ISSN: 0256-307X

中国物理快报
Chinese Physics Letters

Volume 34 Number 7 July 2017

A Series Journal of the Chinese Physical Society
Distributed by IOP Publishing

Online: http://iopscience.iop.org/0256-307X
http://cpl.iphy.ac.cn

CHINESE PHYSICAL SOCIETY
IOP Publishing

CHIN. PHYS. LETT. Vol. 34, No. 7 (2017) 070202

Bright-Dark Mixed N-Soliton Solution of the Two-Dimensional Maccari System *

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(Received 29 March 2017)

The general bright-dark mixed N-soliton solution of the two-dimensional Maccari system is obtained with the KP hierarchy reduction method. The dynamics of single and two solitons are discussed in detail. Asymptotic analysis shows that two solitons undergo elastic collision accompanied by a position shift. Furthermore, our analysis on mixed soliton bound states shows that arbitrary higher-order soliton bound states can take place.

PACS: 02.30.Ik, 05.45.Yv, 02.30.Jr DOI: 10.1088/0256-307X/34/7/070202

The two-dimensional Maccari system is of the form[1]

$$iA_t + A_{xx} + LA = 0, \tag{1}$$
$$iB_t + B_{xx} + LB = 0, \tag{2}$$
$$L_y = (AA^* + \mu BB^*)_x, \tag{3}$$

where $\mu \neq 0, \pm 1$ is a real constant, $A(x, y, t)$ and $B(x, y, t)$ are complex while $L(x, y, t)$ is real, and the asterisk means complex conjugate. This system is usually used to describe the motion of isolated waves, localized in a small part of space, in some fields such as plasma physics, nonlinear optics and hydrodynamics. It reduces to the nonlinear Schrödinger (NLS) equation[2] when $y = x$. The reduction $y = t$ leads to the system of the coupled long-wave resonance equation.[3] When $A = B^*$, it becomes the so-called simplest (2+1)-dimensional extension of the NLS equation introduced by Fokas.[4] Many studies have been carried out on this system. Uthayakumar *et al.*[5] have studied the integrability property of Eqs. (1)–(3) using singularity structure analysis. Based on the technique of coalescence of wavenumbers, its two-dromion solutions were obtained by Lai and Chow.[6] By means of the variable separation approach,[7−9] many coherent soliton structures such as dromions, breathers, foldon and solitoff were obtained by Zhang *et al.*[10,11] Most recently, Yuan *et al.*[12] constructed various rational solutions of Eqs. (1)–(3) through Hirota's bilinear method.

The goal of this work is to construct the mixed N-soliton solution of Eqs. (1)–(3) via the KP hierarchy reduction method. The KP hierarchy reduction method was first developed by the Kyoto school,[13] and has been widely used to derive soliton[14−17] and rogue wave[18,19] solutions of many integrable systems. Assuming that the A component is of bright type and the B component is of dark type, the following dependent variable transformations are introduced,

$$A = \frac{g}{f},\ B = \rho e^{i(\alpha x - \alpha^2 t)}\frac{h}{f},\ L = 2(\log f)_{xx}, \tag{4}$$

where $f(x, y, t)$ is real, $g(x, y, t)$ and $h(x, y, t)$ are complex, α and ρ are two real constants. Then the two-dimensional (2D) Maccari system (1)–(3) can be converted into the bilinear form

$$(D_x^2 + iD_t)g \cdot f = 0, \tag{5}$$
$$(D_x^2 + 2i\alpha D_x + iD_t)h \cdot f = 0, \tag{6}$$
$$D_x D_y f \cdot f = gg^* - \mu\rho^2(f^2 - hh^*), \tag{7}$$

where D is the usual Hirota bilinear operator.[20]

Consider the Gram determinant tau functions of a two-component KP hierarchy

$$\tau_0(k) = \begin{vmatrix} \boldsymbol{A} & \boldsymbol{I} \\ -\boldsymbol{I} & \boldsymbol{B} \end{vmatrix},\ \tau_1(k) = \begin{vmatrix} \boldsymbol{A} & \boldsymbol{I} & \Omega^{\rm T} \\ -\boldsymbol{I} & \boldsymbol{B} & \boldsymbol{0}^{\rm T} \\ \boldsymbol{0} & -\bar{\Psi} & 0 \end{vmatrix},$$
$$\tau_{-1}(k) = \begin{vmatrix} \boldsymbol{A} & \boldsymbol{I} & \boldsymbol{0}^{\rm T} \\ -\boldsymbol{I} & \boldsymbol{B} & \Psi^{\rm T} \\ -\bar{\Omega} & \boldsymbol{0} & 0 \end{vmatrix}, \tag{8}$$

where I is an $N \times N$ identity matrix, $\boldsymbol{0}$ is an N-component zero-row vector, $\boldsymbol{A}$ and $\boldsymbol{B}$ are $N \times N$ matrices, and Ω, Ψ, $\bar{\Omega}$ and $\bar{\Psi}$ are N-component row vectors whose elements are given respectively by

$$a_{ij}(k) = \frac{1}{p_i + \bar{p}_j}\Big(-\frac{p_i - c}{\bar{p}_j + c}\Big)^k e^{\xi_i + \bar{\xi}_j},$$
$$b_{ij} = \frac{1}{q_i + \bar{q}_j}e^{\eta_i + \bar{\eta}_j},$$
$$\Omega = (e^{\xi_1}, e^{\xi_2}, \ldots, e^{\xi_N}),\ \Psi = (e^{\eta_1}, e^{\eta_2}, \ldots, e^{\eta_N}),$$
$$\bar{\Omega} = (e^{\bar{\xi}_1}, e^{\bar{\xi}_2}, \ldots, e^{\bar{\xi}_N}),\ \bar{\Psi} = (e^{\bar{\eta}_1}, e^{\bar{\eta}_2}, \ldots, e^{\bar{\eta}_N}),$$

*Supported by the Global Change Research Program of China under Grant No 2015CB953904, the National Natural Science Foundation of China under Grant Nos 11675054 and 11435005, and the Shanghai Collaborative Innovation Center of Trustworthy Software for Internet of Things under Grant No ZF1213.

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with

$$\xi_i = \frac{1}{p_i - c} x_{-1} + p_i x_1 + p_i^2 x_2 + \xi_{i0},\ \eta_i = q_i y_1 + \eta_{i0},$$
$$\bar{\xi}_j = \frac{1}{\bar{p}_j + c} x_{-1} + \bar{p}_j x_1 - \bar{p}_j^2 x_2 + \bar{\xi}_{j0},\ \bar{\eta}_j = \bar{q}_j y_1 + \bar{\eta}_{j0},$$

in which p_i, $\bar{p}_j$, q_i, $\bar{q}_j$, ξ_{i0}, $\bar{\xi}_{j0}$, η_{i0}, $\bar{\eta}_{j0}$ and c are complex constants. Based on the Sato theory for KP hierarchy,[13] the tau functions (8) satisfy the bilinear equations

$$(D_{x_1}^2 - D_{x_2})\tau_1(k) \cdot \tau_0(k) = 0, \tag{9}$$
$$(D_{x_1}^2 - D_{x_2} + 2cD_{x_1})\tau_0(k+1) \cdot \tau_0(k) = 0, \tag{10}$$
$$D_{x_1} D_{y_1} \tau_0(k) \cdot \tau_0(k) = -2\tau_1(k)\tau_{-1}(k), \tag{11}$$
$$(D_{x_1} D_{x_{-1}} - 2)\tau_0(k) \cdot \tau_0(k) = -2\tau_0(k+1)\tau_0(k-1), \tag{12}$$

which can be proved using the Grammian technique.[20] Assuming that x_1, x_{-1} and y_1 are real; x_2 and c are pure imaginary and taking $p_j^* = \bar{p}_j$, $q_j^* = \bar{q}_j$, $\xi_{j0}^* = \bar{\xi}_{j0}$, and $\eta_{j0}^* = \bar{\eta}_{j0}$, one can check that

$$a_{ij}(k) = a_{ji}^*(k), \quad b_{ij} = b_{ji}^*.$$

Moreover, we let

$$f = \tau_0(0),\ g = \tau_1(0),\ h = \tau_0(1),$$

hence, f is real and

$$g^* = -\tau_{-1}(0),\ h^* = \tau_0(-1),$$

thus the bilinear Eqs. (9)–(12) become

$$(D_{x_1}^2 - D_{x_2})g \cdot f = 0, \tag{13}$$
$$(D_{x_1}^2 - D_{x_2} + 2cD_{x_1})h \cdot f = 0, \tag{14}$$
$$D_{x_1} D_{y_1} f \cdot f = 2gg^*, \tag{15}$$
$$(D_{x_1} D_{x_{-1}} - 2)f \cdot f = -2hh^*. \tag{16}$$

By row and column operations, f can be rewritten as

$$f = \begin{vmatrix} A' & I \\ -I & B' \end{vmatrix}, \tag{17}$$

where the entries in A' and B' are

$$a'_{ij} = \frac{1}{p_i + p_j^*}, \quad b'_{ij} = \frac{1}{q_i + q_j^*} e^{\eta_i + \eta_j^* + \xi_i^* + \xi_j},$$

with

$$\eta_i + \xi_i^* = q_i y_1 + \frac{1}{p_i^* + c} x_{-1} + p_i^* x_1 - p_i^{*2} x_2 + \xi_{i0}^* + \eta_{i0},$$
$$\eta_j^* + \xi_j = q_j^* y_1 + \frac{1}{p_j - c} x_{-1} + p_j x_1 + p_j^2 x_2 + \xi_{j0} + \eta_{j0}^*.$$

Under the reduction conditions

$$-ip_i^{*2} = q_i - \frac{\mu\rho^2}{p_i^* + c},\ ip_j^2 = q_j^* - \frac{\mu\rho^2}{p_j - c}, \tag{18}$$

the following relation holds

$$i\partial_{x_2} b'_{ij} = (\partial_{y_1} - \mu\rho^2 \partial_{x_{-1}}) b'_{ij}, \tag{19}$$

hence we have

$$if_{x_2} = f_{y_1} - \mu\rho^2 f_{x_{-1}}, \tag{20}$$

which also implies

$$if_{x_1 x_2} = f_{x_1 y_1} - \mu\rho^2 f_{x_1 x_{-1}}. \tag{21}$$

In addition, Eqs. (15) and (16) can be expanded as

$$f_{x_1 y_1} f - f_{x_1} f_{y_1} = gg^*, \tag{22}$$

and

$$f_{x_1 x_{-1}} f - f_{x_1} f_{x_{-1}} - f^2 = -hh^*, \tag{23}$$

respectively. By using relations (20) and (21), from Eqs. (22) and (23), we arrive at

$$i(f_{x_1 x_2} f - f_{x_1} f_{x_2}) = gg^* - \mu\rho^2 (f^2 - hh^*). \tag{24}$$

Through the variable transformations

$$x_1 = x,\ x_2 = i(t + \frac{1}{2} y), \tag{25}$$

Eqs. (13) and (14) become Eqs. (5) and (6) by taking $c = i\alpha$, and Eq. (24) is nothing but Eq. (7).

With the variable transformations (25), the variables x_{-1} and y_1 become dummy variables, hence they can be treated as constants. Therefore, we define $e^{\eta_i} = c_i^*$, $e^{\eta_i^*} = c_i$, $(i = 1, 2, \ldots, N)$ and let $C = -(c_1, c_2, \ldots, c_N)$, thus we have obtained the mixed N-soliton solution

$$f = \begin{vmatrix} \boldsymbol{A} & \boldsymbol{I} \\ -\boldsymbol{I} & \boldsymbol{B} \end{vmatrix},\ g = \begin{vmatrix} \boldsymbol{A} & \boldsymbol{I} & \Omega^{\mathrm{T}} \\ -\boldsymbol{I} & \boldsymbol{B} & \boldsymbol{0}^{\mathrm{T}} \\ \boldsymbol{0} & C & 0 \end{vmatrix},$$
$$h = \begin{vmatrix} \boldsymbol{A}^{(1)} & \boldsymbol{I} \\ -\boldsymbol{I} & \boldsymbol{B} \end{vmatrix}, \tag{26}$$

where the entries in $\boldsymbol{A}$, $\boldsymbol{A}^{(1)}$ and $\boldsymbol{B}$ are

$$a_{ij} = \frac{1}{p_i + p_j^*} e^{\xi_i + \xi_j^*},$$
$$a_{ij}^{(1)} = \frac{1}{p_i + p_j^*} \Big(- \frac{p_i - i\alpha}{p_j^* + i\alpha} \Big) e^{\xi_i + \xi_j^*},$$
$$b_{ij} = c_i^* c_j \Big[i(-p_i^{*2} + p_j^2) + \frac{\mu\rho^2 (p_i^* + p_j)}{(p_i^* + i\alpha)(p_j - i\alpha)} \Big]^{-1},$$

meanwhile, Ω and C are given by

$$\Omega = (e^{\xi_1}, e^{\xi_2}, \ldots, e^{\xi_N}),\ C = -(c_1, c_2, \ldots, c_N),$$

with $\xi_i = p_i x + i p_i^2(t + \frac{1}{2}y) + \xi_{i0}$, and p_i, ξ_{i0} and c_i $(i = 1, 2, \ldots, N)$ are complex constants.

Take $N = 1$ in the formula (26) and we can obtain one-soliton solution. In this case, the tau functions can be rewritten as

$$f = 1 + E_{11^*} e^{\xi_1+\xi_1^*},\ g = c_1 e^{\xi_1},\\ h = 1 + F_{11^*} e^{\xi_1+\xi_1^*}, \tag{27}$$

where

$$E_{11^*} = c_1 c_1^* \Big[i(p_1 + p_1^*)^2 (p_1 - p_1^*) + \frac{\mu\rho^2 (p_1 + p_1^*)^2}{(p_1^* + i\alpha)(p_1 - i\alpha)} \Big]^{-1},\\ F_{11^*} = e^{2i\phi} E_{11^*},\quad e^{2i\phi_1} = -\frac{p_1 - i\alpha}{p_1^* + i\alpha}.$$

Note that this solution is nonsingular only if $E_{11^*} > 0$. The tau functions (27) give the one-soliton solution

$$A = \frac{c_1}{2} e^{-\theta_1} e^{i\xi_{1\mathrm{I}}} \mathrm{sech}(\xi_{1\mathrm{R}} + \theta_1),\\ B = \frac{\rho}{2} e^{i(\alpha x - \alpha^2 t)} [1 + e^{2i\phi_1} + (e^{2i\phi_1} - 1)\tanh(\xi_{1\mathrm{R}} + \theta_1)],\\ L = 2p_{1\mathrm{R}}^2 \mathrm{sech}^2(\xi_{1\mathrm{R}} + \theta_1),$$

where $e^{2\theta_1} = E_{11^*}$, $\xi_1 = \xi_{1\mathrm{R}} + i\xi_{1\mathrm{I}}$, and the suffixes R and I denote the real and imaginary parts, respectively. Thus the amplitude of the bright soliton in A component is $\frac{|c_1|}{2} e^{-\theta_1}$ while the amplitude of the bright soliton in L component is $2p_{1\mathrm{R}}^2$. For the dark soliton in B component, $|B|$ approaches $|\rho|$ as $x, y \to \pm\infty$. Moreover, the intensity of the dark soliton is $|\rho| \cos\phi_1$. The mixed one-soliton at time $t = 0$ is displayed in Fig. 1 with the parametric choice $p_1 = 1 - \frac{1}{2}i$, $c_1 = 1 + 2i$, $\rho = \alpha = 1$, $\xi_{10} = y = 0$ and $\mu = 2$.

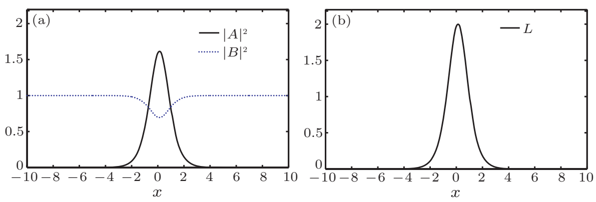

Fig. 1. Mixed one-soliton of the 2D Maccari system.

As a matter of fact, the interaction of nonlinear waves may present some novel phenomena.[17,21] To study the collision of two solitons, we take $N = 2$ in the formula (26). The tau functions can be rewritten as

$$f = 1 + E_{11^*} e^{\xi_1+\xi_1^*} + E_{12^*} e^{\xi_1+\xi_2^*} + E_{21^*} e^{\xi_2+\xi_1^*} + E_{22^*} e^{\xi_2+\xi_2^*} + E_{121^*2^*} e^{\xi_1+\xi_2+\xi_1^*+\xi_2^*},\\ g = c_1 e^{\xi_1} + c_2 e^{\xi_2} + G_{121^*} e^{\xi_1+\xi_2+\xi_1^*} + G_{122^*} e^{\xi_1+\xi_2+\xi_2^*},\\ h = 1 + F_{11^*} e^{\xi_1+\xi_1^*} + F_{1,2^*} e^{\xi_1+\xi_2^*} + F_{21^*} e^{\xi_2+\xi_1^*} + F_{22^*} e^{\xi_2+\xi_2^*} + F_{121^*2^*} e^{\xi_1+\xi_2+\xi_1^*+\xi_2^*}, \tag{28}$$

where

$$E_{ij^*} = c_i c_j^* \Big[i(p_i + p_j^*)^2 (p_i - p_j^*) + \frac{\mu\rho^2 (p_i + p_j^*)^2}{(p_i - i\alpha)(p_j^* + i\alpha)} \Big]^{-1},\\ E_{121^*2^*} = |p_1 - p_2|^2 \Big[\frac{E_{11^*} E_{22^*}}{(p_1 + p_2^*)(p_2 + p_1^*)} - \frac{E_{12^*} E_{21^*}}{(p_1 + p_1^*)(p_2 + p_2^*)} \Big],\\ F_{ij^*} = -\frac{p_i - i\alpha_1}{p_j^* + i\alpha_1} E_{ij^*},\\ F_{121^*2^*} = \frac{(p_1 - i\alpha_1)(p_2 - i\alpha_1)}{(p_1^* + i\alpha_1)(p_2^* + i\alpha_1)} E_{121^*2^*},\\ G_{12i^*} = (p_1 - p_2)\Big(\frac{c_1 E_{2i^*}}{p_1 + p_i^*} - \frac{c_2 E_{1i^*}}{p_2 + p_i^*} \Big).$$

To obtain nonsingular solutions, the denominator f in Eq. (28) must be nonzero. For this purpose, we rewrite f as

$$f = 2e^{\xi_{1\mathrm{R}}+\xi_{2\mathrm{R}}} [e^{\theta_1+\theta_2} \cosh(\xi_{1\mathrm{R}} - \xi_{2\mathrm{R}} + \theta_1 - \theta_2) + e^{\theta_3} \cosh(\xi_{1\mathrm{R}} - \xi_{2\mathrm{R}} + \theta_3) + e^{\zeta_R} \cos(\xi_{1\mathrm{I}} - \xi_{2\mathrm{I}} + \zeta_I)],$$

where

$$e^{2\theta_1} = E_{11^*},\quad e^{2\theta_2} = E_{22^*},\\ e^{2\theta_3} = E_{121^*2^*},\quad e^{\zeta_R + i\zeta_I} = E_{12^*}.$$

Hence, $e^{\theta_1+\theta_2} + e^{\theta_3} > e^{\zeta_R}$ is a sufficient condition to guarantee nonsingular solutions. The asymptotic forms of the soliton s_1, (s_2) before and after collision are of the form: (1) Before collision $(x, y \to -\infty)$ Soliton s_1

$$A_1^- \simeq \frac{c_1}{2} e^{-\theta_1} e^{i\xi_{1\mathrm{I}}} \mathrm{sech}(\xi_{1\mathrm{R}} + \theta_1),\\ B_1^- \simeq \frac{\rho}{2} e^{i(\alpha x - \alpha^2 t)} \cdot [1 + e^{2i\phi_1} + (e^{2i\phi_1} - 1)\tanh(\xi_{1\mathrm{R}} + \theta_1)],\\ L_1^- \simeq 2p_{1\mathrm{R}}^2 \mathrm{sech}^2(\xi_{1\mathrm{R}} + \theta_1).$$

Soliton s_2

$$A_2^- \simeq \frac{1}{2} e^{-\theta_1 - \theta_3} G_{121^*} e^{i\xi_{2\mathrm{I}}} \mathrm{sech}(\xi_{2\mathrm{R}} + \theta_3 - \theta_1),\\ B_2^- \simeq \frac{\rho}{2} e^{i(\alpha x - \alpha^2 t + 2\phi_1)} \cdot [1 + e^{2i\phi_2} + (e^{2i\phi_2} - 1)\tanh(\xi_{2\mathrm{R}} + \theta_3 - \theta_1)],\\ L_2^- \simeq 2p_{2\mathrm{R}}^2 \mathrm{sech}^2(\xi_{2\mathrm{R}} + \theta_3 - \theta_1).$$

(2) After collision $(x, y \to +\infty)$ Soliton s_1

$$A_1^+ \simeq \frac{1}{2} e^{-\theta_2-\theta_3} G_{122^*} e^{i\xi_{1I}} \mathrm{sech}(\xi_{1R}+\theta_3-\theta_2),$$
$$B_1^+ \simeq \frac{\rho}{2} e^{i(\alpha x-\alpha^2 t+2\phi_2)} \cdot [1+e^{2i\phi_1}+(e^{2i\phi_1}-1)\tanh(\xi_{1R}+\theta_3-\theta_2)],$$
$$L_1^+ \simeq 2p_{1R}^2 \mathrm{sech}^2(\xi_{1R}+\theta_3-\theta_2).$$

Soliton s_2

$$A_2^+ \simeq \frac{c_2}{2} e^{-\theta_2} e^{i\xi_{2I}} \mathrm{sech}(\xi_{2R}+\theta_2),$$
$$B_2^+ \simeq \frac{\rho}{2} e^{i(\alpha x-\alpha^2 t)} \cdot [1+e^{2i\phi_2}+(e^{2i\phi_2}-1)\tanh(\xi_{2R}+\theta_2)],$$
$$L_2^+ \simeq 2p_{2R}^2 \mathrm{sech}^2(\xi_{2R}+\theta_2).$$

In the above expressions, $e^{2i\phi_j} = -(p_j - i\alpha_1)/(p_j^* + i\alpha_1)$. The amplitudes of the bright solitons in A component before interaction are $(\frac{c_1}{2} e^{-\theta_1}, \frac{1}{2} e^{-\theta_1-\theta_3} G_{121^*})$, and the amplitudes after interaction are $(\frac{1}{2} e^{-\theta_2-\theta_3} G_{122^*}, \frac{c_2}{2} e^{-\theta_2})$. Substituting various quantities, we can find that $|\frac{c_1}{2} e^{-\theta_1}| = |\frac{1}{2} e^{-\theta_2-\theta_3} G_{122^*}|$ and $|\frac{1}{2} e^{-\theta_1-\theta_3} G_{121^*}| = |\frac{c_2}{2} e^{-\theta_2}|$, which indicate that the intensities of the bright solitons in A component are the same before and after collision. Similarly, the dark solitons in B component and the bright solitons in L component also undergo elastic collision. In addition, both the bright and dark solitons admit the same magnitude position shift. The position shift of soliton s_1, (s_2) is $\Lambda_1 = \theta_3 - \theta_1 - \theta_2$, $(\Lambda_2 = -\Lambda_1)$. The phase shifts of the dark solitons s_1 and s_2 in B component are $2\phi_2$ and $-2\phi_1$, respectively. The above analysis clearly reveals that the collision of two solitons is elastic and energies of solitons in different components completely transmit through. In Fig. 2, the collision of two solitons is displayed for the parameters chosen as $p_1 = 1 - \frac{3}{4}i$, $p_2 = 2 - \frac{1}{4}i$, $c_1 = 1 + \frac{1}{2}i$, $c_2 = \frac{1}{2} + i$, $\rho = \alpha = 1$, $\xi_{10} = \xi_{20} = y = 0$ and $\mu = 2$. It is obvious that the solitons in all the components undergo elastic collision without shape change but just accompanied by a position shift.

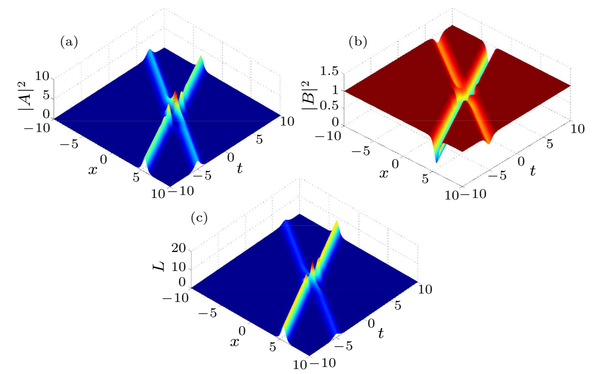

Fig. 2. Collision of mixed two solitons of the 2D Maccari system: (a) the A component, (b) the B component, and (c) the L component.

Soliton bound states are another fascinating class of multi-soliton solutions, which can be viewed as composite solitons moving with the same velocity. Suppose that the wave number of the ith soliton is $p_i = p_{iR} + ip_{iI}$, then one can obtain the mixed two-soliton bound state from Eq. (28) with the restriction $p_{1I} = p_{2I}$. Such a bound state is displayed in Fig. 3 with the parametric choice $p_1 = 1 - \frac{1}{4}i$, $p_2 = 2 - \frac{1}{4}i$, $c_1 = 1 + \frac{1}{2}i$, $c_2 = \frac{1}{2} + i$, $\rho = \alpha = 1$, $\xi_{10} = \xi_{20} = y = 0$ and $\mu = 2$. What is more, the bound state can exist up to an arbitrary order since the same p_{iI} value can give as many distinct p_{iR} values as we want.

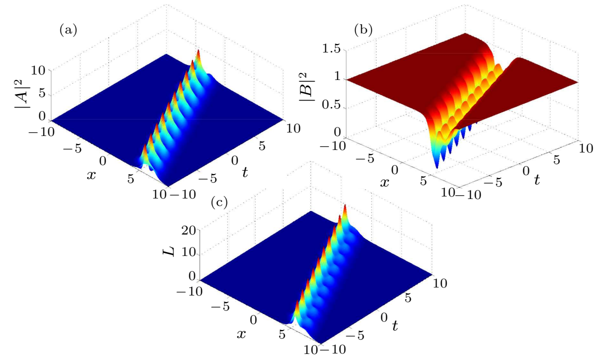

Fig. 3. Mixed two-soliton bound states of the 2D Maccari system: (a) the A component, (b) the B component, and (c) the L component.

We would like to express our sincere thanks to Lou S Y and Chen J C for their valuable comments.

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Chinese Physics Letters

Volume 34 Number 7 July 2017

CONDENSED MATTER: STRUCTURE, MECHANICAL AND THERMAL PROPERTIES

CONDENSED MATTER: ELECTRONIC STRUCTURE, ELECTRICAL, MAGNETIC, AND OPTICAL PROPERTIES