos\left[2\pi f_c t + \frac{2\pi(m-1)}{M}\right] \qquad \begin{array}{c} m = 1, \ldots, M \\ 0 \leq t < T \end{array}$$

  - Es=Energy per symbol, $f_c >> \frac{1}{T}$
  - Basis functions

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$$

$$0 \leq t < T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$$

M-ary digital modulation

- M-ary Phase-shift Keying (**MPSK**)
  - Signal space representation.

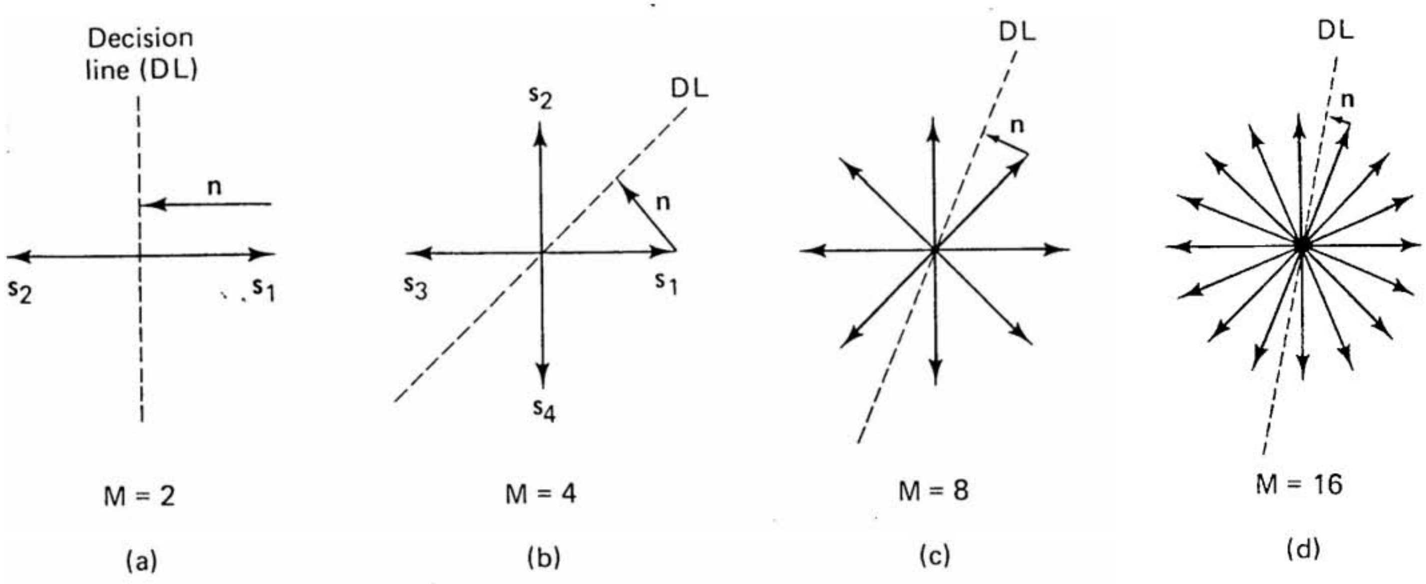
$$
\begin{aligned}
s_m(t) &= \sqrt{\frac{2E_s}{T}}\cos\left[2\pi f_c t + \frac{2\pi(m-1)}{M}\right] \\
&= \sqrt{\frac{2E_s}{T}}\cos(2\pi f_c t)\cos\left[\frac{2\pi(m-1)}{M}\right] \\
&\quad -\sqrt{\frac{2E_s}{T}}\sin(2\pi f_c t)\sin\left[\frac{2\pi(m-1)}{M}\right] \\
&= \sqrt{E_s}\cos\left[\frac{2\pi(m-1)}{M}\right]\phi_1(t) - \sqrt{E_s}\sin\left[\frac{2\pi(m-1)}{M}\right]\phi_2(t)
\end{aligned}
$$

$$
\Rightarrow \quad \mathbf{s}_m = \left[\ \sqrt{E_s}\cos\left(\frac{2\pi(m-1)}{M}\right) \quad \sqrt{E_s}\sin\left(\frac{2\pi(m-1)}{M}\right)\ \right]
$$

$$
m = 1, \ldots, M
$$

M-ary digital modulation

- M-ary Phase-shift Keying (**MPSK**)
  - ➢ Signal constellations.



BPSK QPSK 8PSK 16PSK

M-ary digital modulation

- M-ary Phase-shift Keying (**MPSK**)
  - Euclidean distance

$$d_{mn} = \|\mathbf{s}_m - \mathbf{s}_n\| = \sqrt{2E_s\left(1-\cos\frac{2\pi(m-n)}{M}\right)}$$

  - The minimum Euclidean

$$d_{\min} = \sqrt{2E_s\left(1-\cos\frac{2\pi}{M}\right)} = 2\sqrt{E_s}\sin\frac{\pi}{M}$$

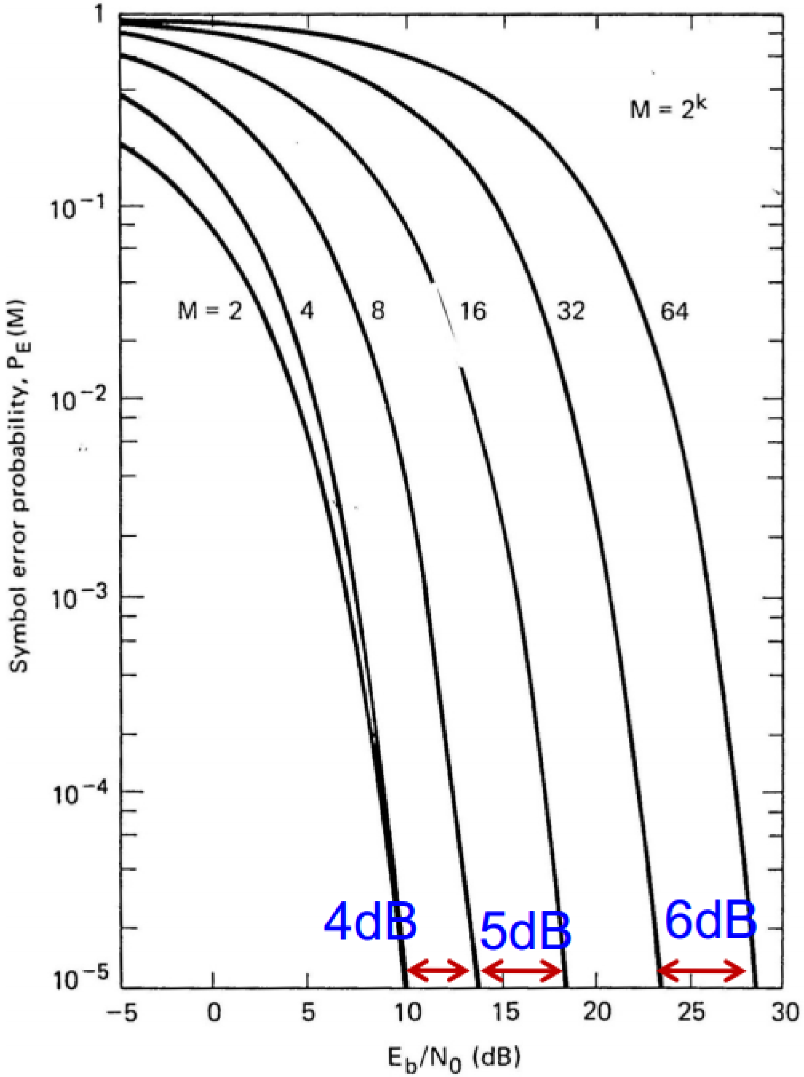
  - $d_{\min}$ plays an important role in determining error performance as discussed previously (union bound)
  - In the case of PSK modulation, the error probability is dominated by the erroneous selection of either one of the two signal points adjacent to the transmitted signal point
  - Consequently, an approximation to the symbol error probability is $P_{MPSK} \approx 2Q\left(\frac{d_{\min}/2}{\sqrt{N_0/2}}\right) = 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{M}\right)$

M-ary digital modulation

- M-ary Phase-shift Keying (**MPSK**)
  - Exercise: Consider the M=2, 4, 8 PSK signal constellations. All have the same transmitted signal energy Es.
  - Determine the minimum distance $d_{\min}$ between adjacent signal points
  - For M=8, determine by how many dB the transmitted signal energy Es must be increased to achieve the same $d_{\min}$ as M=4.

M-ary digital modulation

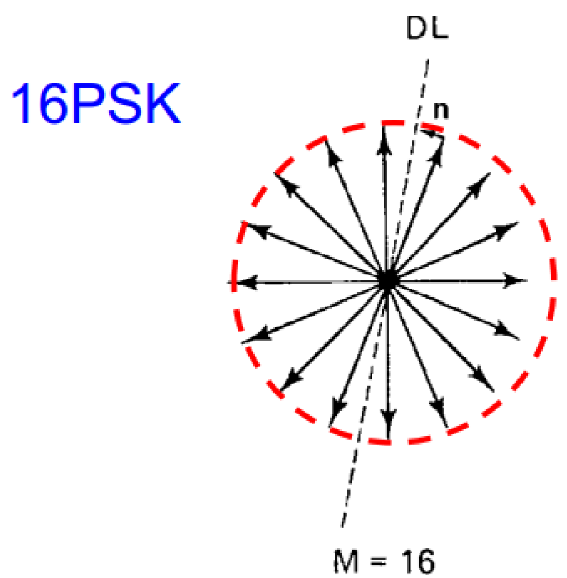
- M-ary Phase-shift Keying (**MPSK**)

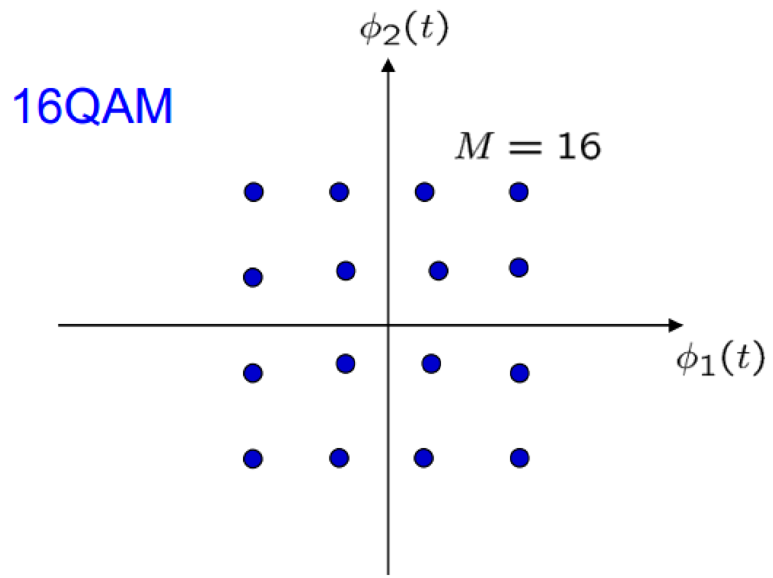


For large M, doubling the number of phases requires an additional 6 dB/bit to achieve the same performance

M-ary digital modulation

- M-ary Quadrature Amplitude Modulation (**MQAM**)
  - In MPSK, in-phase and quadrature components are interrelated in such a way that the envelope is constant (circular constellation)
  - If we relax this constraint, we get M-ary QAM





M-ary digital modulation

- M-ary Quadrature Amplitude Modulation (**MQAM**)
  - Modulation:

$$s_i(t) = \sqrt{\frac{2E_0}{T}} a_i \cos(2\pi f_c t) + \sqrt{\frac{2E_0}{T}} b_i \sin(2\pi f_c t)$$

  - E_0 is the energy of the signal with the lowest amplitude
  - a_i, b_i are a pair of independent integers
  - Basis functions

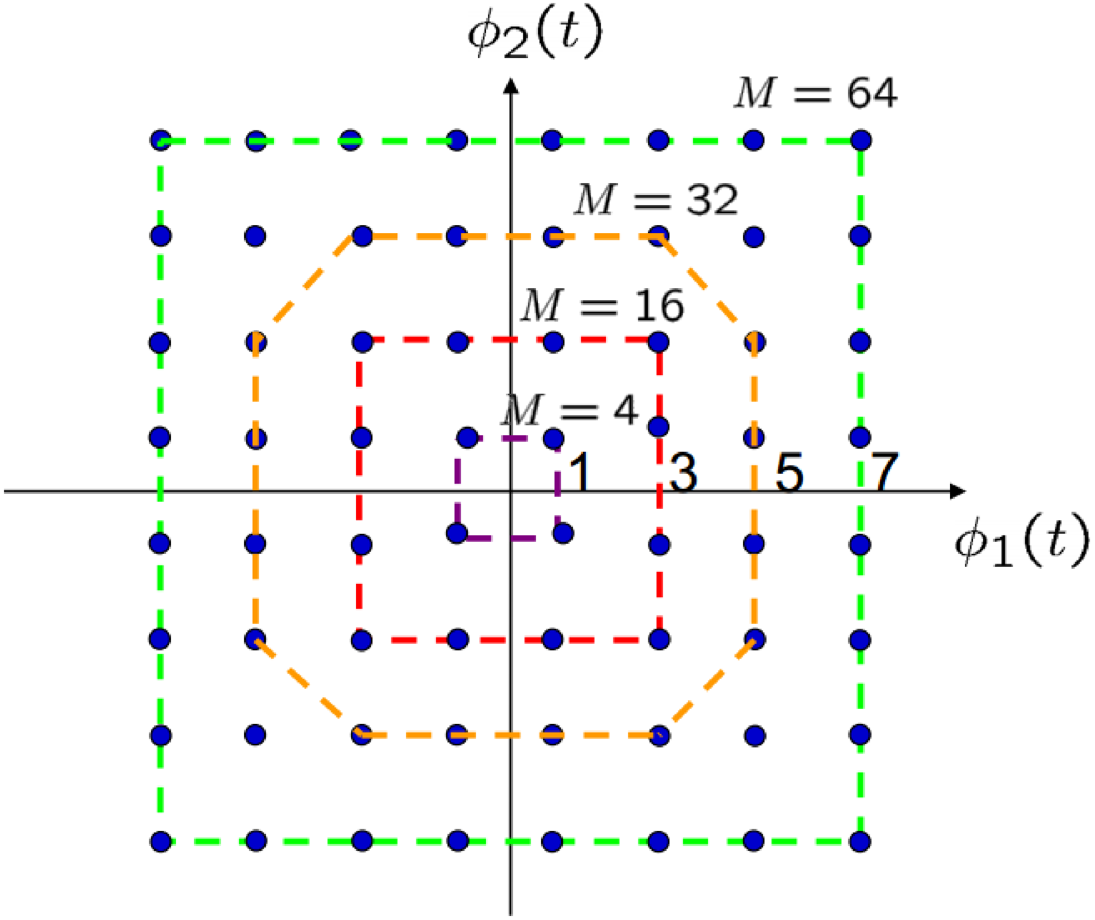
$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \quad \phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \quad 0 \le t < T$$

  - Signal space representation

$$\vec{s}_i = [\sqrt{E_0} a_i \quad \sqrt{E_0} b_i]$$

M-ary digital modulation

- M-ary Quadrature Amplitude Modulation (**MQAM**)
  - Signal constellation.



M-ary digital modulation

- M-ary Quadrature Amplitude Modulation (**MQAM**)
  - Probability of error analysis.
  - Upper bound of the symbol error probability

$$P_e \le 4Q\left(\sqrt{\frac{3kE_b}{(M-1)N_0}}\right) \qquad \text{(for } M = 2^k \text{)}$$

Think about the increase in Eb required to maintain the same error performance if the number of bits per symbol is increased from k to k+1, where k is large.

M-ary digital modulation

- M-ary Frequency-shift Keying (**MFSK**) (Multitone Signaling)
  - Signal set:

$$s_m(t) = \sqrt{\frac{2E_s}{T}} \cos\{2\pi(f_c + (m-1)\triangle f)t\} \quad \begin{array}{l} m = 1, \ldots, M \\ 0 \le t < T \end{array}$$

    where $\triangle f = f_m - f_{m-1}$ with $f_m = f_c + m\triangle f$

  - Correlation between two symbols

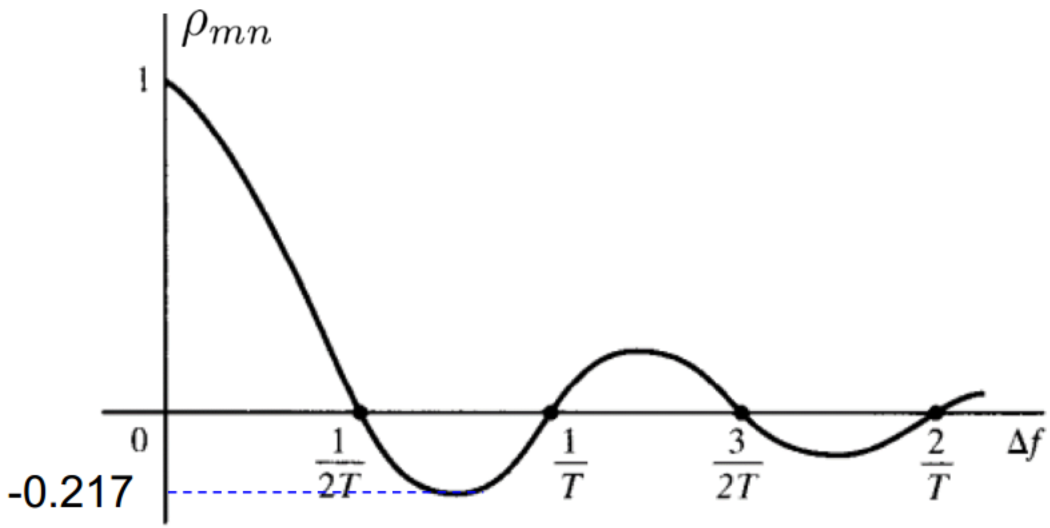
$$\rho_{mn} = \frac{1}{E_s} \int_0^T s_m(t) s_n(t) dt$$

$$= \frac{\sin[2\pi(m-n)\triangle f T]}{2\pi(m-n)\triangle f T}$$

$$= \text{sinc}[2(m-n)\triangle f T]$$

M-ary digital modulation

- M-ary Frequency-shift Keying (**MFSK**) (Multitone Signaling)



  - For orthogonality, the minimum frequency separation is

$$\Delta f = \frac{1}{2T}$$

M-ary digital modulation

- M-ary Frequency-shift Keying (**MFSK**) (Multitone Signaling)
  - Geometrical representation.

$$\mathbf{s}_0 = \left(\sqrt{E_s}, 0, 0, \cdots, 0\right)$$

$$\mathbf{s}_1 = \left(0, \sqrt{E_s}, 0, \cdots, 0\right)$$

$$\vdots$$

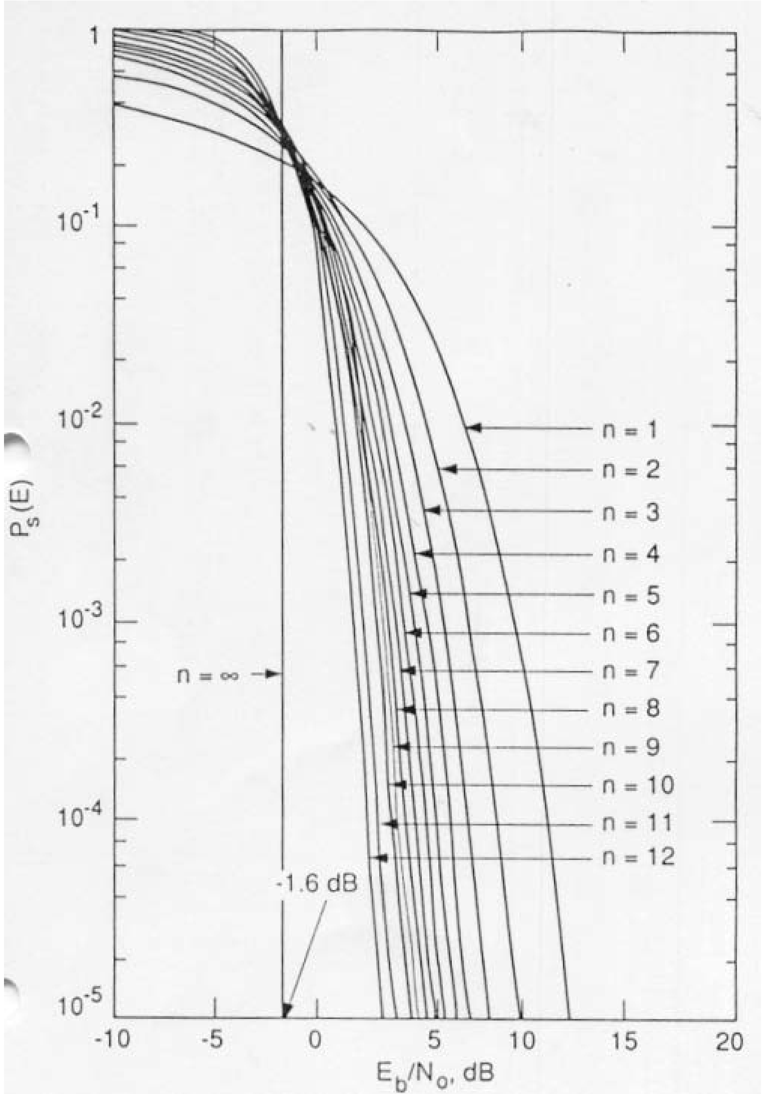
$$\mathbf{s}_{M-1} = \left(0, 0, \cdots, 0, \sqrt{E_s}\right)$$

  - Basis functions.

$$\phi_m = \sqrt{\frac{2}{T}} \cos 2\pi \left(f_c + m\Delta f\right) t$$

M-ary digital modulation

- M-ary Frequency-shift Keying (**MFSK**) (Multitone Signaling)
  - Probability of error.



M-ary digital modulation

- Notes
  - Pe is found by integrating conditional probability of error over the decision region, which is difficult to compute but can be simplified using union bound
  - Pe depends only on the distance profile of the signal constellation

M-ary digital modulation

- **Gray Code**
  - Symbol errors are different from bit errors
  - When a symbol error occurs, all k bits could be in error
  - In general, we can find BER using

$$P_b = \sum_{i=1}^{M} P(\vec{s}_i) \sum_{j=1, j\neq i}^{M} \frac{n_{i,j}}{\log_2 M} P\left(\hat{\vec{s}} = \vec{s}_j | \vec{s}_i\right)$$

n_{ij} the number of different bits between $\mathbf{s}_i$ and $\mathbf{s}_j$

  - Gray coding is a bit-to-symbol mapping, where two adjacent symbols differ in only one bit out of the k bits
  - An error between adjacent symbol pairs results in one and only one bit error

M-ary digital modulation

- **Gray Code**

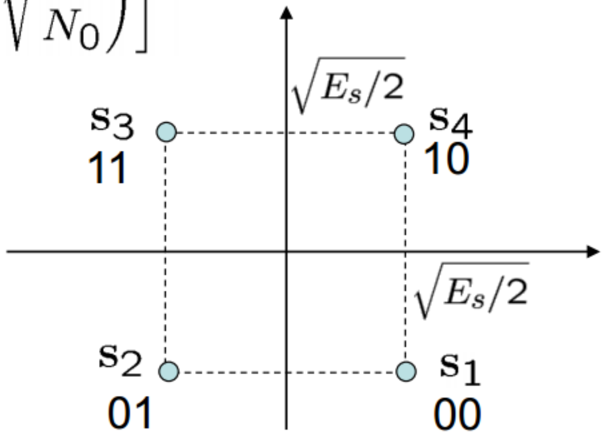
$$P_b = \sum_{i=1}^{M} \frac{1}{4} \sum_{j=1, j \neq i}^{M} \frac{n_{i,j}}{\log_2 M} P\left(\hat{\vec{s}} = \vec{s}_j | \vec{s}_i\right)$$

$$= \frac{1}{2} P\left(\hat{\vec{s}} = \vec{s}_1 | \vec{s}_4\right) + \frac{2}{2} P\left(\hat{\vec{s}} = \vec{s}_2 | \vec{s}_4\right) + \frac{1}{2} P\left(\hat{\vec{s}} = \vec{s}_3 | \vec{s}_4\right)$$

$$= \left[1 - Q\left(\sqrt{\frac{E_s}{N_0}}\right)\right] \cdot Q\left(\sqrt{\frac{E_s}{N_0}}\right) + \left[Q\left(\sqrt{\frac{E_s}{N_0}}\right)\right]^2$$

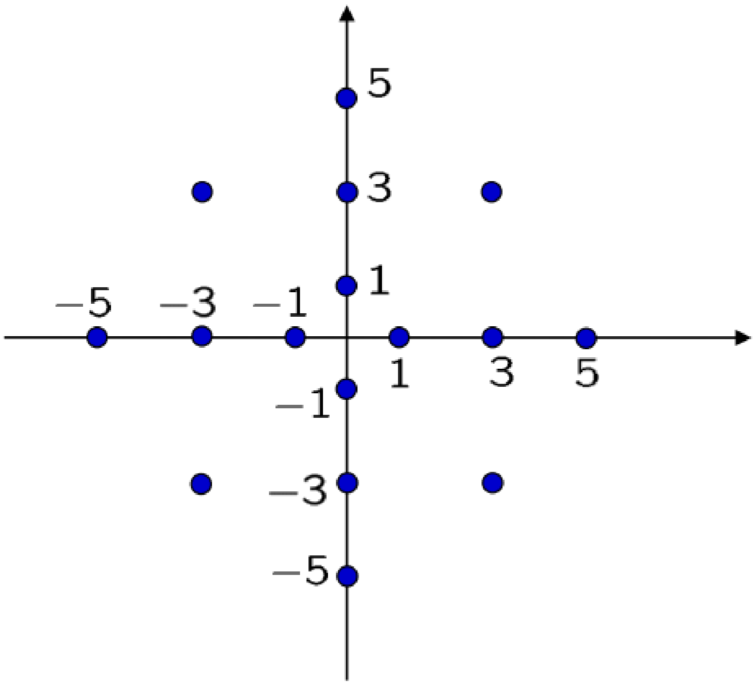
$$= Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$



M-ary digital modulation

- Example
  - The 16-QAM signal constellation shown right is an international standard for telephone-line modems (called V.29)
  - Determine the optimum decision boundaries for the detector
  - Derive the union bound of the probability of symbol error assuming that the SNR is sufficiently high so that errors only occur between adjacent points
  - Specify a Gray code for this 16-QAM V.29 signal constellation



- **Gray Code**
  - For MPSK with Gray coding, we know that an error between adjacent symbols will most likely occur. Thus, bit error probability can be approximated by

$$P_b \approx \frac{P_e}{\log_2 M}$$

  - For MFSK, when an error occurs, anyone of the other symbols may result equally likely. Thus, k/2 bits every k bits will on average be in error when there is a symbol error. The bit error rate is approximately half of the symbol error rate

$$P_b \cong \frac{1}{2} P_e$$

Think about why MQAM is more preferrable?

M-ary digital modulation

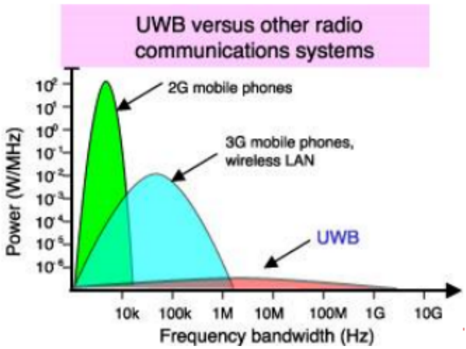
- Channel bandwidth and transmit power are two primary communication resources and have to be used as efficient as possible
  - Power utilization efficiency (energy efficiency): measured by the required E_b/N_0 to achieve a certain bit error probability
  - Spectrum utilization efficiency (bandwidth efficiency): measured by the achievable data rate per unit bandwidth R_b/B
- It is always desired to maximize bandwidth efficiency at a minimal required E_b/N_0

M-ary digital modulation

- Consider for example you are a system engineer in Huawei/ZTE, designing a part of the communication systems. You are required to design a modulation scheme for three systems using MFSK, MPSK or MQAM only. State the modulation level M to be low, medium or high

An ultra-wideband system

- Large amount of bandwidth
- Band overlays with other systems
- Purpose: high data rate



A wireless remote control system

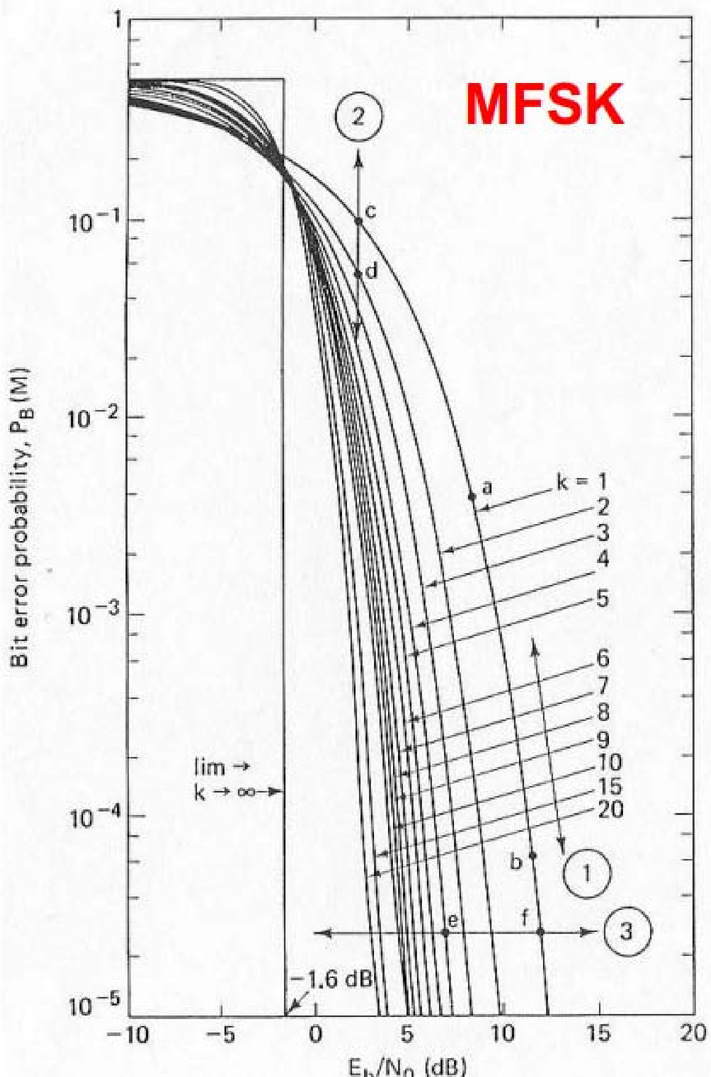
- Use unlicensed band
- Purpose: control devices remotely

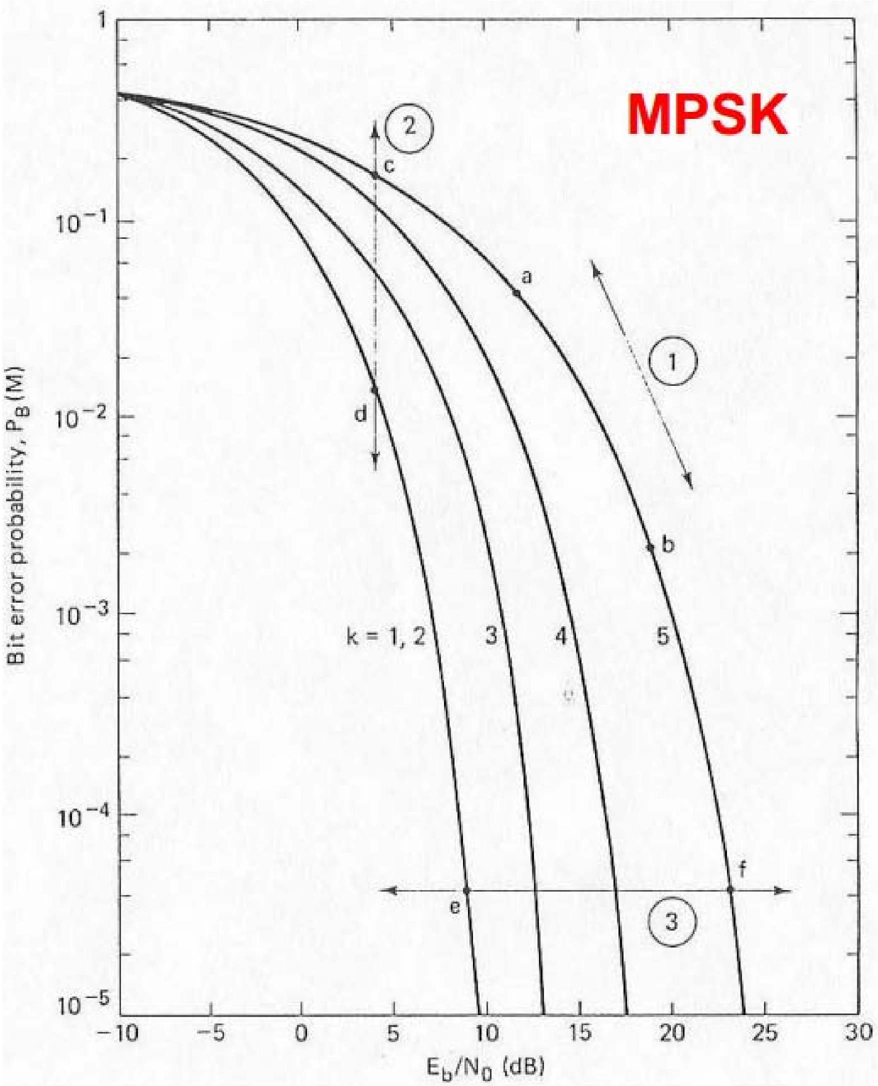
A fixed wireless system

- Use licensed band
- Transmitter and receiver fixed with power supply
- Voice and data connections in rural areas

M-ary digital modulation

- Energy efficiency comparison





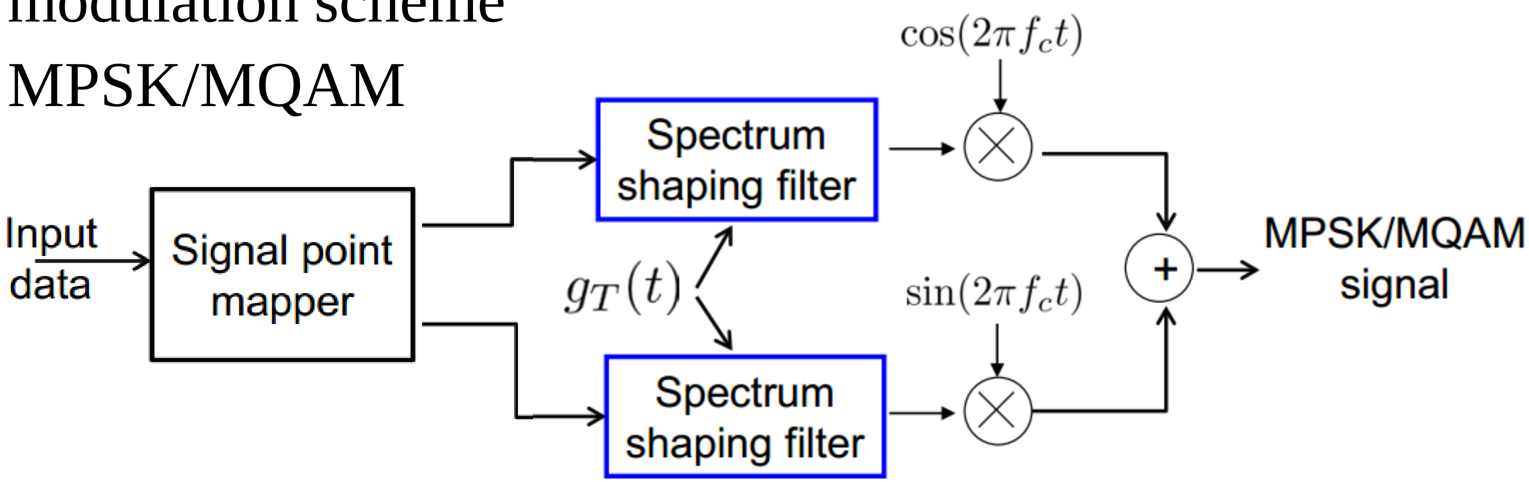
M-ary digital modulation

- Energy efficiency comparison
  - MFSK: At fixed E_b/N_0, increasing M can provide an improvement on P_b; At fixed P_b, increasing M can provide a reduction in the E_b/N_0
  - MPSK: BPSK and QPSK have the same energy efficiency. At fixed E_b/N_0, increasing M degrades P_b; At fxied P_b, increasing M increases the E_b/N_0 requirement

MFSK is more energy efficient than MPSK

M-ary digital modulation

- Bandwidth efficiency comparison
  - To compare bandwidth efficiency, we need to know the power spectral density (power spectra) of a given modulation scheme
  - MPSK/MQAM



  - If $g_T(t)$ is rectangular, the bandwidth of main-lobe is $B = \frac{2}{T_s}$
  - If it has a raised cosine spectrum, the bandwidth is $B = \frac{1+\alpha}{T_s}$

M-ary digital modulation

- Bandwidth efficiency comparison
  - In general, bandwidth required to pass MPSK/MQAM signal is approximately given by $B = \frac{1}{T_s}$.
  - The bit rate is $$R_b = \frac{\log_2 M}{T_s}$$
  - So the bandwidth efficiency may be expressed as

$$\rho = \frac{R_b}{B} = \log_2 M \ \text{(bits/sec/Hz)}$$

  - But for MFSK, bandwidth required to transmit MSFK signal is $B = \frac{M}{2T}$
  - Bandwidth efficiency

$$\rho = \frac{R_b}{B} = \frac{2\log_2 M}{M} \ \text{(bits/s/Hz)}$$

Adjacent frequencies need to be separated by 1/2T to maintain orthogonality

M-ary digital modulation

- Bandwidth efficiency comparison
  - In general, bandwidth required to pass MPSK/MQAM signal is approximately given by $B = \frac{1}{T_s}$
  - The bit rate is $$R_b = \frac{\log_2 M}{T_s}$$
  - So the bandwidth efficiency may be expressed as

MPSK/MQAM is more bandwidth efficient than MFSK

signal is $B = \frac{M}{2T}$

Adjacent frequencies need to be separated by 1/2T to maintain orthogonality

  - Bandwidth efficiency

$$\rho = \frac{R_b}{B} = \frac{2\log_2 M}{M} \quad \text{(bits/s/Hz)}$$

M-ary digital modulation

- Fundamental tradeoff: Bandwidth Efficiency vs. Energy Efficiency
  - To see the ultimate power-bandwidth tradeoff, we need to use Shannon's channel capacity theorem:

Channel capacity is the theoretical upperbound for the maximum rate at which information could be transmitted without error (Shannon 1948)

  - Specifically, for a bandlimited channel corrupted by AWGN, the maximum achievable rate is given by

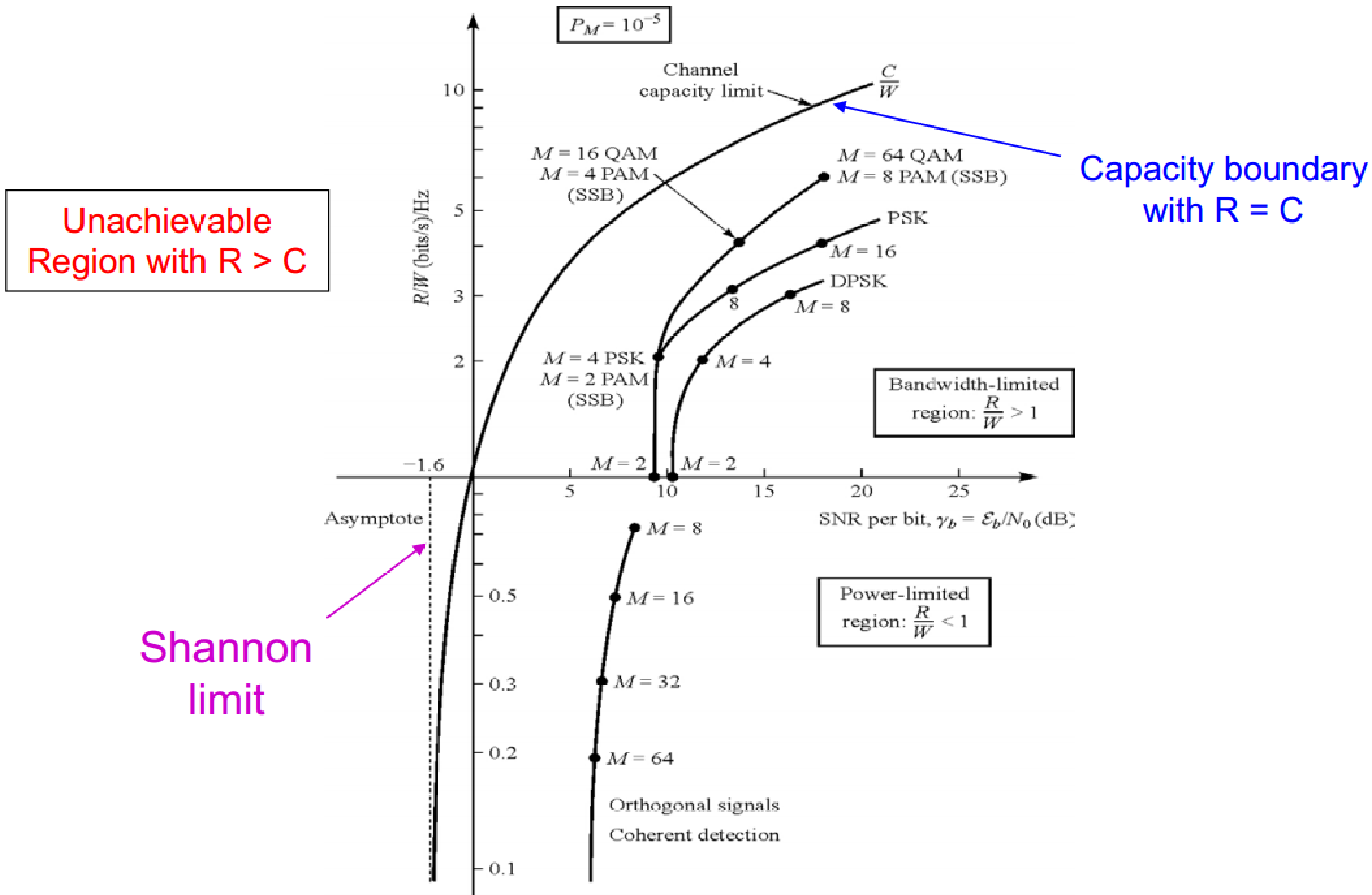
$$R \le C = B\log_2(1+SNR) = B\log_2\left(1+\frac{P_s}{N_0 B}\right)$$

  - Note that $\frac{E_b}{N_0} = \frac{P_s T}{N_0} = \frac{P_s}{RN_0} = \frac{P_s B}{RN_0 B} = SNR\frac{B}{R}$
  - Thus,

$$\frac{E_b}{N_0} = \frac{B}{R}(2^{R/B}-1)$$

M-ary digital modulation

- Fundamental tradeoff: Bandwidth Efficiency vs. Energy Efficiency



M-ary digital modulation

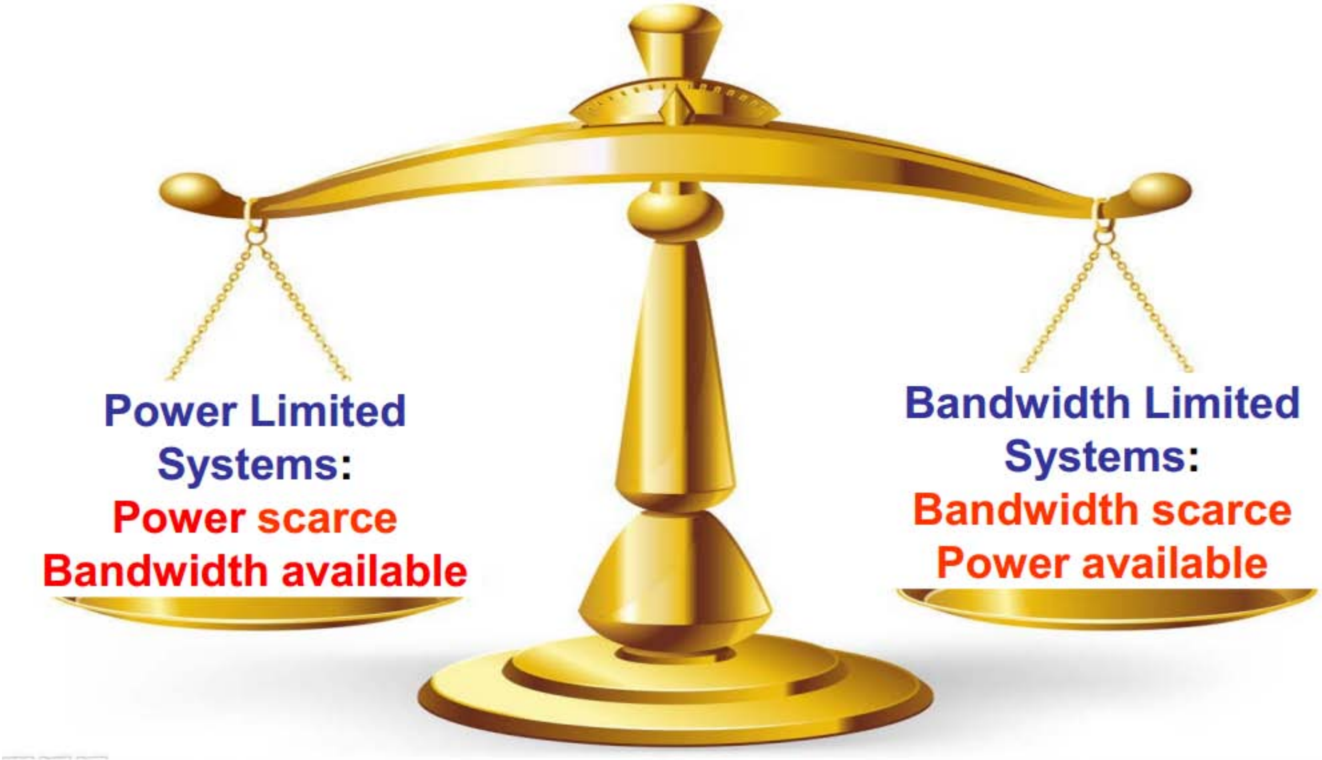
- Fundamental tradeoff: Bandwidth Efficiency vs. Energy Efficiency
  - In the limits as R/B goes to 0, we get

$$\frac{E_b}{N_0} = \ln 2 = 0.693 = -1.59dB$$

  - This value is called the Shannon limit. Received E_b/N_0 must be **>-1.59** dB to ensure reliable communication
  - BPSK and QPSK require the same E_b/N_0 of 9.6 dB to achieve $P_e=10^{-5}$. However, QPSK has a better bandwidth efficiency.
  - MQAM is superior to MPSK
  - MPSK/MQAM increases bandwidth efficiency at the cost of energy efficiency
  - MFSK trades energy efficiency at reduced bandwiidth efficiency

M-ary digital modulation

- Fundamental tradeoff: Bandwidth Efficiency vs. Energy Efficiency
  - Which modulation to use?



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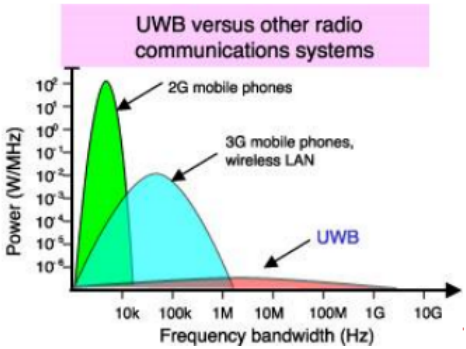

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M-ary digital modulation

- Practical applications
  - BPSK: WLAN IEEE 802.11b (1 Mbps)
  - QPSK:
    1. WLAN IEEE 802.11b (2 Mbps, 5.5 Mbps, 11 Mbps)
    2. 3G WCDMA
    3. DVB-T (with OFDM)
  - QAM:
    1. Telephone modem (16-QAM)
    2. Downstream of Cable modem (64-QAM, 256-QAM)
    3. WLAN IEEE 802.11 a/g (16-QAM for 24 Mbps, 36 Mbps; 64-QAM for 38 Mbps and 54 Mbps)
    4. LTE cellular Systems
    5. 5G
  - FSK:
    1. Cordless telephone
    2. Paging system