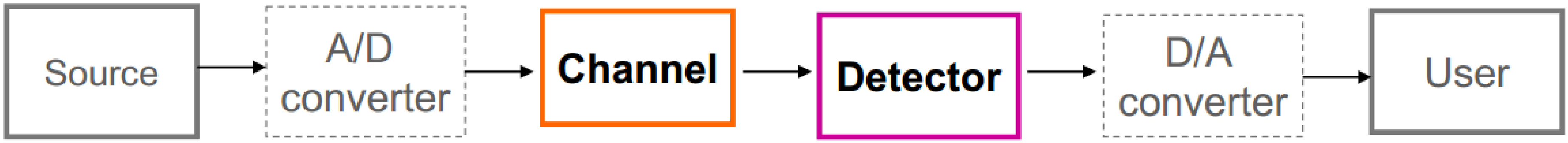
Outline

- Introduction
- Signal, random variable, random process and spectra
- Analog modulation
- Analog to digital conversion
- **Digital transmission through bandlimited channels**
- Signal space representation
- Optimal receivers
- Digital modulation techniques
- Channel coding
- Synchronization
- Information theory

Digital transmission through bandlimited channels



- Digital waveforms over band-limited baseband channels
- Band-limited channel and Inter-symbol interference
- Signal design for band-limited channel
- System design
- Channel equalization

Chapter 10.1-10.5

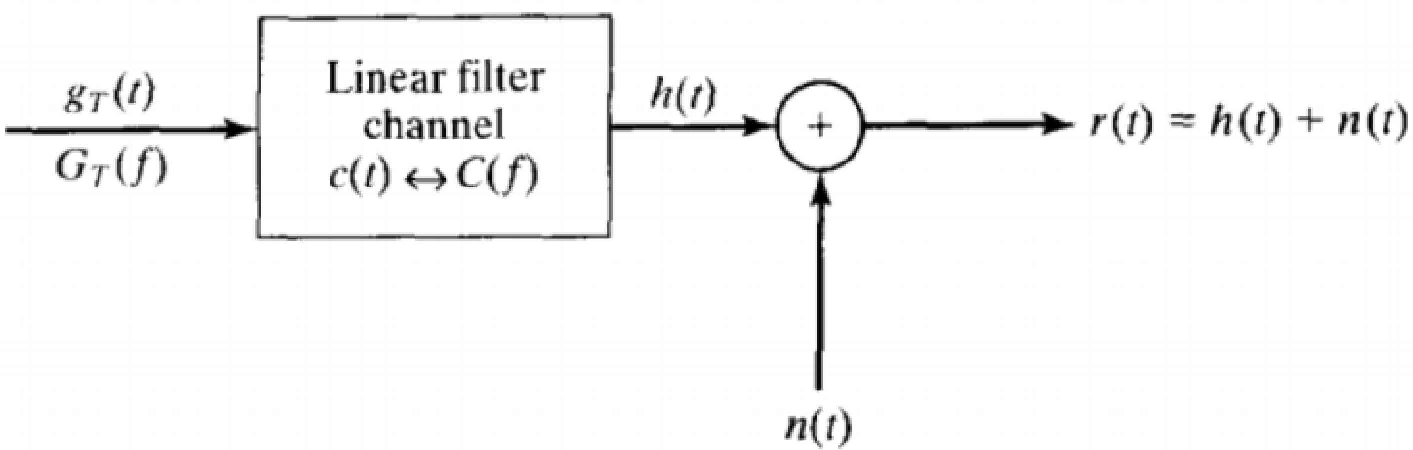
Digital transmission through bandlimited channels

- Band-limited channel

20MHz

5MHz, 10MHz
15MHz, 20MHz

- Modeled as a linear filter with frequency response limited to certain frequency range



Digital transmission through bandlimited channels

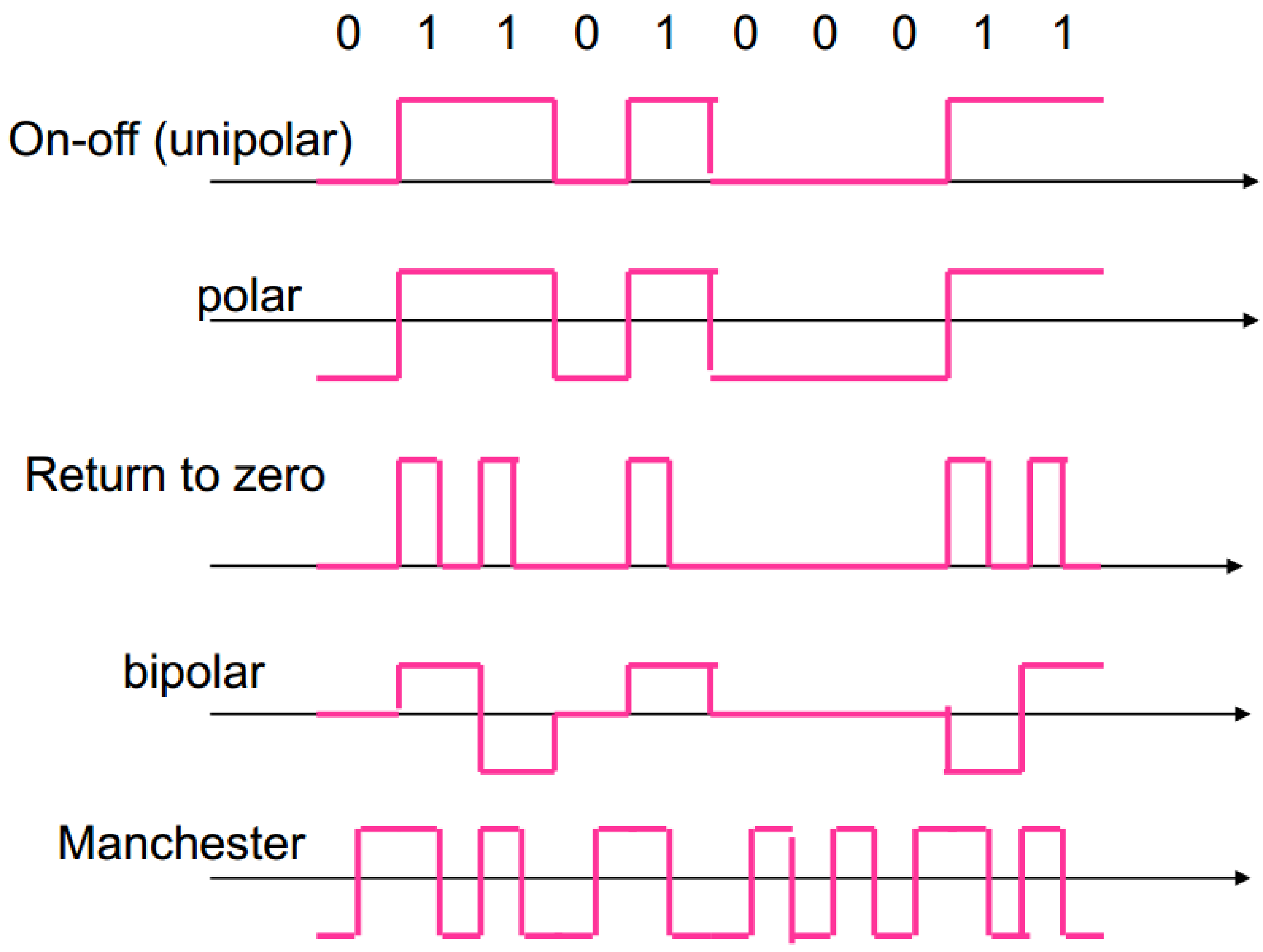
- Baseband signaling waveforms
  - To send the encoded digital data over a baseband channel, we require the use of format or waveform for representing the data
  - System requirement on digital waveforms
    1. Easy to synchronize
    2. High spectrum utilization efficiency
    3. Good noise immunity
    4. No DC component and little low frequency component
    5. Self-error-correction capability
    6. Et al.

Digital transmission through bandlimited channels

- Basic waveforms
  - On-off or unipolar signaling
  - Polar signaling
  - Return-to-zero signaling
  - Bipolar signaling (useful because no DC)
  - Split-phase or Manchester code (no DC)
  - Et al.

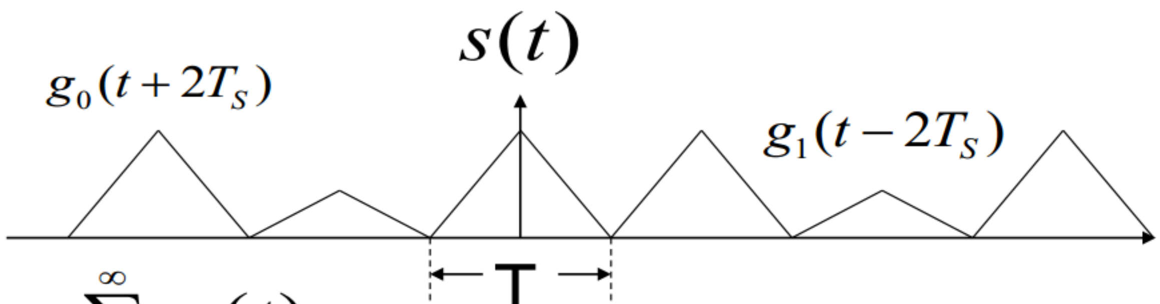
Digital transmission through bandlimited channels

- Basic waveforms



Digital transmission through bandlimited channels

- Spectra of baseband signals
  - Consider a random binary sequence $g_0(t)$ – 0, $g_1(t)$ – 1
  - The pulses $g_0(t)$ and $g_1(t)$ occur independently with probabilities given by p and 1-p, respectively. The duration of each pulse is given by Ts.



$$s(t) = \sum_{n=-\infty}^{\infty} s_n(t)$$

$$s_n(t) = \begin{cases} g_0(t - nT_s), & \text{with prob. } P \\ g_1(t - nT_s), & \text{with prob. } 1 - P \end{cases}$$

Digital transmission through bandlimited channels

- Spectra of baseband signals
  - PSD of the baseband signal s(t) is[1]

$$S(f)=\frac{1}{T_s}p(1-p)\left|G_0(f)-G_1(f)\right|^2+\frac{1}{T_s^2}\sum_{m=-\infty}^{\infty}\left|pG_0\left(\frac{m}{T_s}\right)+(1-p)G_1\left(\frac{m}{T_s}\right)\right|^2\delta\left(f-\frac{m}{T_s}\right)$$

    1st term is the continuous freq. component

    2nd term is the discrete freq. component

  - For polar signaling with $g_0(t)=-g_1(t)=g(t)$ and p=1/2

$$S(f)=\frac{1}{T}\left|G(f)\right|^2$$

  - For unipolar signaling with $g_0(t)=0$ $g_1(t)=g(t)$ and p=1/2, and g(t) is a rectangular pulse

$$G(f)=T\left[\frac{\sin\pi fT}{\pi fT}\right] \quad \Rightarrow \quad S_x(f)=\frac{T}{4}\left[\frac{\sin\pi fT}{\pi fT}\right]^2+\frac{1}{4}\delta(f)$$

1. R.C. Titsworth and L. R. Welch, "**Power spectra of signals modulated by random and pseudrandom sequences**," JPL, CA, Technical Report, Oct. 1961.

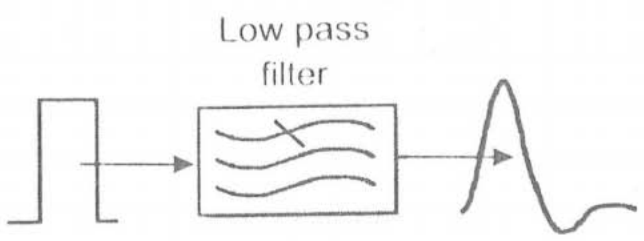
Digital transmission through bandlimited channels

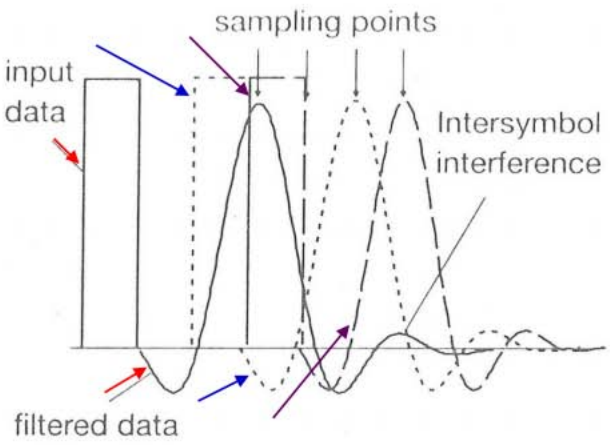
- Spectra of baseband signals
  - For return-to-zero unipolar signaling $\tau = T/2$

$$S_x(f) = \frac{T}{16}\left[\frac{\sin \pi fT/2}{\pi fT/2}\right]^2 + \frac{1}{16}\delta(f) + \frac{1}{4}\sum_{\text{odd } m}\frac{1}{[m\pi]^2}\delta(f - \frac{m}{T})$$

Digital transmission through bandlimited channels

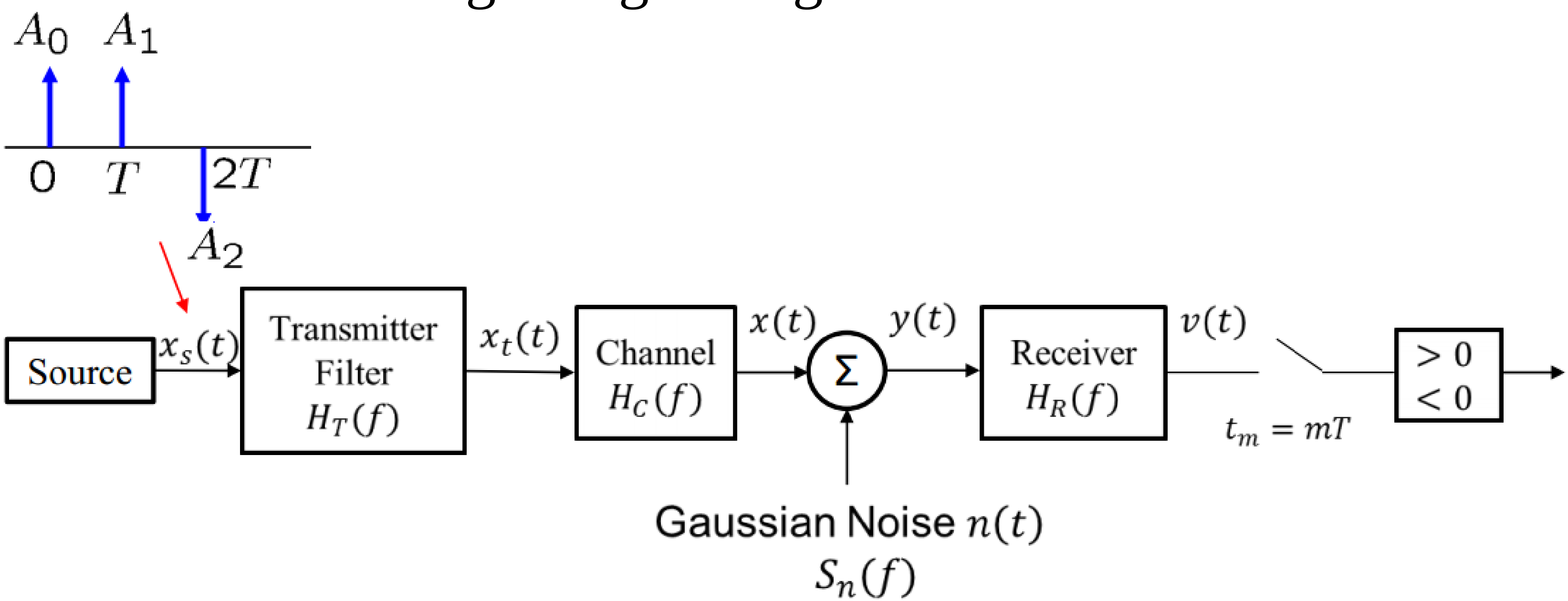
- Inter-symbol interference
  - The filtering effect of the band-limited channel will cause a spreading of individual data symbols passing through
  - For consecutive symbols, this spreading causes part of symbol energy to overlap with neighboring symbols, causing inter-symbol interference





Digital transmission through bandlimited channels

- Baseband signaling through band-limited channels



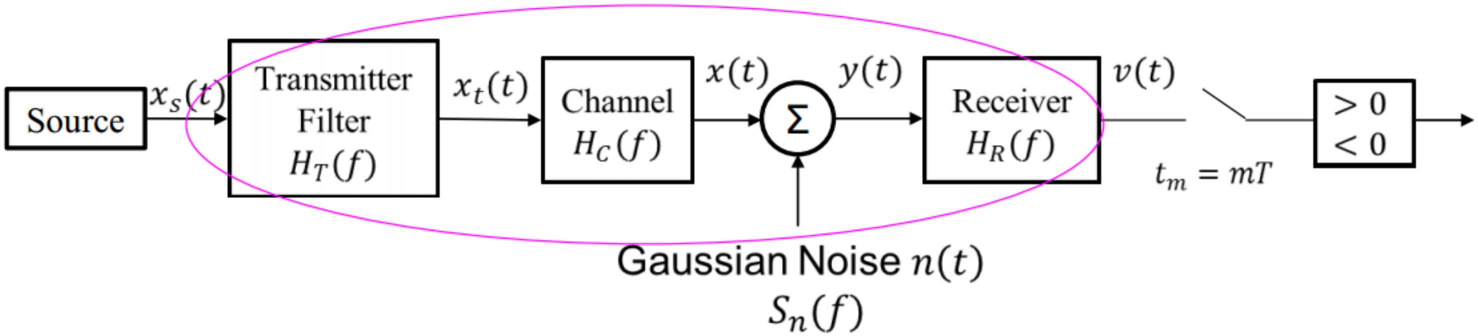
Input to tx filter $$x_s(t) = \sum_{i=-\infty}^{\infty} A_i \delta(t - iT)$$

Output of tx filter $$x_t(t) = \sum_{i=-\infty}^{\infty} A_i h_T(t - iT)$$

Output of rx filter $$v(t) = x_s(t) * h_T(t) * h_c(t) * h_R(t) + n(t) * h_R(t)$$

Digital transmission through bandlimited channels

- Baseband signaling through band-limited channels



  - Pulse shape at the receiver filter output

$$p(t) = h_T(t) * h_c(t) * h_R(t)$$

  - Overall frequency response

$$P(f) = H_T(f) H_C(f) H_R(f)$$

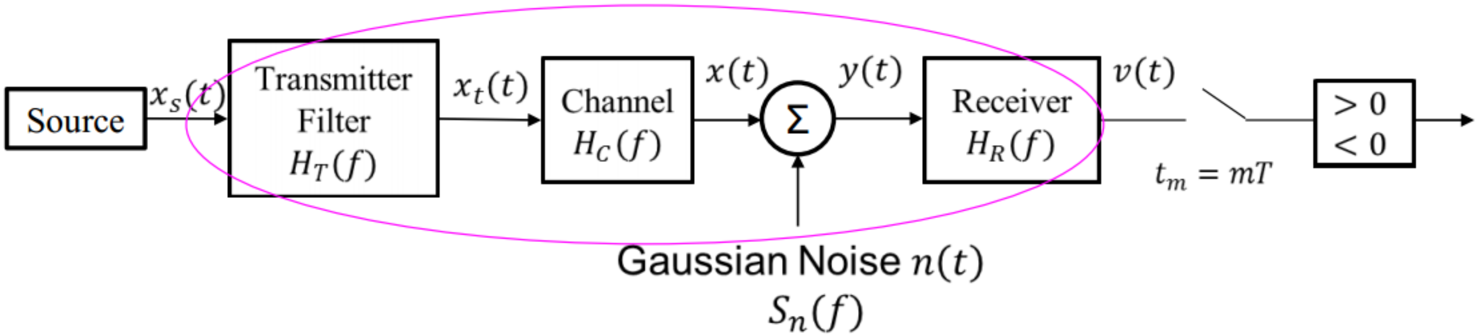
  - Receiving filter output

$$v(t) = \sum_{k=-\infty}^{\infty} A_k p(t - kT) + n_o(t)$$

$$n_o(t) = n(t) * h_R(t)$$

Digital transmission through bandlimited channels

- Baseband signaling through band-limited channels

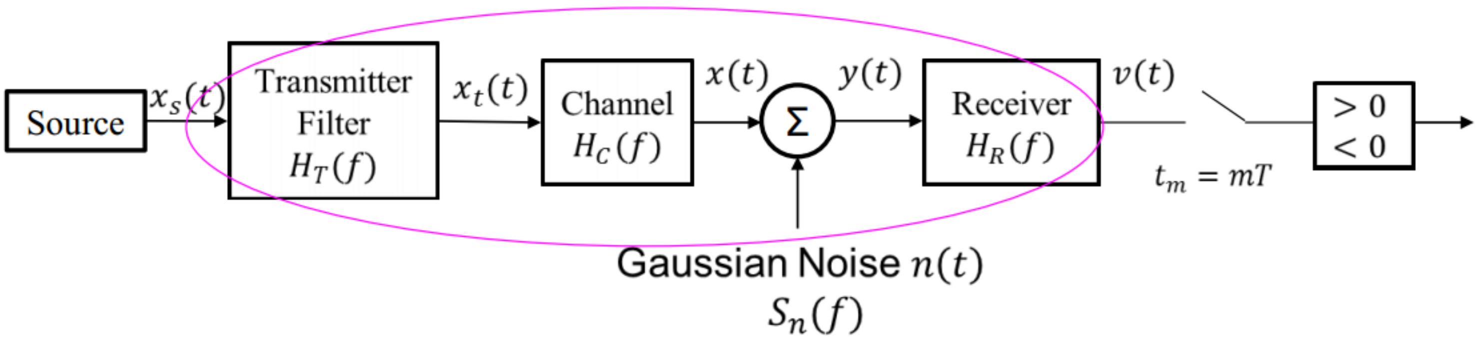


  - ➢ Sample the receiver filter output v(t) at $t_m = mT$ to detect A_m

$$
\begin{aligned}
v(t_m) &= \sum_{k=-\infty}^{\infty} A_k p(mT - kT) + n_o(t_m) \\
&= \underbrace{A_m p(0)}_{\text{Desired signal}} + \underbrace{\sum_{\substack{k=-\infty \\ k \neq m}}^{\infty} A_k p[(m-k)T]}_{\text{intersymbol interference (ISI)}} + \underbrace{n_o(t_m)}_{\text{Gaussian noise}}
\end{aligned}
$$

Digital transmission through bandlimited channels

- Baseband signaling through band-limited channels



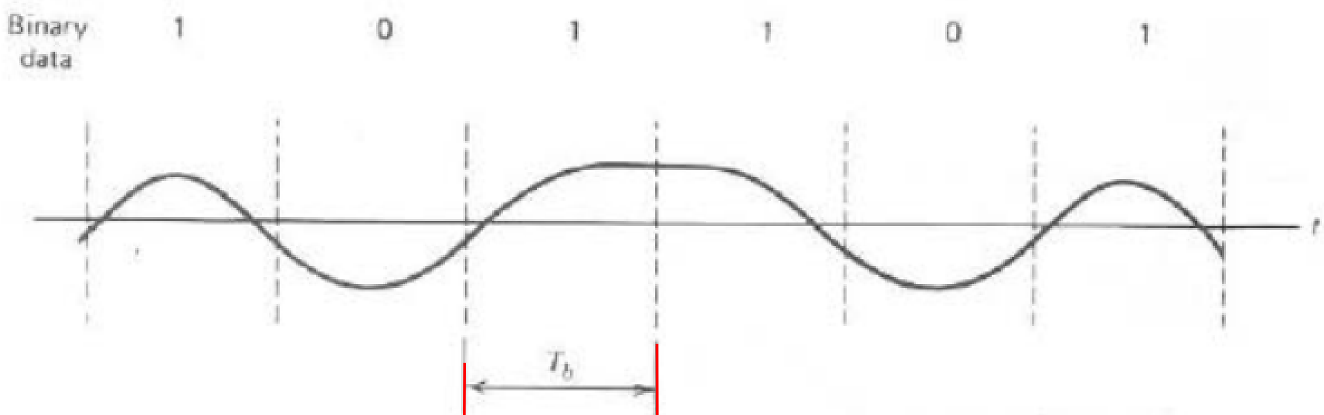
➢ Sample the receiver filter output v(t) at t_m=mT to detect A_m

$$v(t_m) = \sum_{k=-\infty}^{\infty} A_k p(mT - kT) + n_o(t_m)$$

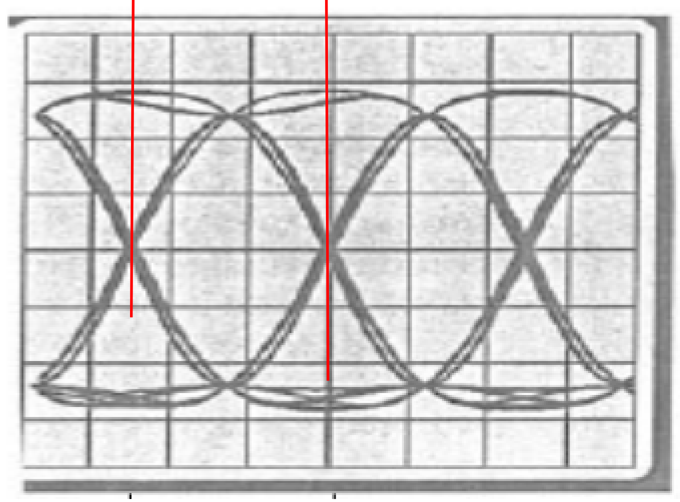
$$= \underbrace{A_m p(0)}_{\text{Desired signal}} + \underbrace{\sum_{k \neq m}^{\infty} A_k p[(m-k)T]}_{\text{intersymbol interference (ISI)}} + \underbrace{n_o(t_m)}_{\text{Gaussian noise}}$$

Digital transmission through bandlimited channels

- Eye diagram
  - Distorted binary wave



  - Eye diagram



Digital transmission through bandlimited channels

- ISI minimization
  - Choose transmitter and receiver filters which shape the received pulse function to eliminate or minimize interference between adjacent pulses, hence not to degrade the bit error rate performance of the link

Digital transmission through bandlimited channels

- Signal design for band-limited channel zero ISI
  - Nyquist condition for zero ISI for pulse shape p(t)

$$p(nT) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

Echos made to be zero at sampling points

or $$\sum_{k=-\infty}^{\infty} P\left(f + \frac{k}{T}\right) = \text{constant}$$

  - With the above condition, the receiver output simplifies to

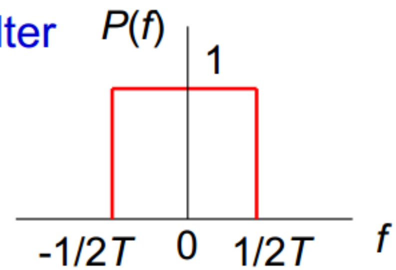
$$v(t_m) = A_m + n_o(t_m)$$

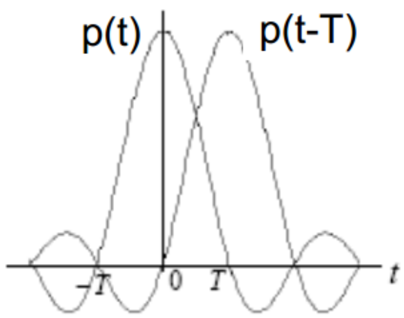
Digital transmission through bandlimited channels

- Signal design for band-limited channel zero ISI
  - Nyquist's first method for eliminating ISI is to use

$$P(f) = \begin{cases} 1 & |f| < \frac{1}{2T} \\ 0 & \text{otherwise} \end{cases} \Longleftrightarrow p(t) = \frac{\sin(\pi t/T)}{\pi t/T} = \operatorname{sinc}\left(\frac{t}{T}\right)$$

"brick wall" filter





$$B_0 = \frac{1}{2T} = \frac{R_s}{2}$$ = Nyquist bandwdith,

The minimum transmission bandwidth for zero ISI. A channel with bandwidth B_0 can support a maximal transmission rate of $2B_0$ symbols/sec

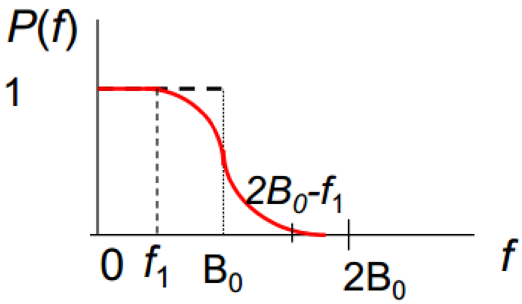
Digital transmission through bandlimited channels

- Signal design for band-limited channel zero ISI
  - Challenges of designing such p(t) or P(f)
    1. P(f) is physically unrealizable due to the abrupt transitions at B0
    2. p(t) decays slowly for large t, resulting in little margin of error in sampling times in the receiver
    3. This demands accurate sample point timing-a major challenge in current modem/data receiver design
    4. Inaccuracy in symbol timing is referred to as timing jitter.

Digital transmission through bandlimited channels

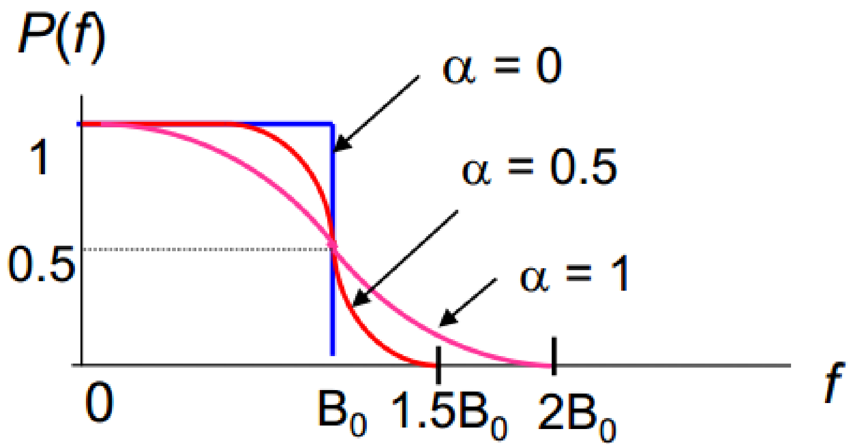
- Signal design for band-limited channel zero ISI
  - Raised cosine filter.
  - P(f) is made up of 3 parts: pass band, stop band, and transition band. The transition band is shaped like a cosine wave.

$$P(f)=\begin{cases} 1 & 0\le |f|<f_1 \\ \dfrac{1}{2}\left\{1+\cos\left[\dfrac{\pi(|f|-f_1)}{2B_0-2f_1}\right]\right\} & f_1\le |f|<2B_0-f_1 \\ 0 & |f|\ge 2B_0-f_1 \end{cases}$$



Digital transmission through bandlimited channels

- Signal design for band-limited channel zero ISI
  - ➢ Raised cosine filter.



Roll-off factor

$$\alpha = 1 - \frac{f_1}{B_0}$$

  - ➢ The sharpness of the filter is controlled by *a*.
  - ➢ Required bandwidth $B=B_0(1+a)$.

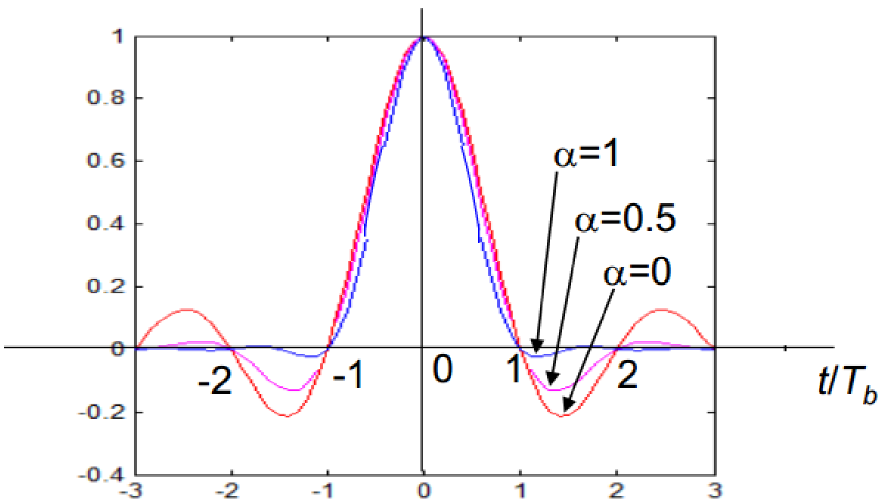
Digital transmission through bandlimited channels

- Signal design for band-limited channel zero ISI
  - Raised cosine filter.
  - Taking the inverse Fourier transform

$$p(t) = \mathrm{sinc}(2B_0 t)\frac{\cos(2\pi\alpha B_0 t)}{1-16\alpha^2 B_0^2 t^2}$$

Ensures zero crossing at desired sampling instants

Decreases as $1/t^2$, such that the data receiving is relatively insensitive to sampling time error



Digital transmission through bandlimited channels

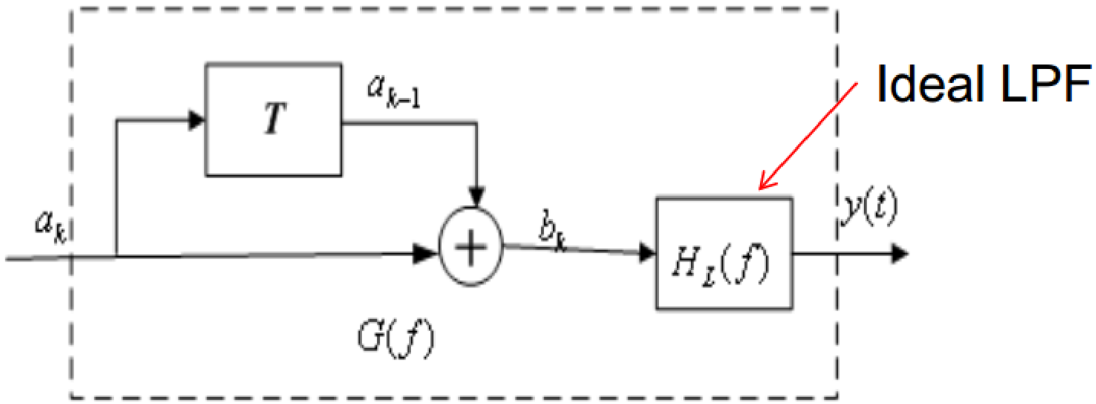
- Signal design for band-limited channel zero ISI
  - Raised cosine filter.
  - Small a: higher bandwidth efficiency
  - Large a: simpler filter with fewer stages hence easier to implement; less sensitive to symbol timing accuracy

Digital transmission through bandlimited channels

- Signal design with controlled ISI – partial response signals
  - Relax the condition of zero ISI and allow a controlled amount of ISI
  - Then we can achieve the maximal symbol rate of 2W symbols/sec
  - The ISI we introduce is deterministic or controlled; hence it can be taken into account at the receiver

Digital transmission through bandlimited channels

- Signal design with controlled ISI – partial response signals
  - Duobinary signal.
  - Let $\{a_k\}$ be the binary sequence to be transmitted. The pulse duration is T.
  - Two adjacent pulses are added together, i.e., $b_k = a_k + a_{k-1}$



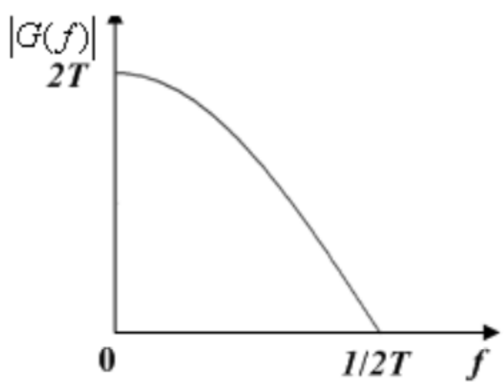
  - The resulting sequence $\{b_k\}$ is called duobinary signal

Digital transmission through bandlimited channels

- Signal design with controlled ISI – partial response signals
  - Duobinary signal: frequency domain.

$$G(f) = \left(1 + e^{-j2\pi fT}\right) H_L(f) \qquad H_L(f) = \begin{cases} T & (|f| \le 1/2T) \\ 0 & (\text{otherwise}) \end{cases}$$

$$= \begin{cases} 2Te^{-j\pi fT} \cos \pi fT & (|f| \le 1/2T) \\ 0 & (\text{otherwise}) \end{cases}$$



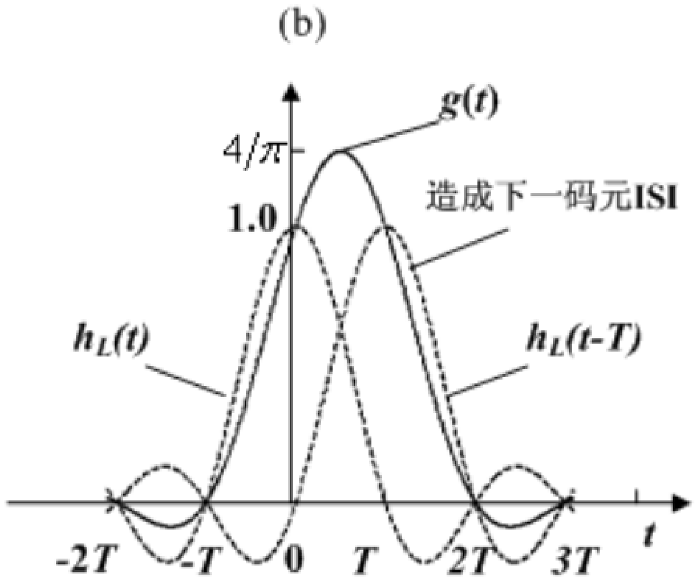
Digital transmission through bandlimited channels

- Signal design with controlled ISI – partial response signals
  - Duobinary signal: time domain.

$$g(t) = [\delta(t) + \delta(t-T)] * h_L(t) = \frac{\sin \pi t / T}{\pi t / T} + \frac{\sin \pi (t-T)/T}{\pi (t-T)/T}$$

$$= \mathrm{sinc}\left(\frac{t}{T}\right) + \mathrm{sinc}\left(\frac{t-T}{T}\right) = \frac{T^2}{\pi t} \cdot \frac{\sin \pi t / T}{(T-t)}$$

  - g(t) is called a duobinary signal pulse
  - $g(0)=g_0=1$
  - $g(T)=g_1=1$
  - $g(iT)=g_i=0, i \neq 1$



Digital transmission through bandlimited channels

- Signal design with controlled ISI – partial response signals
  - Duobinary signal: decoding.
  - Without noise, the received signal is the same as the transmitted signal

$$y_k = \sum_{i=0}^{\infty} a_i g_{k-i} = a_k + a_{k-1} = b_k$$

A 3-level sequence

  - When {a_k} is a polar sequence with values +1 or -1

$$y_k = b_k = \begin{cases} 2 & (a_k = a_{k-1} = 1) \\ 0 & (a_k = 1, a_{k-1} = -1 \text{ or } a_k = -1, a_{k-1} = 1) \\ -2 & (a_k = a_{k-1} = -1) \end{cases}$$

  - When {a_k} is a unipolar sequence with values 1 or 0

$$y_k = b_k = \begin{cases} 0 & (a_k = a_{k-1} = 0) \\ 1 & (a_k = 0, a_{k-1} = 1 \text{ or } a_k = 1, a_{k-1} = 0) \\ 2 & (a_k = a_{k-1} = 1) \end{cases}$$

Digital transmission through bandlimited channels

- Signal design with controlled ISI – partial response signals
  - Duobinary signal: decoding.
  - To recover the transmitted sequence, we can use

    $$\hat{a}_k = b_k - \hat{a}_{k-1} = y_k - \hat{a}_{k-1}$$

    although the detection of the current symbol relies on the detection of the previous symbol→ error propagation will occur
  - How to solve the ambiguity problem and error propagation?
  - Precoding: Apply differential encoding on {ak} so that

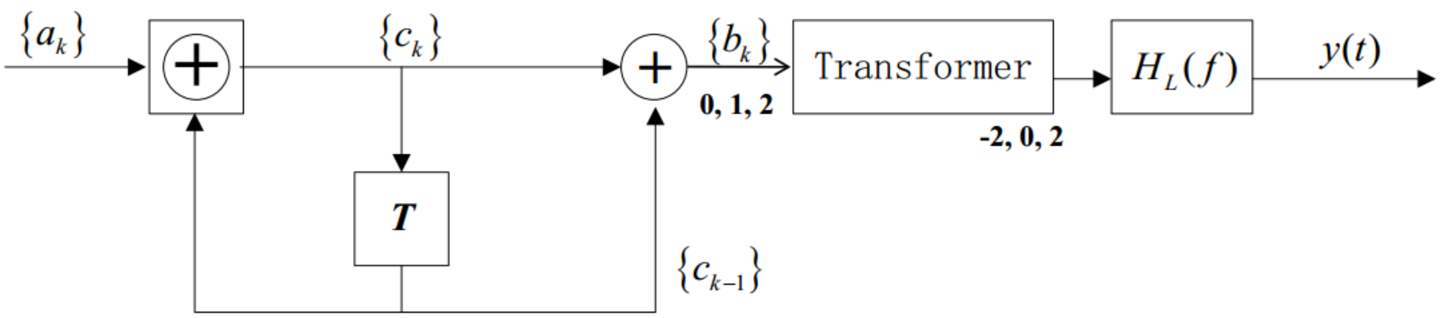
    $$c_k = a_k \oplus c_{k-1}$$

  Then the output of the duobinary signal system is

$$b_k = c_k + c_{k-1}$$

Digital transmission through bandlimited channels

- Signal design with controlled ISI – partial response signals
  - Duobinary signal: decoding.
  - Block diagram of precoded duobinary signal



Digital transmission through bandlimited channels

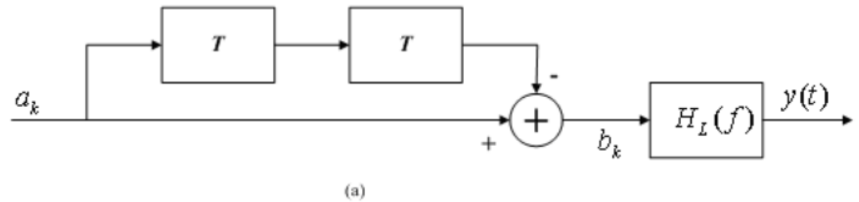
- Signal design with controlled ISI – partial response signals
  - Modified duobinary signal

$$b_k = a_k - a_{k-2}$$

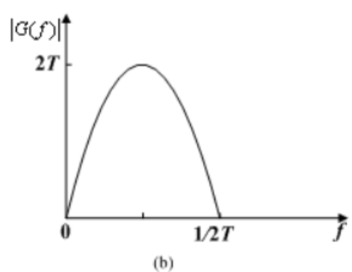
  - After LPF H(f), the overall response is

$$G(f) = (1 - e^{-j4\pi fT})H_L(f) \quad = \begin{cases} 2Tje^{-j2\pi fT}\sin 2\pi fT & (|f| \le 1/2T) \\ 0 & \text{otherwise} \end{cases}$$

$$g(t) = \frac{\sin \pi t/T}{\pi t/T} - \frac{\sin \pi(t-2T)/T}{\pi(t-2T)/T} \quad = -\frac{2T^2 \sin \pi t/T}{\pi t(t-2T)}$$



(a)



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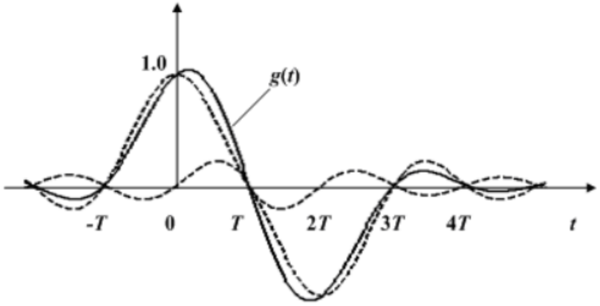


Digital transmission through bandlimited channels

- Signal design with controlled ISI – partial response signals
  - Modified duobinary signal.
  - The magnitude spectrum is a half-sin wave and hence easy to implement
  - No DC component and small low freq. component
  - At sampling interval T, the sampled values are

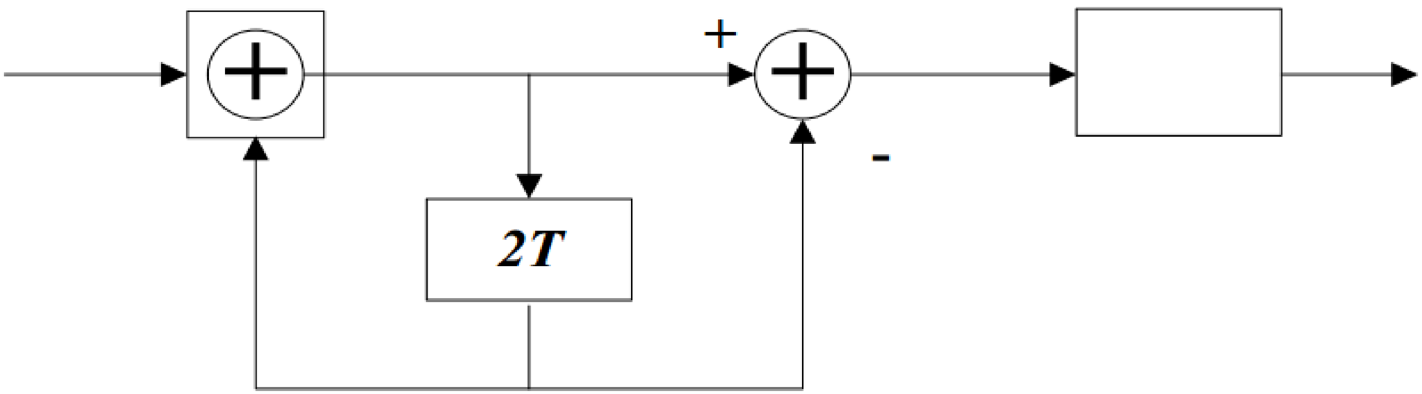
$$g(0) = g_0 = 1$$
$$g(T) = g_1 = 0$$
$$g(2T) = g_2 = -1$$
$$g(iT) = g_i = 0, i \neq 0,1,2$$

  - g(t) decays as $1/t^2$. But time offset may cause significant problem.

Digital transmission through bandlimited channels

- Signal design with controlled ISI – partial response signals
  - Modified duobinary signal: decoding.
  - To overcome error propagation, precoding is also needed $c_k = a_k \oplus c_{k-2}$
  - The coded signal is

$$b_k = c_k - c_{k-2}$$



Digital transmission through bandlimited channels

- Update
  - We have discussed
    1. Pulse shapes of baseband signal and their power spectrum
    2. ISI in band-limited channels
    3. Signal design for zero ISI and controlled ISI
  - We will discuss system design in the presence of channel distortion
    1. Optimal transmitting and receiving filters
    2. Channel equalizer

Digital transmission through bandlimited channels

- Optimal transmit/receive filter
  - Recall that when zero-ISI condition is satisfied by p(t) with raised cosine spectrum P(f), then the sampled output of the receiver filter is Vm=Am+Nm (assume p(0)=1)
  - Consider binary PAM transmission: Am = $\pm$d
  - Variance of $N_m = \sigma^2 = \int_{-\infty}^{\infty} S_n(f)|H_R(f)|^2 df$

    with $P(f) = H_T(f)H_C(f)H_R(f)$ $\quad p(t) = h_T(t) * h_C(t) * h_R(t)$

$$\Longrightarrow \quad P_e = Q\left(\frac{d}{\sigma}\right)$$

Error Probability can be minimized through a proper choice of H_R(f) and H_T(f) so that d/σ is maximum
(assuming H_C(f) fixed and P(f) given)

Digital transmission through bandlimited channels

- Optimal transmit/receive filter
  - Compensate the channel distortion equally between the transmitter and receiver filters

$$\begin{cases} |H_T(f)| = \dfrac{\sqrt{P(f)}}{|H_c(f)|^{1/2}} \\ |H_R(f)| = \dfrac{\sqrt{P(f)}}{|H_c(f)|^{1/2}} \end{cases} \quad \text{for } |f| \leq W$$

  - Then, the transmit signal energy is given by

$$E_{av} = \int_{-\infty}^{\infty} d^2 h_T^2(t)dt = \int_{-\infty}^{\infty} d^2 H_T^2(f)df = \int_{-W}^{W} \frac{d^2 P(f)}{|H_C(f)|}df$$

By Parseval's theorem

  - Hence $d^2 = E_{av} \cdot \left[\int_{-W}^{W} \frac{P(f)}{|H_C(f)|}df\right]^{-1}$

Digital transmission through bandlimited channels

- Optimal transmit/receive filter
  - Noise variance at the output of the receive filter is

$$\sigma^2 = \frac{N_0}{2}\int_{-\infty}^{\infty} |H_R(f)|^2 df = \frac{N_0}{2}\int_{-W}^{W} \frac{P(f)}{|H_C(f)|} df$$

$$\Rightarrow \quad P_{e,\min} = Q\left[\sqrt{\frac{2E_{av}}{N_0}\underbrace{\left\{\int_{-W}^{W}\frac{P(f)}{|H_c(f)|}df\right\}}_{\text{Performance loss due to channel distortion}}^{-1}}\right]$$

  - Special case: Hc(f)=1 for |f|≤W
    1. This is the idea case with “flag” fading
    2. No loss, same as the matched filter receiver of AWGN channel

Digital transmission through baseband channels

- Optimal transmit/receive filter
  - Exercise.
  - Determine the optimum transmitting and receiving filters for a binary communication system that transmits data at a rate R=1/T=4800 bps over a channel with a frequency response $|H_c(f)| = \dfrac{1}{\sqrt{1+(\frac{f}{W})^2}}$, $|f| \leq W$ where W=4800 Hz
  - The additive noise is zero mean white Gaussian with spectral density $N_0/2=10^{-15}$ Watt/Hz

Digital transmission through baseband channels

- Optimal transmit/receive filter
  - Exercise.
  - Since W=1/T=4800, we use a signal pulse with a raised cosine spectrum and a roll-off factor =1.
  - Thus, $$P(f) = \frac{1}{2}[1 + \cos(\pi|f|)] = \cos^2\left(\frac{\pi|f|}{9600}\right)$$
  - Therefore

$$|H_T(f)| = |H_R(f)| = \cos\left(\frac{\pi|f|}{9600}\right)\left[1 + \left(\frac{f}{4800}\right)^2\right]^{1/4}, \text{for } |f| \leq 4800$$

  - One can now use these filters to determine the amount of transmit energy required to achieve a specified error probability.

Digital transmission through bandlimited channels

- Performance with ISI
  - If zero-ISI condition is not met, then

$$V_m = A_m + \sum_{k \neq m} A_k p[(m-k)T] + N_m$$

  - Let

$$A_I = \sum_{k \neq m} I_k = \sum_{k \neq m} A_k p[(m-k)T]$$

  - Then

$$V_m = A_m + A_I + N_m$$

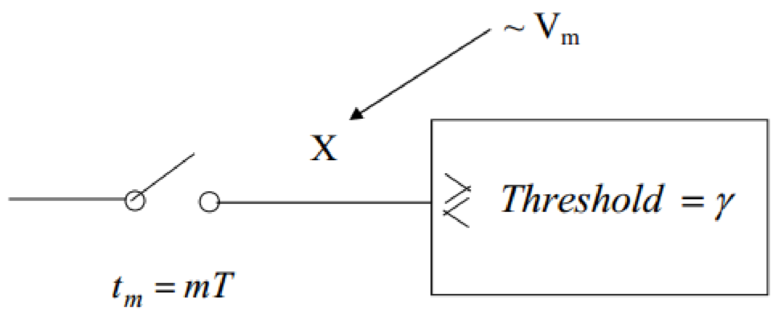
  - Often only 2M significant terms are considered. Hence

$$V_m = A_m + A_I' + N_m \quad \text{with} \quad A_I' = \sum_{k=-M}^{M} A_k p[(m-k)T]$$

  - Finding the probability of error?

Digital transmission through bandlimited channels

- Performance with ISI
  - Monte Carlo simulation.



Let

$$I(x) = \begin{cases} 1 & error\ occurs \\ 0 & else \end{cases}$$

$$\therefore \quad \left\| P_e = \frac{1}{L} \sum_{l=1}^{L} I\left(X^{(l)}\right) \right.$$

where $X^{(1)}, X^{(2)}, \ldots, X^{(L)}$ are i.i.d. (*independent and identically distributed*) random samples

Digital transmission through bandlimited channels

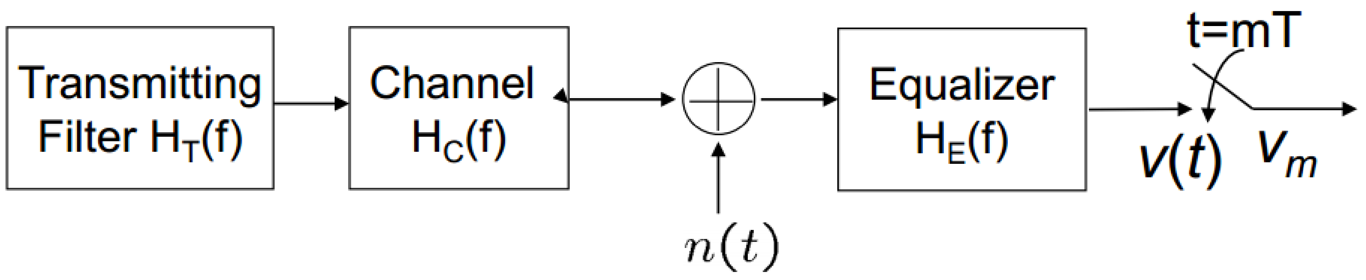
- Performance with ISI
  - Monte Carlo simulation.
  - If one want Pe to be within 10% accuracy, how many independent simulation runs do we need?
  - If Pe~10-9 (this is typically the case for optical communication systems), and assume each simulation run takes 1 ms, how long will the simulation take?

Digital transmission through bandlimited channels

- Update
  - Monte Carlo simulation.
  - We have shown that by properly designing the transmitting and receiving filters, one can guarantee zero ISI at sampling instants, thereby minimizing Pe.
  - Appropriate when the channel is precisely known and its characteristics do not change with time.
  - In practice, the channel is unknown or time-varying
  - We next consider channel equalizer.

Digital transmission through bandlimited channels

- Equalizer
  - A receiving filter with adjustable frequency response to minimize/eliminate inter-symbol interference



  - Overall frequency response

$$H_o(f) = H_T(f)H_C(f)H_E(f)$$

  - Nyquist criterion for zero-ISI

$$\sum_{k=-\infty}^{\infty} H_o\left(f + \frac{k}{T}\right) = \text{constant}$$

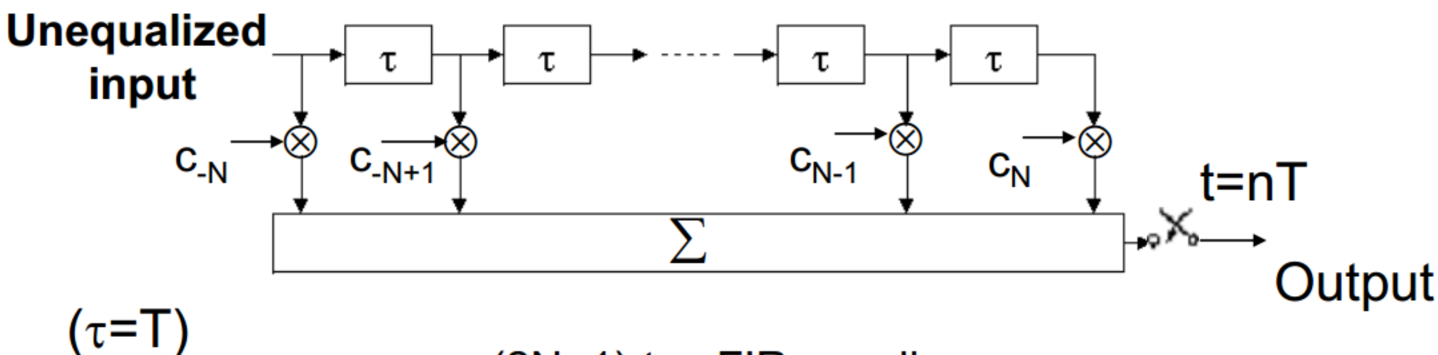
  - Thus, ideal zero-ISI equalizer is an inverse channel filter

$$H_E(f) \propto \frac{1}{H_T(f)H_C(f)} \qquad |f| \leq 1/2T$$

Digital transmission through bandlimited channels

- Equalizer
  - ➢ Linear transversal filter
  - ➢ Finite impulse response (FIR) filter

$$h_E(t) = \sum_{n=-N}^{N} c_n \delta(t - nT)$$



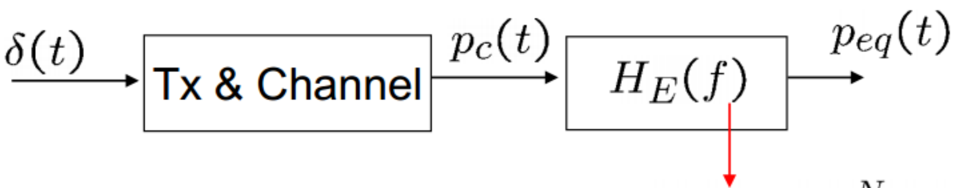
(2N+1)-tap FIR equalizer

- $\{c_n\}$ are the adjustable $2N+1$ equalizer coefficients
- N is sufficiently large to span the length of ISI

Digital transmission through bandlimited channels

- Equalizer
  - Zero-forcing (ZF) equalizer

$P_c(t)$ the received pulse from a channel to be equalized



$$p_{eq}(t) = p_c(t) * h_E(t) = \sum_{n=-N}^{N} c_n p_c(t - nT)$$

$$h_E(t) = \sum_{n=-N}^{N} c_n \delta(t - nT)$$

$$t = mT$$

$$p_{eq}(mT) = \sum_{n=-N}^{N} c_n p_c[(m-n)t] = \begin{cases} 1, & m = 0 \\ 0, & m = \pm 1, \ldots, \pm N \end{cases}$$

To suppress 2N adjacent interference terms

Digital transmission through bandlimited channels

- Equalizer
  - ➢ Zero-forcing (ZF) equalizer
  - ➢ In matrix form

$$\mathbf{p}_{eq} = \mathbf{P}_c \cdot \mathbf{c}$$

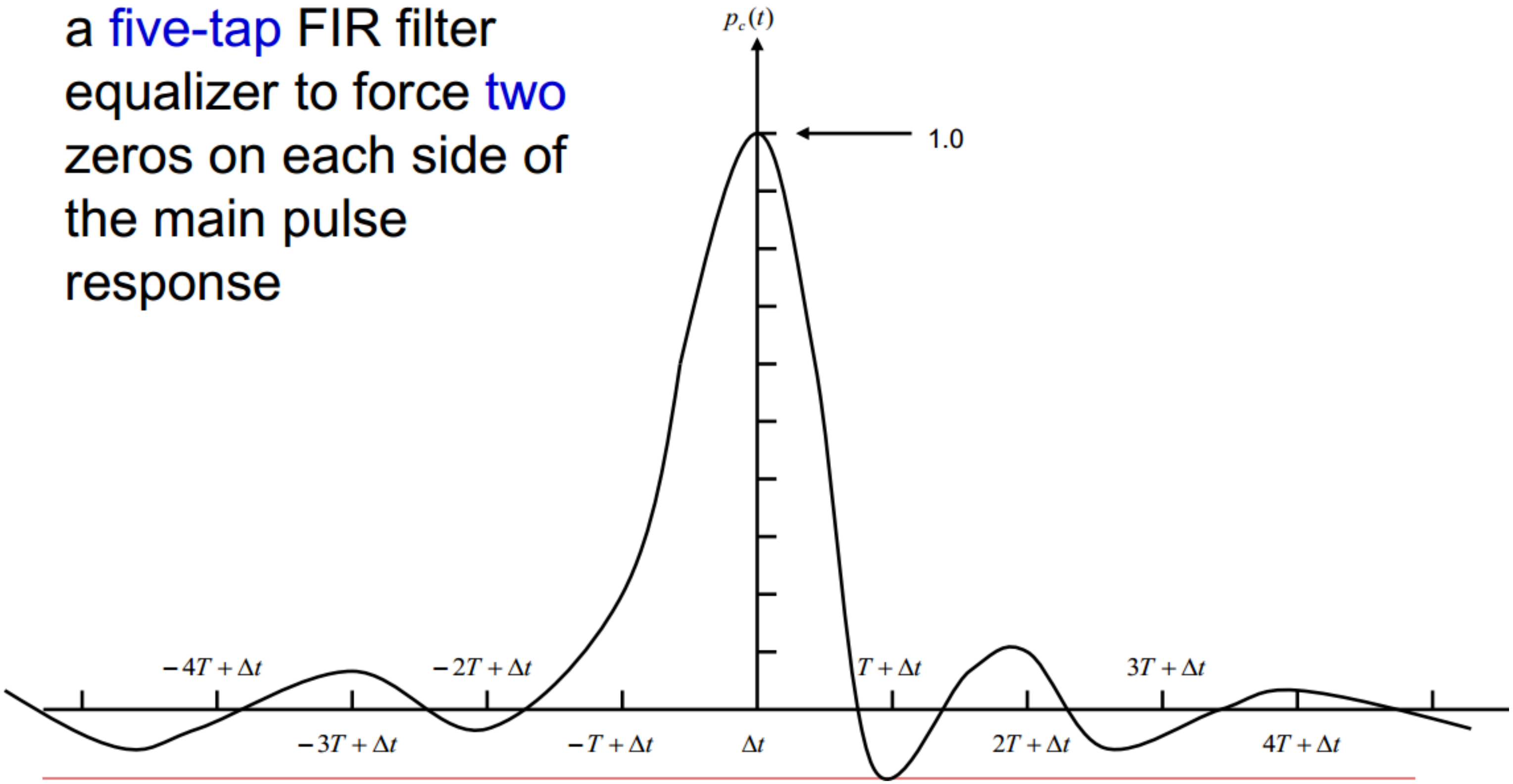
$(2N+1) \times (2N+1)$ channel response matrix

$$\mathbf{p}_{eq} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} c_{-N} \\ c_{-N+1} \\ \vdots \\ c_{-1} \\ c_0 \\ c_1 \\ \vdots \\ c_N \end{bmatrix} \quad \mathbf{P}_c = \begin{bmatrix} p_c(0) & p_c(-1) & \cdots & p_c(-2N) \\ p_c(1) & p_c(0) & \cdots & p_c(-2N+1) \\ \vdots & \vdots & \ddots & \vdots \\ p_c(2N) & p_c(2N-1) & \cdots & p_c(0) \end{bmatrix}$$

$\Rightarrow$ $c = \mathbf{P}_c^{-1}\mathbf{p}_{eq}$ or the middle-column of $\mathbf{P}_c^{-1}$

- Equalizer
  - Example.

- Find the coefficients of a five-tap FIR filter equalizer to force two zeros on each side of the main pulse response



Digital transmission through bandlimited channels

- Equalizer
  - Example.
  - By inspection

$$p_c(-4) = -0.02 \qquad p_c(0) = 1$$
$$p_c(-3) = 0.05 \qquad p_c(1) = -0.1$$
$$p_c(-2) = -0.1 \qquad p_c(2) = 0.1$$
$$p_c(-1) = 0.2 \qquad p_c(3) = -0.05$$
$$p_c(4) = 0.02$$

  - The channel response matrix

$$[P_c] = \begin{bmatrix} 1.0 & 0.2 & -0.1 & 0.05 & -0.02 \\ -0.1 & 1.0 & 0.2 & -0.1 & 0.05 \\ 0.1 & -0.1 & 1.0 & 0.2 & -0.1 \\ -0.05 & 0.1 & -0.1 & 1.0 & 0.2 \\ 0.02 & -0.05 & 0.1 & -0.1 & 1.0 \end{bmatrix}$$

Digital transmission through bandlimited channels

- Equalizer
  - Example.
  - The inverse of this matrix

$$[P_c]^{-1} = \begin{bmatrix} 0.966 & -0.170 & 0.117 & -0.083 & 0.056 \\ 0.118 & 0.945 & -0.158 & 0.112 & -0.083 \\ -0.091 & 0.133 & 0.937 & -0.158 & 0.117 \\ 0.028 & -0.095 & 0.133 & 0.945 & -0.170 \\ -0.002 & 0.028 & -0.091 & 0.118 & 0.966 \end{bmatrix}$$

  - Therefore, $c_1=0.117$, $c_{-1}=-0.158$, $c_0 = 0.937$, $c_1 = 0.133$, $c_2 = -0.091$
  - Equalized pulse response $p_{eq}(m) = \sum_{n=-2}^{2} c_n p_c(m-n)$
  - It can be verified

$$p_{eq}(0) = 1.0 \qquad p_{eq}(m) = 0, \;\; m = \pm 1, \pm 2$$

Digital transmission through bandlimited channels

- Equalizer
  - Example.
  - Note that values of $p_{eq}(n)$ for n<-2 or n>2 are not zero. For example

$$
\begin{aligned}
p_{eq}(3) &= (0.117)(0.005) + (-0.158)(0.02) + (0.937)(-0.05) \\
&\quad + (0.133)(0.1) + (-0.091)(-0.1) \\
&= -0.027
\end{aligned}
$$

$$
\begin{aligned}
p_{eq}(-3) &= (0.117)(0.2) + (-0.158)(-0.1) + (0.937)(-0.05) \\
&\quad + (0.133)(0.1) + (-0.091)(-0.01) \\
&= 0.082
\end{aligned}
$$

Digital transmission through bandlimited channels

- Equalizer
  - Minimum mean-square error equalizer.
  - Drawback of ZF equalizer: ignores the additive noise
  - Suppose we relax zero ISI condition, and minimize the combined power in the residual ISI and additive noise at the output of the equalizer.
  - Then, we have MMSE equalizer, which is a channel equalizer optimized based on the minimum mean square error (MMSE) criterion

- Equalizer
  - Minimum mean-square error equalizer.

Output from the channel

$y(t) \longrightarrow$ **Transversal filter Equalizer** $h_E(t) = \sum_n c_n \delta(t - nT)$ $\longrightarrow z(t)$

$$y(t) = \sum_{n=-\infty}^{\infty} A_n g_c(t - nT) + n(t)$$

$$z(t) = \sum_{n=-N}^{N} c_n y(t - nT)$$

  - The output is sampled at t=mT:

$$z(mT) = \sum_{n=-N}^{N} c_n y[(m-n)T]$$

  - Let Am=desired equalizer output

$$MSE = E\left[\left(z(mT) - A_m\right)^2\right] = \text{Minimum}$$

Digital transmission through bandlimited channels

- Equalizer
  - Minimum mean-square error equalizer.

$$MSE = E\left[\left(\sum_{n=-\infty}^{\infty} c_n y[(m-n)T] - A_m\right)^2\right]$$
$$= \sum_{n=-N}^{N}\sum_{k=-N}^{N} c_n c_k R_Y(n-k) - 2\sum_{k=-N}^{N} c_k R_{AY}(k) + E(A_m^2)$$

where

$$\begin{cases} R_Y(n-k) = E[y(mT-nT)y(mT-kT)] \\ R_{AY}(k) = E[y(mT-kT)A_m] \end{cases}$$

E is taken over A_m and the additive noise

- MMSE solution is obtained by $\dfrac{\partial MSE}{\partial c_n} = 0$

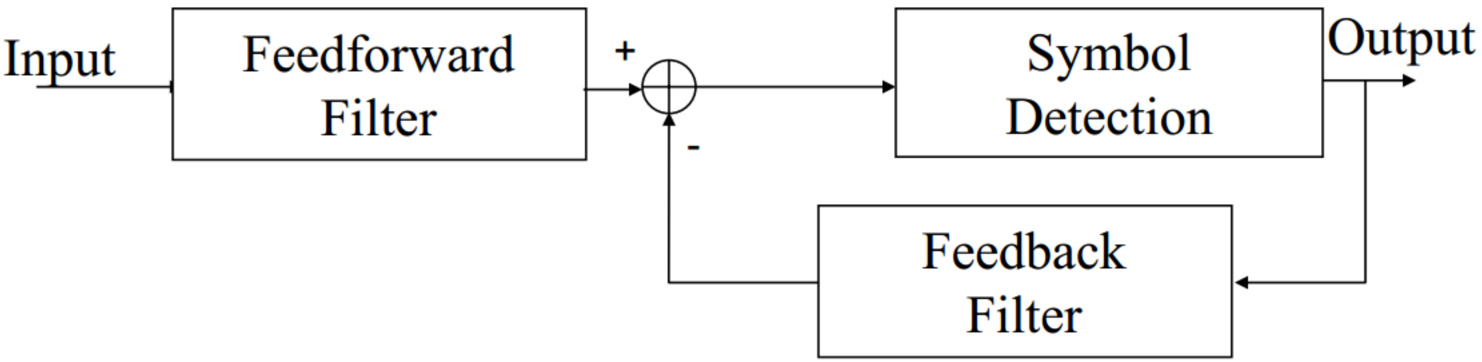
$$\Rightarrow \sum_{n=-N}^{N} c_n R_Y(n-k) = R_{AY}(k), \quad \text{for } k = 0, \pm 1, \ldots, \pm N.$$

Digital transmission through bandlimited channels

- Equalizer
  - MMSE equalizer vs. ZF equalizer.
  - Both can be obtained by solving similar equations.
  - ZF equalizer does not consider the effects of noise
  - MMSE equalizer is designed so that mean-square error (consisting of ISI terms and noise at the equalizer output) is minimized
  - Both equalizers are known as linear equalizers

Digital transmission through bandlimited channels

- Equalizer
  - Decision feedback equalizer (DFE)
  - DFE is a nonlinear equalizer which attempts to subtract from the current symbol to be detected the ISI created by previously detected symbol



Digital transmission through bandlimited channels

- Equalizer
  - Example of channels with ISI.

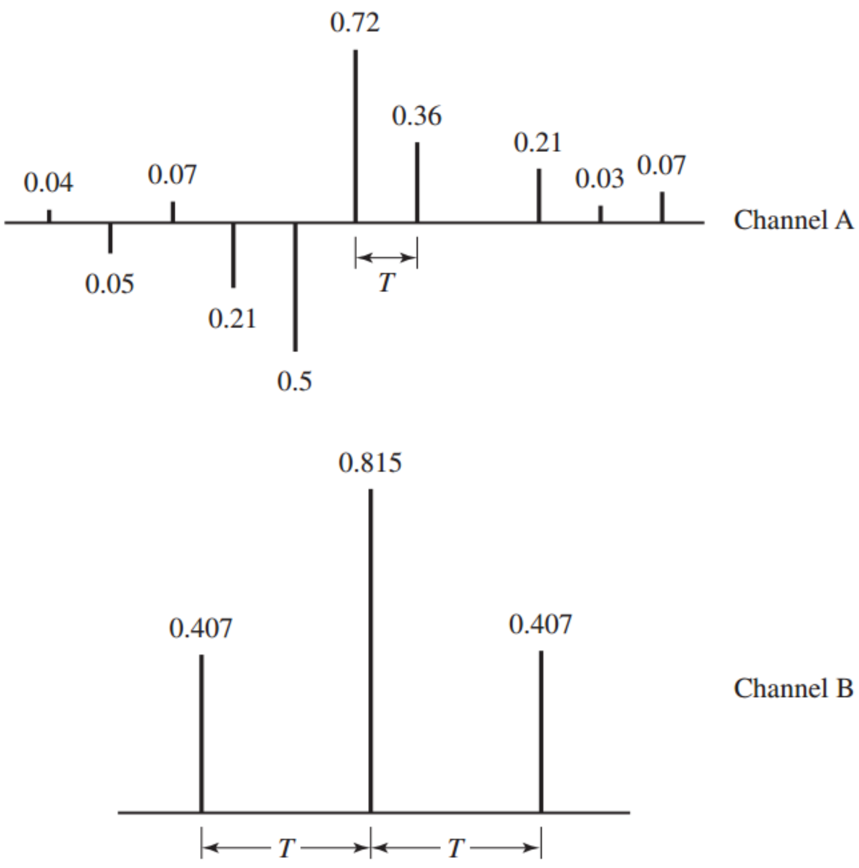


Figure 8.42 Two channels with ISI.

- Equalizer
  - Frequency response.

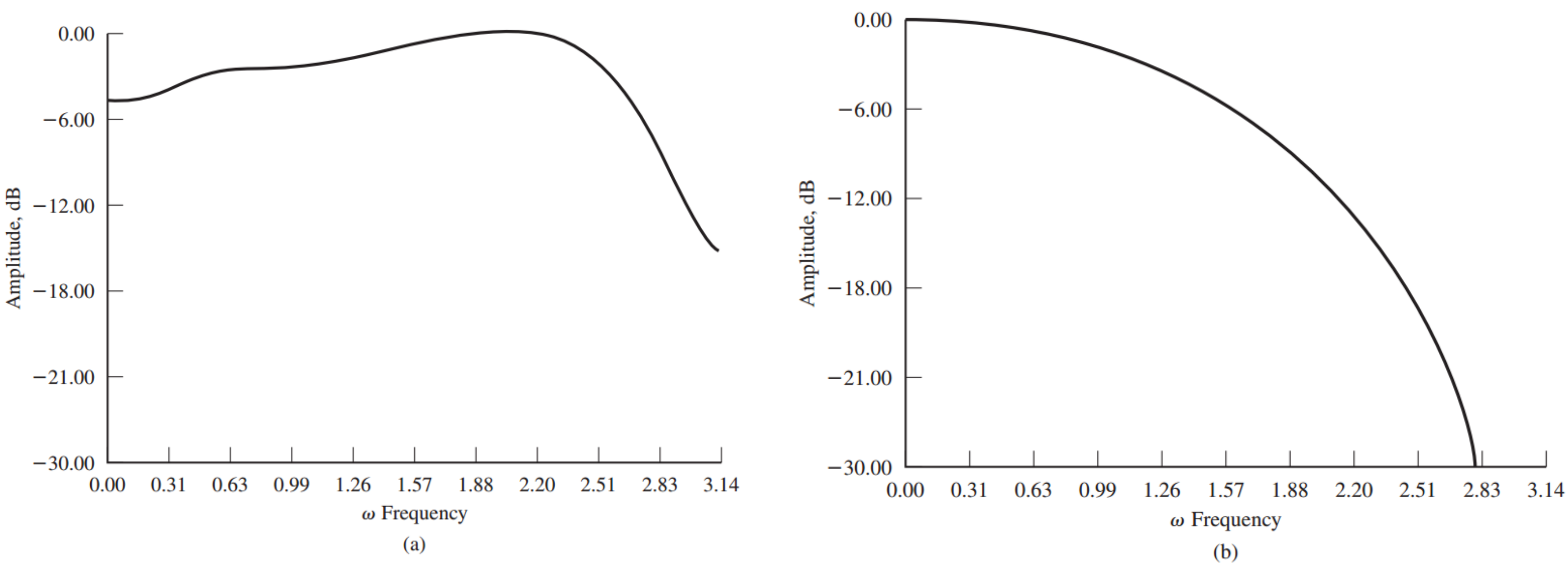


Figure 8.43 Amplitude spectra for (a) channel A shown in Figure 8.42(a) and (b) channel B shown in Figure 8.42(b).

Digital transmission through bandlimited channels

- Equalizer
  - Performance of MMSE equalizer.

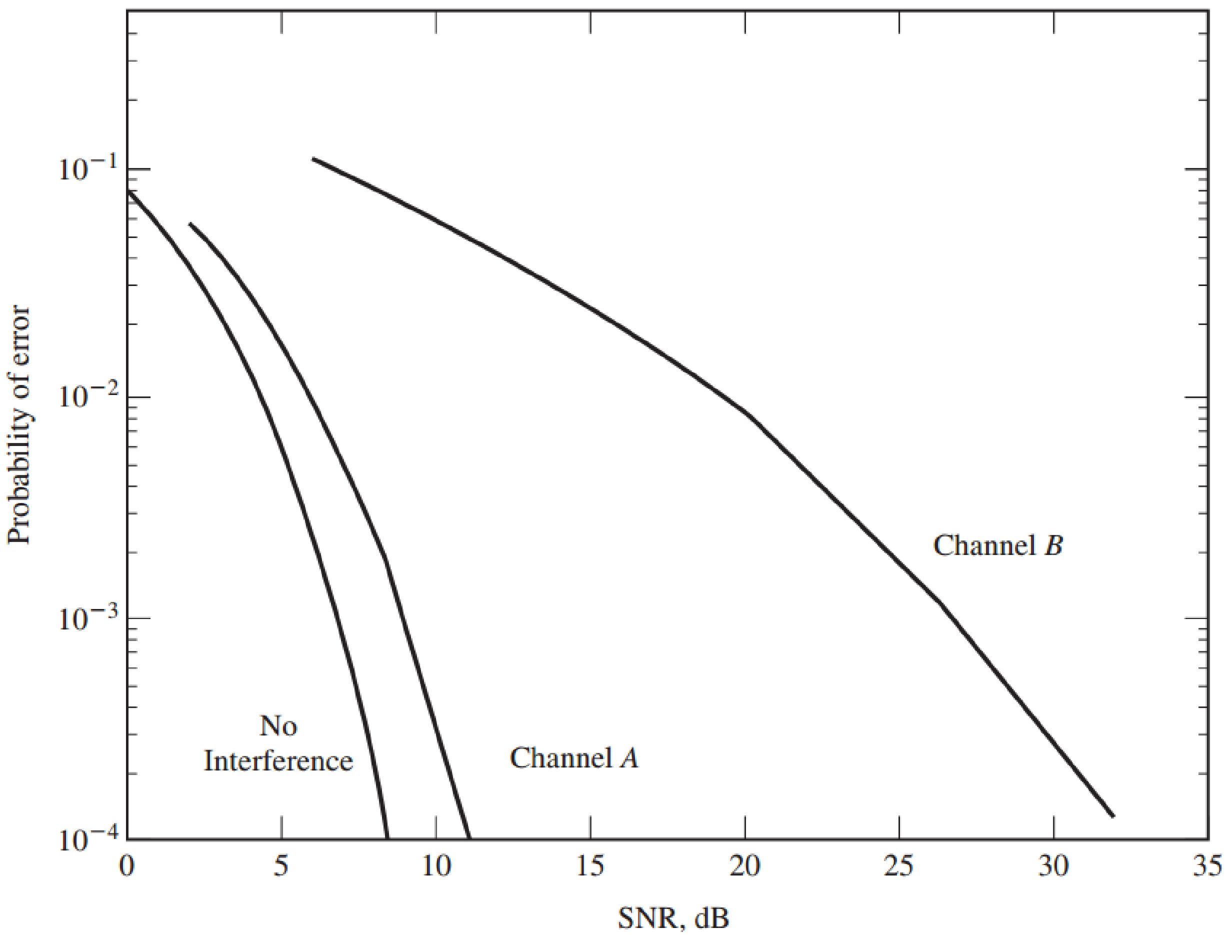


Figure 8.44 Error-rate performance of linear MSE equalizer.

- Equalizer
  - Performance of DFE equalizer.

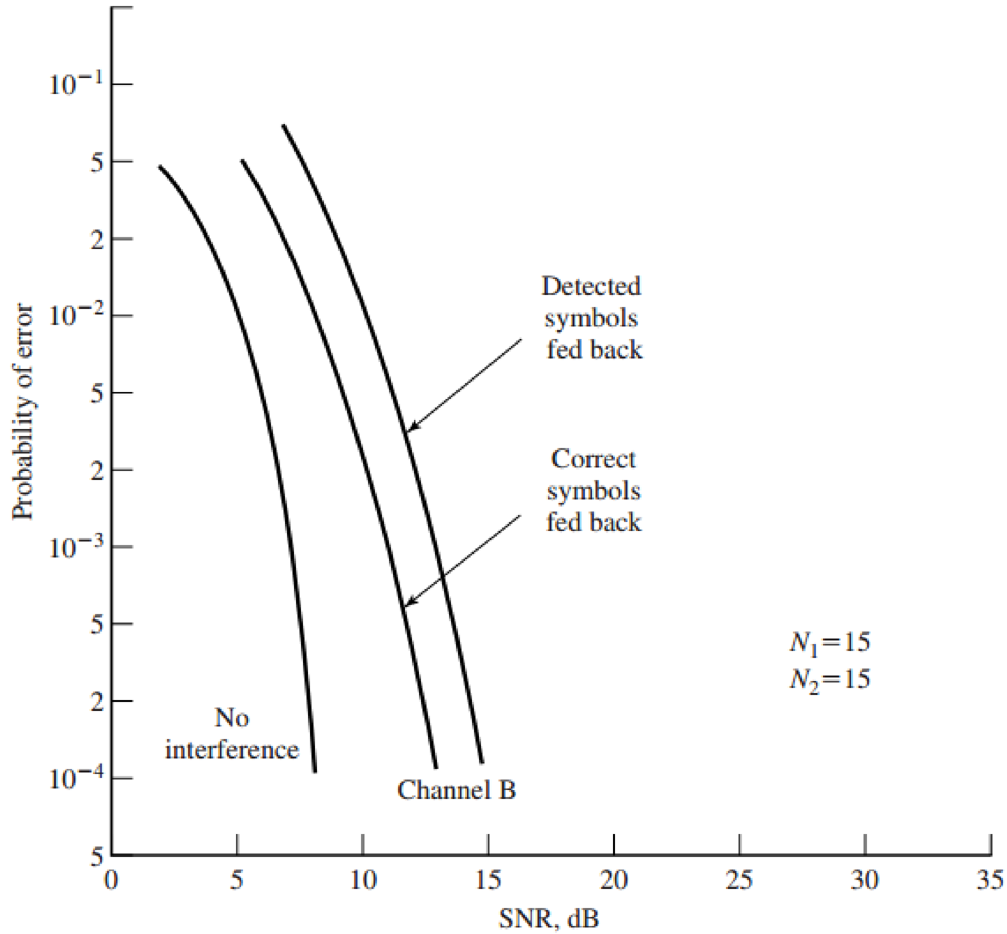
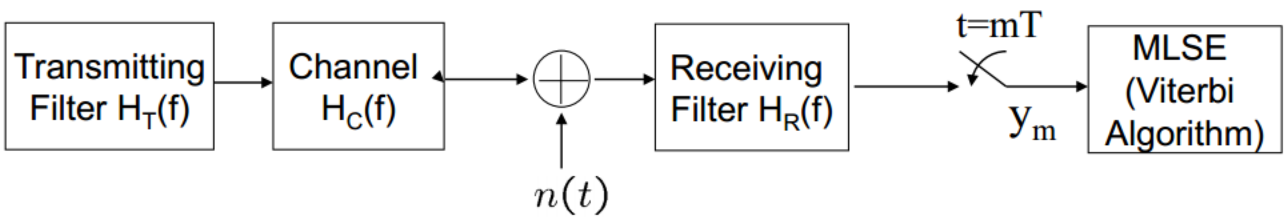


Figure 8.47 Performance of DFE with and without error propagation.

Digital transmission through bandlimited channels

- Equalizer
  - Maximum likelihood sequence estimation (MLSE).



  - Let the transmitting filter have a square root raised cosine frequency response

$$|H_T(f)| = \begin{cases} \sqrt{P(f)} & |f| \le W \\ 0 & |f| > W \end{cases}$$

  - The receiving filter is matched to the transmitter filter with

$$|H_R(f)| = \begin{cases} \sqrt{P(f)} & |f| \le W \\ 0 & |f| > W \end{cases}$$

  - The sampled output from receiving filter is

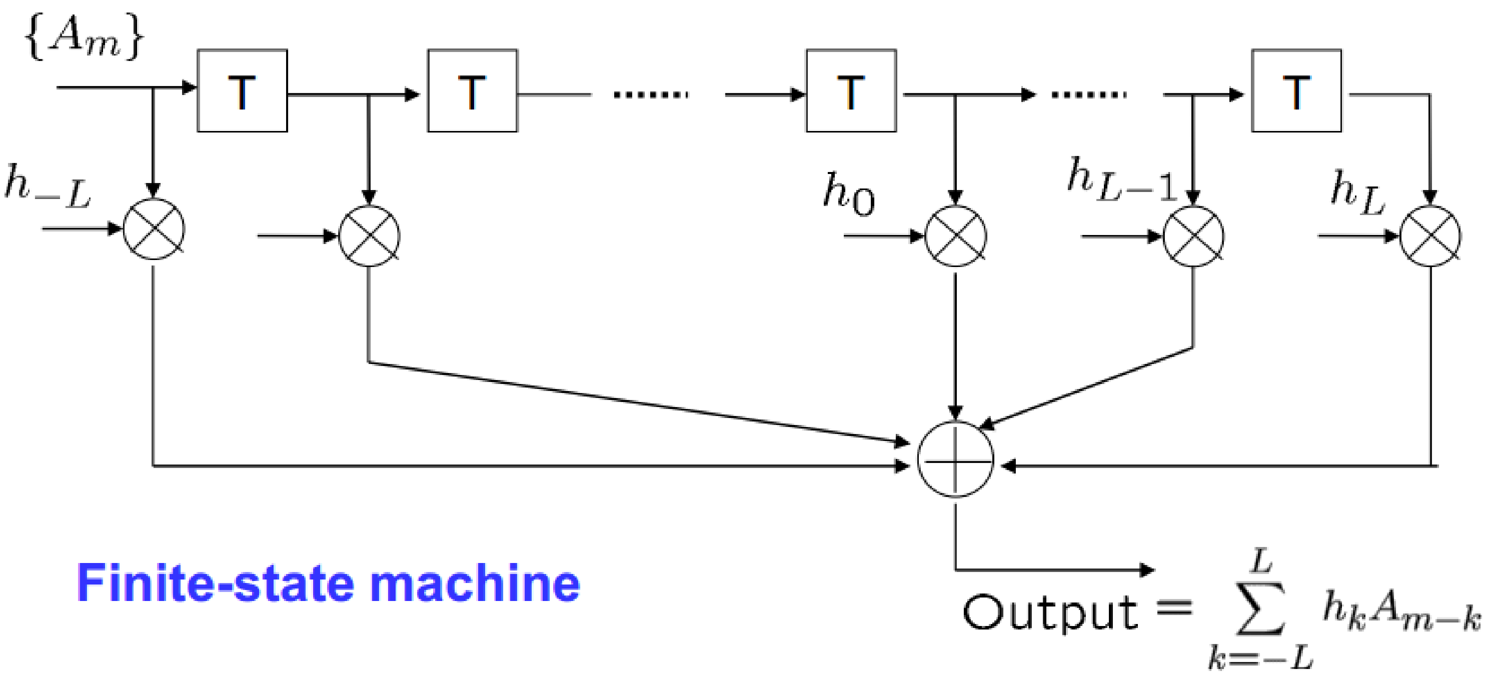
$$y_m = h_0 A_m + \sum_{\substack{n=-\infty \\ n \neq m}}^{\infty} h_{m-n} A_n + v_m$$

Digital transmission through bandlimited channels

- Equalizer
  - Maximum likelihood sequence estimation (MLSE).
  - Assume ISI affects finite number of symbols with

$$h_n = 0 \text{ for } |n| > L$$

  - Then, the channel is equivalent to a FIR discrete-time filter



Finite-state machine

Digital transmission through bandlimited channels

- Equalizer
  - Performance of MLSE

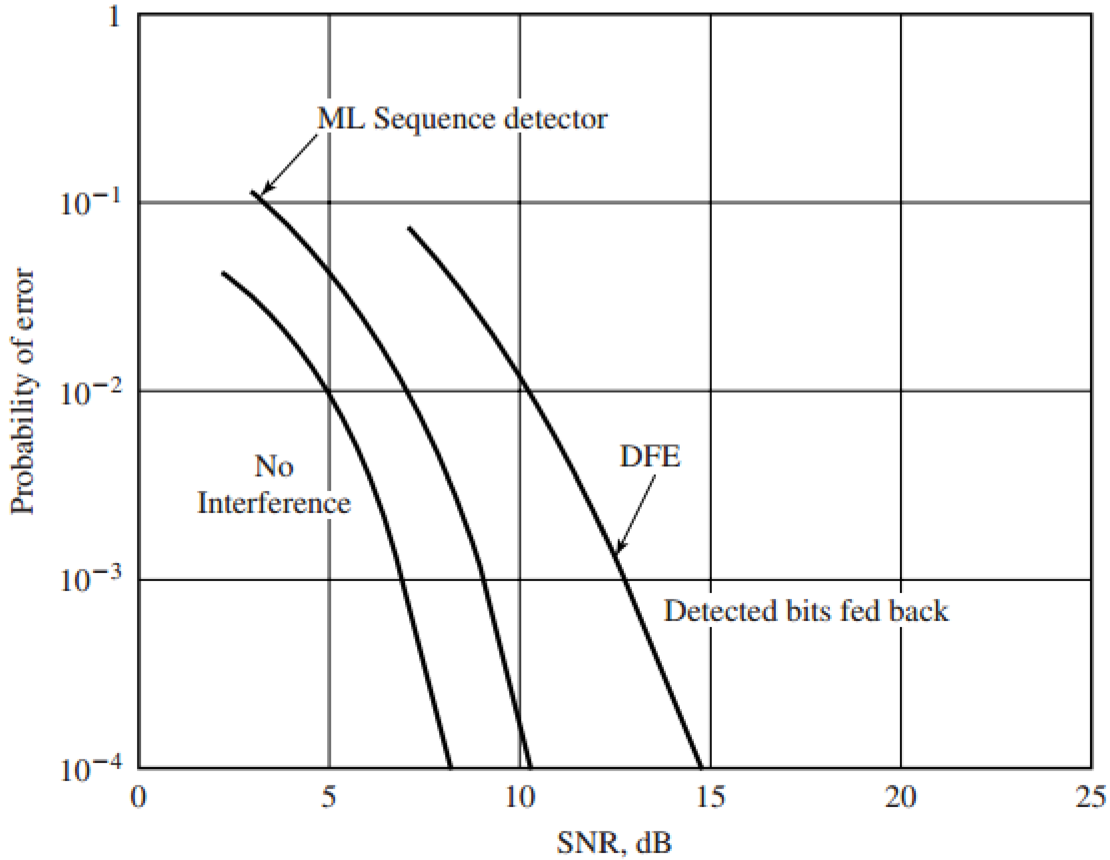
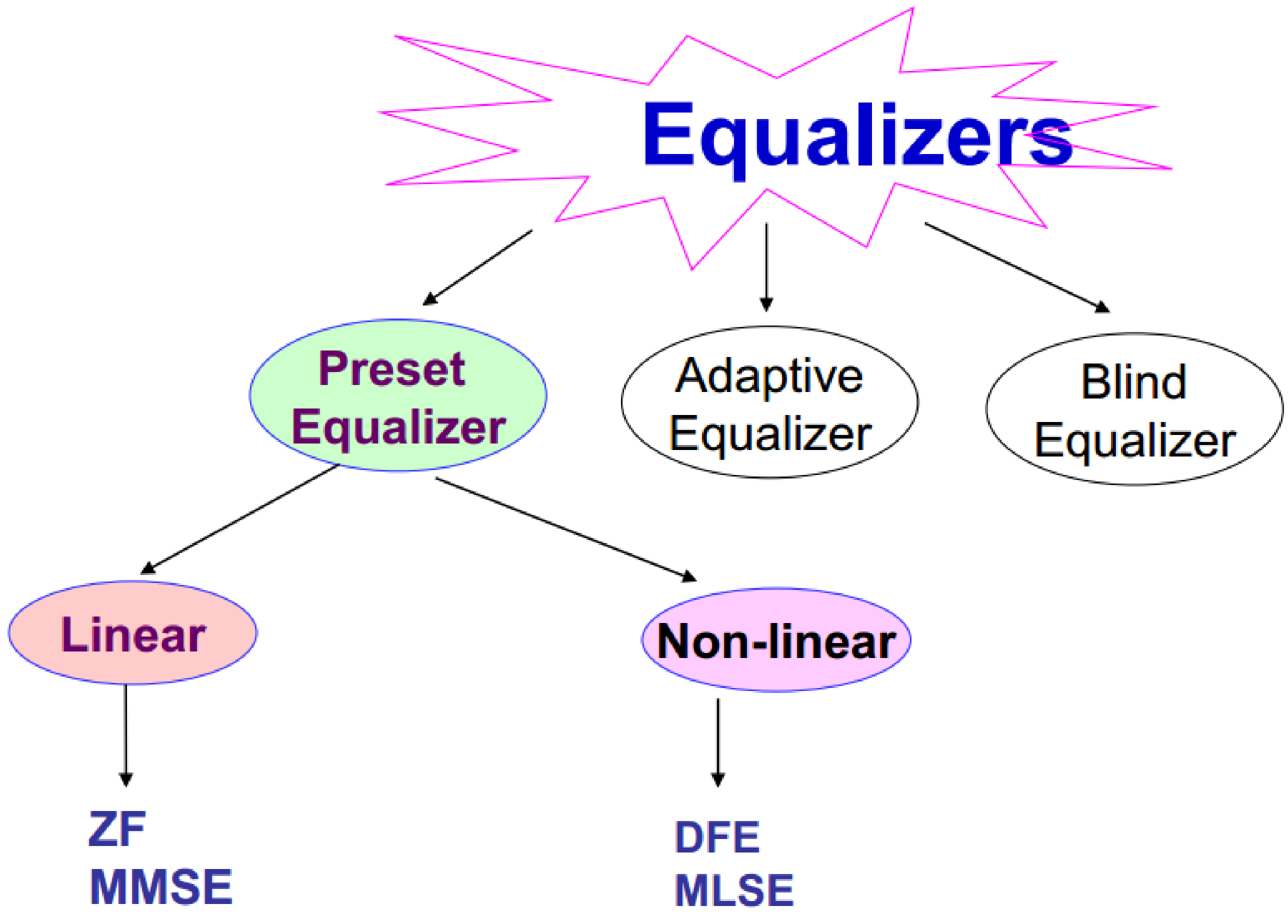


Figure 8.48 Performance of Viterbi detector and DFE for channel B.

Digital transmission through bandlimited channels

- Equalizer



Digital transmission through bandlimited channels

- Suggested reading
  - Chapter 10.1-10.5