Reductions of Darboux Transformations for the $\mathcal{PT}$-Symmetric Nonlocal Davey-Stewartson Equations

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Abstract

In this letter, a study of the reductions of the Darboux transformations(DTs) for the $\mathcal{PT}$-symmetric nonlocal Davey-Stewartson(DS) equations is presented. Firstly, a binary DT is constructed in integral form for the $\mathcal{PT}$-symmetric nonlocal DS-I equation. Secondly, an elementary DT is constructed in differential form for the $\mathcal{PT}$-symmetric nonlocal DS-II equation. Afterwards, a new binary DT in integral form is also found for the nonlocal DS-II equation. Moreover, it is shown that the symmetry properties in the corresponding Lax-pairs of the equations are well preserved through these DTs. Thirdly, based on above DTs, the fundamental rogue waves and rational travelling waves are obtained.

Keywords: $\mathcal{PT}$-symmetric Nonlocal Davey-Stewartson equations, Darboux transformation, Rogue waves

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1. Introduction

In the last several years, $\mathcal{PT}$-symmetric systems which allow for lossless-like propagation due to their balance of gain and loss have attracted considerable attention and triggered renewed interest in integrable systems. These nonlocal integrable equations are different from local integrable equations and could produce novel patterns of solution dynamics and intrigue new physical applications [1–17]. As an integrable multidimensional versions of the nonlocal nonlinear Schrödinger equation, a new integrable nonlocal Davey-Stewartson (DS) equation is recently introduced in Refs.[7, 10]:

$$iu_t + \frac{1}{2}\alpha^2 u_{xx} + \frac{1}{2}u_{yy} + (uv - w)u = 0, \tag{1}$$

$$w_{xx} - \alpha^2 w_{yy} - 2\left[(uv)\right]_{xx} = 0, \tag{2}$$

where $v(x,y,t) = \epsilon\bar{u}(-x,-y,t)$, $\epsilon = \pm 1$. u, v and w are functions of x, y, t, $\alpha^2 = \pm 1$ is the equation-type parameter ($\alpha^2 = 1$ being the DS-I and $\alpha^2 = -1$ being DS-II). Here the sign $\bar{u}$ represents the complex conjugation of this function, and ϵ is the sign of nonlinearity. For this equation, the corresponding auxiliary linear system has the form as

$$L\Phi = 0, \quad L = \partial_y - J\partial_x - P, \tag{3}$$

$$M\Phi = 0, \quad M = \partial_t - \sum_{j=0}^{2} V_{2-j}\partial^j = \partial_t - i\alpha^{-1}J\partial_x^2 - i\alpha^{-1}P\partial_x - \alpha^{-1}V, \tag{4}$$

$$J = \alpha^{-1}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \; P = \begin{pmatrix} 0 & u \\ -v & 0 \end{pmatrix}, \; V = \frac{\mathrm{i}}{2}\begin{pmatrix} \omega_1 & u_x + \alpha u_y \\ -v_x + \alpha v_y & \omega_2 \end{pmatrix}, \tag{5}$$

with

$$w = uv - \frac{1}{2\alpha}(\omega_1 - \omega_2). \tag{6}$$

The compatibility condition means that $[L, M] = 0$ if and only if u, v, w satisfy the DS system.

For the local DS equations, several forms of Darboux transformations are given in [18, 21–23]. Especially, for the local DS-II equation, a new reduction of DT is given in [18], and the solutions are expressed in terms of Grammian type

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determinants. In this $\mathcal{PT}$-symmetric nonlocal DS system, a binary DT is constructed for the nonlocal DS-I equation, and an elementary DT is obtained for the nonlocal DS-II equation. Moreover, inherited from the idea proposed in [18], a new binary DT in integral form for the nonlocal DS-II equation is also proposed under certain reductions.

This letter is organized as follows. In section 2, for the $\mathcal{PT}$-symmetric nonlocal DS-II equation, an elementary DT in differential form and a new binary DT in integral form are constructed. In section 3, for the $\mathcal{PT}$-symmetric nonlocal DS-I equation, the elementary DT is not enough, so we construct a binary DT in integral form. In addition, all the symmetry properties in the corresponding Lax-pairs are shown to be well preserved through these DTs. Moreover, as applications of these DTs, with certain reductions on the eigenfunctions and parameters in our DT procedure, some interesting solutions are obtained and discussed in section 4, including rogue waves and rational travelling waves.

2. Darboux transformations for the $\mathcal{PT}$-symmetric nonlocal DS-II equation

As we known, the local DS-II equation possess a Darboux transformation in differential form. Moreover, for the partially $\mathcal{PT}$-symmetric nonlocal DS-II equation, a differential form DT has been constructed in [12]. For this $\mathcal{PT}$-symmetric nonlocal DS-II equation, one can also constructs Darboux transformation in the differential form.

It is already shown in [24] that for any invertible matrix θ such that $L(\theta) = M(\theta) = 0$, the operator

$$G_\theta = \theta\partial\theta^{-1}, \ \ \partial = \partial_x, \tag{7}$$

makes L and M form invariant under the elementary Darboux transformation:

$$L \to \tilde{L} = G_\theta L G_\theta^{-1}, \ \ M \to \tilde{M} = G_\theta M G_\theta^{-1}.$$

The potential matrix P in (3)-(4) satisfies the following symmetric reduction

$$-\sigma_\epsilon P(x,y,t)\sigma_\epsilon^{-1} = \overline{P}(-x,-y,t), \ \ \sigma_\epsilon = \begin{pmatrix} 0 & \epsilon \\ 1 & 0 \end{pmatrix}. \tag{8}$$

Considering the following zero curvature condition:

$$P_t - V_{2,y} + [P, V_2] + \Delta = 0, \tag{9}$$

where $\Delta = JV_{2,x} - \mathrm{i}\alpha^{-1}PP_x - \mathrm{i}\alpha^{-1}JP_{xx}$. This equation is deduced from the compatibility condition. Therefore, with the uniqueness of V_2 in the lax-pair, it leads to: $\sigma_\epsilon V_2(x,y,t)\sigma_\epsilon^{-1} = \overline{V}_2(-x,-y,t)$. Moreover, these give rise to the same symmetry constraint in operators L and M:

$$-\sigma_\epsilon L\sigma_\epsilon^{-1} = \overline{L}_{(x\to -x,\ y\to -y)}, \ \sigma_\epsilon M\sigma_\epsilon^{-1} = \overline{M}_{(x\to -x,\ y\to -y)}. \tag{10}$$

Supposing $\theta_c = \begin{pmatrix} \xi(x,y,t), \eta(x,y,t) \end{pmatrix}^T$ is a vector solution of equations (3)-(4), then it is inferred from symmetry (10) that $\sigma_\epsilon\theta_c = \begin{pmatrix} -\epsilon\bar{\eta}(-x,-y,t), \bar{\xi}(-x,-y,t) \end{pmatrix}^T$ also satisfies these equations. Hence $\theta = (\theta_c, \sigma_\epsilon\theta_c)$ can be chosen as the solution matrix, and it admits the following symmetry

$$\overline{\theta}(x,y,t) = \sigma_\epsilon\theta(-x,-y,t)\sigma_\epsilon^{-1}. \tag{11}$$

Since the n-fold DT is nothing but a n-times iteration of one-fold DT, so the elementary DT is merely considered here. With the action of elementary DT, a relation between the potential matrices is obtained as:

$$\widetilde{P} = P + [J, S], \ S = \theta_x\theta^{-1}, \tag{12}$$

$$\widetilde{V}_2 = V_2 + V_{1,x} + 2V_0 S_x + [V_0, S]S + [V_1, S]. \tag{13}$$

Thus, with relation (11)-(12), one can verify that G_θ keeps the symmetry (8) invariant in the new potential matrix $\widetilde{P}$:

$$-\sigma_\epsilon\widetilde{P}(x,y,t)\sigma_\epsilon^{-1} = \overline{\widetilde{P}}(-x,-y,t),$$

and one can also verify the symmetry in matrix $\widetilde{V}_2(x,y,t)$ through (13), which is $\sigma_\epsilon\widetilde{V}_2(x,y,t)\sigma_\epsilon^{-1} = \overline{\widetilde{V}_2}(-x,-y,t)$. These imply that symmetry (10) are well preserved in the new operators $\overline{\widetilde{L}}$ and $\overline{\widetilde{M}}$. Moreover, the Bäcklund transformations between potentials for the $\mathcal{PT}$-symmetric nonlocal DS-II equation are:

$$\widetilde{u} = u + 2\alpha^{-1}S_{1,2}, \tag{14}$$

$$\widetilde{w} = w - 2\alpha^2[\mathbf{tr}(S)]_x = w - 2\alpha^2[\ln(\det(\theta))]_{xx}, \tag{15}$$

here the subscript $_{1,2}$ stands the row 1 and column 2 element in matrix S, and $\mathbf{tr}$ is the trace of a matrix.

Ref. [18] proposes a new DT in terms of Grammian type determinants of solutions for the local DS-II equation. Inspired by this work, we can also construct a new Darboux transformation in integral form for this equation, i.e., the binary Darboux transformation (BDT). The standard BDT scheme was introduced in [21]. Especially, for this operator L, a standard BDT has been constructed in [24], which can be written explicitly as

$$G_{\theta,\phi} = I - \theta\Omega^{-1}(\theta,\phi)\partial^{-1}\phi^\dagger, \quad \Omega(\theta,\phi) = \partial^{-1}(\phi^\dagger\theta), \tag{16}$$

where θ satisfy $L(\theta) = 0$, and ϕ admits the (formal) adjoint operator: $L^\dagger(\phi) = 0$. Then the operator

$$\hat{L} = G_{\theta,\phi}LG_{\theta,\phi}^{-1} \tag{17}$$

can be directly verified to be a new operator which has the same form as L, and so is for $\widehat{M}$. Furthermore, this Darboux transformation makes sense for any $m\times k$ matrices θ and ϕ, and we only need $\Omega(\theta,\phi)$ to be an invertible square matrix. By the combination of an elementary DT with its inverse [24], one derive the new potential matrix:

$$\hat{P} = P + [J, \theta\Omega^{-1}(\theta,\phi)\phi^\dagger]. \tag{18}$$

Next, to reduced (16) to the BDT for the partially $\mathcal{PT}$-symmetric nonlocal DS-II equation, some special reductions on the eigenfunction and the adjoint-eigenfunction must be taken, separately, which are

$$(\sigma_\epsilon)\theta(x,y,t) = \overline{\theta}(-x,-y,t), \tag{19}$$

$$(\sigma_\epsilon)\phi(x,y,t) = \overline{\phi}(-x,-y,t). \tag{20}$$

With this condition, it can be directly verified that matrix $\hat{P}$ also keeps symmetry (8) (and the same for $\widehat{V}_2$):

$$-\sigma_\epsilon\hat{P}(x,y,t)\sigma_\epsilon^{-1} = \overline{\hat{P}}(-x,-y,t),$$

noticing the variable $x\to -x$ in the integral operator ∂^{-1}. Therefore, the new operator $\hat{L}$ and $\hat{M}$ keeps the symmetry (8) as well. Moreover, the Bäcklund transformations between potentials for the nonlocal DS-II equation are

$$\hat{u} = u + \alpha^{-1}[\theta\Omega^{-1}(\theta,\phi)\phi^\dagger]_{1,2}, \tag{21}$$

$$\hat{w} = w + 2\alpha^{-2}[\mathbf{tr}(\theta\Omega^{-1}(\theta,\phi)\phi^\dagger)]_x. \tag{22}$$

3. Binary Darboux transformation for the $\mathcal{PT}$-symmetric nonlocal DS-I equation

In this section, we construct the corresponding Darboux transformation for the $\mathcal{PT}$-symmetric nonlocal DS-I equation (when $\alpha^2 = 1$). Firstly, it is noticed that potential matrices P and V_2 have the following symmetry properties:

$$\kappa P(-x,-y,t)^\dagger\kappa^{-1} = P(x,y,t), \ \ \kappa V_2(-x,-y,t)^\dagger\kappa^{-1} = -V_2(x,y,t) + \mathrm{i}\alpha P_x(x,y,t), \ \ \kappa = \begin{pmatrix} 1 & 0 \\ 0 & -\epsilon \end{pmatrix}. \tag{23}$$

Afterwards, symmetry (23) can be further transmitted to operators L and M:

$$\kappa L^\dagger\kappa^{-1} = L_{(x\to -x, y\to -y)}, \ \ \kappa M^\dagger\kappa^{-1} = -M_{(x\to -x, y\to -y)}. \tag{24}$$

However, in this case, one can not find a suitable matrix solution θ to construct the elementary DT in the differential form as (7) to preserve this constraint (23). In order to overcome this problem one still need to use the binary Darboux transformation, which the standard form has already been given by (16) and (18) in section 2. Now, to reduced them to the BDT for the $\mathcal{PT}$-symmetric nonlocal DS-I equation, the eigenfunction and adjoint eigenfunction are needed to satisfy the following relation

$$\phi(x,y,t) = \kappa\theta(-x,-y,t). \tag{25}$$

With condition (25), it can be verified that the new potential matrix in (18) indeed preserves the symmetry:

$$\kappa\hat{P}(-x,-y,t)^{\dagger}\kappa^{-1} = \hat{P}(x,y,t),$$

and so is for $\hat{V}_2$, and also for the new operators $\hat{L}$ and $\hat{M}$. Therefore, in this reduction, the binary DT becomes the one for the $\mathcal{PT}$-symmetric nonlocal DS-I equation. Moreover, under reduction (25), the Bäcklund transformations for generating new solutions in this equation are also given by (21)-(22).

4. Rational solutions from special parameters reductions

In this section, as an concrete example of the application on above DT reduction, we intent to generate some rational solutions for the $\mathcal{PT}$-symmetric nonlocal DS systems.

First of all, the general form of eigenfunction $\theta(x,y,t)$ solved from the system (3)-(4) when the initial potential solution u is taken as a real constant ρ is

$$\theta(x,y,t) = \begin{pmatrix} \xi_1(x,y,t) \\ \eta_1(x,y,t) \end{pmatrix} = \begin{pmatrix} \rho_1 \exp\left[\omega_1(x,y,t)\right] \\ \frac{\lambda_1\rho_1}{\rho}\exp\left[\omega_1(x,y,t)\right] \end{pmatrix}, \; \omega_1(x,y,t) = \alpha_1 x + \beta_1 y + \gamma_1 t, \tag{26}$$
$$\alpha_1 = -\frac{1}{2}\alpha\left(\lambda_1 + \frac{\epsilon\rho^2}{\lambda_1}\right),\; \beta_1 = \frac{1}{2}\left(\lambda_1 - \frac{\epsilon\rho^2}{\lambda_1}\right),\; \gamma_1 = \mathrm{i}\alpha^{-1}\alpha_1\beta_1,\; \lambda_1 = r_1\exp\left(\mathrm{i}\varphi_1\right),$$

where r_1, φ_1 and ρ are free real parameters, ρ_1 is set to be complex. Since ρ here stands for the constant background, it is set as $\rho = 1$ in the following discussion.

Generally, to derive rational type solutions, one needs to take derivative to parameter φ_1, and this can subtract polynomials from the original eigenfunction. Thus, a more general eigenfunction can be defined through superposition principle:

$$\theta_1(x,y,t) := C_1\theta(x,y,t) + C_2\partial_{\varphi_1}\theta(x,y,t),\;\; C_1, C_2 \in \mathbb{C}. \tag{27}$$

In addition, to apply the binary DT formula, the adjoint eigenfunction $\phi(x,y,t)$ is still needed. For the nonlocal DS-I equation, it only needs us to choose adjoint eigenfunction according to the reduction given in (25). For the nonlocal DS-II equation, because u is a real constant in L and M, it leads the following symmetry property:

$$\tau_\epsilon L^{\dagger}\tau_\epsilon^{-1} = -\overline{L}_{(t\to -t)},\;\; \tau_\epsilon M^{\dagger}\tau_\epsilon^{-1} = \overline{M}_{(t\to -t)},\;\; \tau_\epsilon = \begin{pmatrix} 1 & 0 \\ 0 & \epsilon \end{pmatrix}, \tag{28}$$

which can be used to construct solution for the adjoint operator. Actually, assuming $\phi(x,y,t)$ satisfying $L^{\dagger}(\phi) = M^{\dagger}(\phi) = 0$, then by using (28), we get $L\left(\tau_\epsilon\overline{\phi}(x,y,-t)\right) = M\left(\tau_\epsilon\overline{\phi}(x,y,-t)\right) = 0$. Therefore, in this case, the general adjoint-eigenfunction is constructed as (27) with

$$\phi_1(x,y,t) = \tau_\epsilon\overline{\theta_1}\left(x,y,-t,r_1\to\widehat{r}_1,\varphi_1\to\widehat{\varphi}_1,\rho_1\to\widehat{\rho}_1,C_1\to\widehat{C}_1,C_2\to\widehat{C}_2\right),$$

i.e., changing all the free parameters in wave-function θ_1 into a new set of parameters.

Moreover, to make functions θ_1 and ϕ_1 satisfying the reductions proposed in (19)-(20), one still needs some parameters to meet the certain conditions:

$$\epsilon = 1,\; r_1 = \widehat{r}_1 = 1,\; C_1, C_2, \widehat{C}_1, \widehat{C}_2 \in \mathbb{R}. \tag{29}$$

For example, with condition (29) and taking $C_2 = \widehat{C_2} = 0$ in formula (21), we derive a linear rogue wave solution for the nonlocal DS-II equation, which can be simplified into the form:

$$u_{II} = 1 - \frac{\mathrm{i}}{\mathrm{i}x\sin\varphi + \mathrm{i}y\cos\varphi - 2t\cos 2\varphi + C_1}. \tag{30}$$

When $t \to -\infty$, this solution approaches the constant background $u = 1$, and the singularity of this solution occurs at a critical time $t_c = \frac{C_1}{2\cos 2\varphi}$ with $(x, y) = (0, 0)$ on the spatial plane.

Furthermore, if C_2 and $\widehat{C_2}$ are taken not to be zero (here they all taken to be one for convenience), it generates the fundamental line rogue wave with three free real parameters. After a shifting of time: $t \to \hat{t} = t - \frac{C_1 - \widehat{C}_1}{4\cos 2\varphi}$ with changes of parameters: $C_1 \to \hat{x}_0 = \frac{C_1}{2\sin\varphi}$, $\widehat{C}_1 \to \hat{y}_0 = \frac{\widehat{C}_1}{2\sin\varphi}$, this solution can be written in a more compact form:

$$u_{II} = 1 + \frac{4 + 16\mathrm{i}\hat{t}\cos\varphi}{1 + 4\left[(x - \mathrm{i}\hat{x}_0)\sin\varphi + (y - \mathrm{i}\hat{y}_0)\cos\varphi\right]^2 + 16\hat{t}^{\,2}\cos^2\varphi}, \tag{31}$$

as one could easily see, if $\hat{x}_0 = \hat{y}_0 = 0$, solution (31) degenerates into the fundamental line rogue wave for the local DS-II equation, which is obtained in [19] via the KP reduction. The graphs of our rogue waves are qualitatively similar to those in the local DS equation, so they won't be exhibited here. In this solution, two new parameters $\hat{x}_0$, $\hat{y}_0$ are contained which could dominate the singularities for the solution, to be exactly:

(i). when $\varphi = \frac{k\pi}{2}$ $(k = 0, \pm 1, \pm 2, \cdots)$, solution (31) becomes singular at $x = 0$ with $\hat{x}_0 = \pm 1/2$,

(ii). When $\varphi \neq \frac{k\pi}{2}$ $(k = 0, \pm 1, \pm 2, \cdots)$, if $(\hat{x}_0 \sin\varphi + \hat{y}_0\cos\varphi)^2 \geq 1$, then solution (31) becomes singular at critical time point: $\hat{t}_c = \pm\frac{\sqrt{(\hat{x}_0\sin\varphi + \hat{y}_0\cos\varphi)^2 - 1}}{4\cos\varphi}$ and locate at: $x\sin\varphi + y\cos\varphi = 0$ on the spatial plane.

Moreover, for the $\mathcal{PT}$-symmetric nonlocal system, the general rogue waves formulas have been studied in [14] by using the Hirota bilinear method. It has been shown that our new binary DT reduction can reproduced the fundamental rogue waves in the nonlocal DS-II equation.

Next, for the nonlocal DS-I equation, if one takes $\varphi = \pi/2$ with $C_2 = 1$ in (18) with (25), it produce a rational solution:

$$u_I = 1 - \frac{2L_1L_2}{L_1L_2 - \mathrm{i}r_1^2(L_1 + L_2)}, \tag{32}$$

where,

$$\begin{aligned} L_1 &= -r_1^3(x - y) - \epsilon r_1(x + y) + 2(r_1^4 + 1)t + r_1^2(\mathrm{i} - 2C_1), \\ L_2 &= r_1^3(x - y) + \epsilon r_1(x + y) + 2(r_1^4 + 1)t + r_1^2\left(\mathrm{i} - 2\overline{C}_1\right). \end{aligned}$$

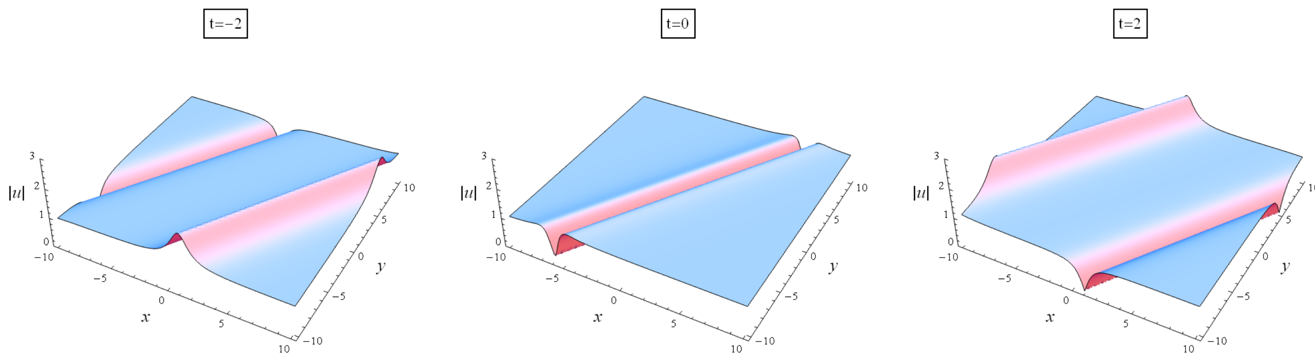

Figure 1: The dark and anti-dark rational travelling waves interactions in the $\mathcal{PT}$-symmetric nonlocal DS-II equation with parameters chosen from (32) with $\epsilon = 1, C_1 = \mathrm{i}, r_1 = 2$.

By analysing its denominator, we find that if $\mathrm{Im}(C_1) \neq 0$, this solution is a nonsingular rational travelling wave. If $\mathrm{Im}(C_1) = 0$, it becomes a singular solution. Therefore, by choosing certain values in parameter C_1, we derive travelling waves with different dynamic patterns. Here we choose one of them and show the time evolution for this solution in Fig.1.

5. Conclusions

In conclusion, we study the reductions of Darboux transformations for the $\mathcal{PT}$-symmetric nonlocal DS system. For the $\mathcal{PT}$-symmetric nonlocal DS-I equation, we find its binary Darboux transformation with a suitable reduction which relates the eigenfunction and the adjoint-eigenfunction that can preserve the symmetry property of the Lax operator. For the $\mathcal{PT}$-symmetric nonlocal DS-II equation, we first propose a reduction for an elementary DT in the differential form. Then, a new reduction is also found, which leads to a binary DT in the integral form. Especially, the eigenfunction and the adjoint eigenfunction are totally independent in this reduction. Moreover, as applications for these Darboux transformations, the fundamental rogue waves and rational travelling waves are constructed for these two nonlocal systems, respectively. Since partially $\mathcal{PT}$-symmetric physical systems has been shown possible applications in optics. We expect our results could have interesting applications for $\mathcal{PT}$-symmetric in multi-dimensions.

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