Commun Nonlinear Sci Numer Simulat 57 (2018) 276–289

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Darboux transformation of the coupled nonisospectral Gross–Pitaevskii system and its multi-component generalization

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ARTICLE INFO

Article history:
Received 26 May 2017
Accepted 12 September 2017
Available online 21 September 2017

Keywords:
Coupled Gross–Pitaevskii system
Darboux transformation
Soliton
Breather
Rogue wave

ABSTRACT

In this paper, we extend the one-component Gross–Pitaevskii (GP) equation to the two-component coupled GP system including damping term, linear and parabolic density profiles. The Lax pair with nonisospectral parameter and infinitely-many conservation laws of this coupled GP system are presented. Actually, the Darboux transformation (DT) for this kind of nonautonomous system is essentially different from the autonomous case. Consequently, we construct the DT of the coupled GP equations, besides, nonautonomous multi-solitons, one-breather and the first-order rogue wave are also obtained. Various kinds of one-soliton solution are constructed, which include stationary one-soliton and nonautonomous one-soliton propagating along the negative (positive) direction of x-axis. The interaction of two solitons and two-soliton bound state are demonstrated respectively. We get the nonautonomous one-breather on a curved background and this background is completely controlled by the parameter β. Using a limiting process, the nonautonomous first-order rogue wave can be obtained. Furthermore, some dynamic structures of these analytical solutions are discussed in detail. In addition, the multi-component generalization of GP equations are given, then the corresponding Lax pair and DT are also constructed.

1. Introduction

In recent years, analytic solutions of nonlinear evolution equations, such as solitons [1–3], breathers [4–6] and rogue waves [7–10], which have received a large of research activities in many realms. When the effects of dispersion and nonlinearity are balanced in nonlinear waves during propagating, solitons will be formed. These waves keep some features (amplitudes, speeds, etc.) unchanged during propagating. In many cases, soliton is considered as an ideal solution model in physics [11]. As the particular solutions of nonlinear systems, breathers propagate steadily and localize in either time or space, such as Akhmediev breathers (ABs) [12,13] and Kuznetsov–Ma breathers (KMBs) [14]. ABs are periodic in space and localized in time, while KMBs are periodic in time and localized in space. Another special type of analytic solution is the rogue wave localized in both space and time, and it has peak amplitude usually more than twice of the background wave height [15,16]. Besides, rogue waves always appear from nowhere and disappear without a trace, and they can be written in terms of the rational functions of coordinates.

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https://doi.org/10.1016/j.cnsns.2017.09.009
1007-5704/

In many documents, these above three types of analytical solutions had been greatly discussed in the autonomous system whose coefficients are all constants [17,18]. However, there are a lot of variable-coefficients models consisted of time or (both) space-dependent nonlinearity, dispersion and external potentials. These models are called nonautonomous systems [19–21]. The analytic solutions in the nonautonomous system are greatly different from those in the autonomous system. When solitons of the nonautonomous system propagate, their amplitudes, speeds and widths will change [22,23].

In the plasma, when both high frequency and low frequency waves have the same group velocity, they can be nonlinearly coupled through density depletion caused by the high and low frequency fields as well as via the generation of high frequency side bands and low frequency second harmonic. In this case, the propagation of waves can be controlled by the coupled nonautonomous Gross–Pitaevskii (GP) equations [19,24] and this system can be written as

$$\begin{aligned} &iq_{1t}+q_{1xx}+2\mu^2(|q_1|^2+|q_2|^2)q_1+(i\beta-\alpha x+\beta^2x^2)q_1=0,\\ &iq_{2t}+q_{2xx}+2\mu^2(|q_1|^2+|q_2|^2)q_2+(i\beta-\alpha x+\beta^2x^2)q_2=0, \end{aligned} \tag{1}$$

where $q_1(x, t)$ and $q_2(x, t)$ are the complex envelops of two fields in the inhomogeneous plasma and μ is the nonlinearity parameter, meanwhile, $\mu,\ \alpha,$ and β are all real numbers. Besides, β^2x^2 and αx refer to parabolic and linear density profiles respectively, and $i\beta$ denotes the damping term. Setting $\mu=1$ and $\alpha=\epsilon$, the coupled nonisospectral GP Eqs. (1) become the coupled system in [24], where the soliton solutions were constructed by Bäcklund transformation and its N-coupled damped generalization were also obtained.

In [19], the one-component GP equation was studied by Darboux transformation (DT) and its multi solitons, breather and higher-order rogue waves were obtained. However, we test this kind of DT in [19] by Maple software and find that it does not admit the t-part of the Lax pair. Here, we extend the one-component GP equation in [19] to the multi-component system and reconstruct the corresponding DT. In [25], Yong et al. tested that the DT about a generalized inhomogeneous higher-order nonlinear Schrödinger (NLS) equation in [26] was incorrect. The inhomogeneous NLS equation with a non-isospectral nonlinear eigenvalue problem was mistaken for having a constant spectral parameter in [26]. The modified DT about the same inhomogeneous higher-order NLS equation was reconstructed and some novel solitons were also obtained in [25]. There had been many other articles about one- and multi-component nonautonomous models. The variable-coefficients coupled Hirota equation was studied in [27], then some new types of rogue waves were obtained, such as dark-bright, composite, three-sister, quadruple and sextuple rogue waves. In [28], the periodic rogue wave, composite rogue wave and oscillating rogue wave were discussed via the variable-coefficient modified DT. Utilizing Hirota bilinear method, nonautonomous matter waves were also constructed in a spin-1 Bose–Einstein condensate [29].

In this paper, we construct the DT of the two-component coupled GP system with nonisospectral linear eigenvalue problem and present its some special solutions, such as multi-soliton, one-breather and rogue wave. It is obvious that the coupled GP system (1) admits the AKNS (Ablowitz–Kaup–Newell–Segur) spectral problem. Gu [30] found a unified approach to construct DT for this kind of isospectral AKNS system. However, the spectral parameter of Eq. (1) is dependent of time variable t, so we can not utilize Gu's method directly. In [31,32], Zhou generalized Gu's formula for the DT to the nonisospectral AKNS hierarchy. In the following contents, based on Zhou's method, we construct the DTs of this two-component coupled GP system and its multi-component generalization [24] respectively. The spectral parameter λ of the coupled nonisospectral GP equations holds $\lambda=\frac{\alpha}{4\beta}+\xi e^{-2\beta t}$ with ξ being an arbitrary constant, thus we can take ξ as a new spectral parameter [19]. In order to make sure that the coupled nonisospectral GP Eqs. (1) are integrable, the infinitely-many conservation laws are constructed. Nonautonomous multi-soliton, one-breather and first-order rogue wave are constructed through the DT. In nonautonomous multi-soliton solutions, some free parameters play the important roles in dynamic structures of solitons. The parameter μ and the imaginary parts of spectral parameters ξ_j $(j=1,2,3,\dots)$ determine the amplitudes of solitons. Additionally, the real parts of spectral parameters ξ_j affect the directions of solitons' propagation and α influences the localization of these solitons. Choosing $Re(\xi_1)=Re(\xi_2)$, the two-soliton bound state can be generated. Starting from the nonzero background seed solution, the nonautonomous one-breather on a curved background is acquired. This type of breather has some deformations along the direction of t. Besides, the parameter β determines the degree of the curved background. The amplitude of the breather decreases as t increases till being zero. Through taking a limiting process in the special vector solutions of the Lax pair, the nonautonomous first-order rogue wave is given. Besides, the multi-component generalization of GP equations are given, then the corresponding Lax pair and DT are also constructed.

The paper is organized as follows. In Section 2, the Lax pair and conservation laws of the coupled nonisospectral GP equations are given. In Section 3, the DT of this coupled GP system with nonisospectral Lax pair is constructed and its multi-soliton is also given. In Section 4, breather and rogue waves of the coupled GP system are revealed. In Section 5, the DT and Lax pair of multi-component generalization of the two-component coupled GP equations are discussed. The last section includes several conclusions and discussions.

2. Integrability: Lax pair and conservation laws

Based on Su et al. [19] and Uthayakumar et al. [24], the Lax pair of Eq. (1) can be written as follow

$$\Psi_x=U\Psi=(i\lambda U_0+\mu U_1)\Psi, \tag{2}$$

$$\Psi_t=V\Psi=(2i\lambda^2U_0+2\lambda(-i\beta xU_0+\mu U_1)+iV_1)\Psi, \tag{3}$$

Where

$$U_0=\begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix},\quad U_1=\begin{bmatrix}0&Q_1&Q_2\\-Q_1^*&0&0\\-Q_2^*&0&0\end{bmatrix},$$

$$V_1=\begin{bmatrix}\mu^2(|Q_1|^2+|Q_2|^2)-\frac{\alpha x}{2} & \mu Q_{1x}+2i\mu\beta xQ_1 & \mu Q_{2x}+2i\mu\beta xQ_2\\ \mu Q_{1x}^*-2\mu i\beta xQ_1^* & -\mu^2|Q_1|^2+\frac{\alpha x}{2} & -\mu^2Q_2Q_1^*\\ \mu Q_{2x}^*-2\mu i\beta xQ_2^* & -\mu^2Q_1Q_2^* & -\mu^2|Q_2|^2+\frac{\alpha x}{2}\end{bmatrix},$$

with $Q_1(x,t)=q_1(x,t)e^{-\frac{i\beta x^2}{2}}$, $Q_2(x,t)=q_2(x,t)e^{-\frac{i\beta x^2}{2}}$ and $\lambda(t)=\frac{\alpha}{4\beta}+\xi e^{-2\beta t}$ (ξ is an arbitrary complex constant). Here, the sign * denotes the complex conjugate, and λ is dependent of time variable t. The column vector $\Psi=(\psi_1,\psi_2,\psi_3)^T$ is the eigenfunction with λ being the spectral parameter. Besides, the coupled GP system (1) can be directly deduced by the compatibility condition $U_t-V_x+[U,V]=0$.

In the following contents, the infinitely-many conservation laws [33–35] of the coupled GP system (1) are given, which further indicates its integrability. For this purpose, the two functions $\Gamma_1(x,t)=\frac{\psi_2}{\psi_1}$ and $\Gamma_2(x,t)=\frac{\psi_3}{\psi_1}$ are introduced. According to Eq. (2), the following two equalities can be derived, namely

$$\Gamma_{1x}=-\mu Q_1^*+2i\lambda\Gamma_1-\mu Q_1\Gamma_1^2-\mu Q_2\Gamma_1\Gamma_2, \tag{4}$$

$$\Gamma_{2x}=-\mu Q_2^*+2i\lambda\Gamma_2-\mu Q_2\Gamma_2^2-\mu Q_1\Gamma_1\Gamma_2. \tag{5}$$

Here, we set $Q_1\Gamma_1=\sum_{n=1}^{\infty}\Gamma_1^{(n)}\lambda^{-n}$ and $Q_2\Gamma_2=\sum_{n=1}^{\infty}\Gamma_2^{(n)}\lambda^{-n}$, where $\Gamma_1^{(n)}$'s and $\Gamma_2^{(n)}$'s $(n=1,2,3,\dots)$ are all functions of x and t to be determined. Substituting these two expansions into Eq. (4) and Eq. (5) respectively, then equating the coefficients of the same power of λ, the following equations can be obtained as

$$\lambda^0:\Gamma_1^{(1)}=-\frac{i}{2}\mu Q_1Q_1^*, \tag{6a}$$

$$\lambda^{-1}:\Gamma_1^{(2)}=-\frac{i}{2}\left(\Gamma_{1x}^{(1)}-\frac{Q_{1x}}{Q_1}\Gamma_1^{(1)}\right)=-\frac{1}{4}\mu Q_1Q_{1x}^*, \tag{6b}$$

$$\begin{aligned}\lambda^{-2}:\Gamma_1^{(3)}&=-\frac{i}{2}\left[-\frac{Q_{1x}}{Q_1}\Gamma_1^{(2)}+\Gamma_{1x}^{(2)}+\mu\Gamma_1^{(1)}(\Gamma_1^{(1)}+\Gamma_2^{(2)})\right]\\&=\frac{i\mu}{8}(Q_1Q_{1xx}^*+\mu^2Q_1^2Q_1^{*2}+\mu^2Q_1Q_2Q_1^*Q_2^*),\end{aligned} \tag{6c}$$

$$\lambda^{-k}:\Gamma_1^{(k+1)}=-\frac{i}{2}\left[-\frac{Q_{1x}}{Q_1}\Gamma_1(k)+\Gamma_{1x}^{(k)}+\mu\sum_{j=1}^{k-1}\Gamma_1^{(j)}(\Gamma_1^{(k-j)}+\Gamma_2^{(k-j)})\right]\ (k=3,4,5,\dots), \tag{6d}$$

and

$$\lambda^0:\Gamma_2^{(1)}=-\frac{i}{2}\mu Q_2Q_2^*, \tag{7a}$$

$$\lambda^{-1}:\Gamma_2^{(2)}=-\frac{i}{2}\left(\Gamma_{2x}^{(1)}-\frac{Q_{2x}}{Q_2}\Gamma_2^{(1)}\right)=-\frac{1}{4}\mu Q_2Q_{2x}^*, \tag{7b}$$

$$\begin{aligned}\lambda^{-2}:\Gamma_2^{(3)}&=\frac{i\mu}{8}\left[-\frac{Q_{2x}}{Q_2}\Gamma_2^{(2)}+\Gamma_{2x}^{(2)}+\mu\Gamma_2^{(1)}(\Gamma_1^{(1)}+\Gamma_2^{(1)})\right]\\&=\frac{i\mu}{8}(Q_2Q_{2xx}^*+\mu^2Q_2^2Q_2^{*2}+\mu^2Q_1Q_2Q_1^*Q_2^*),\end{aligned} \tag{7c}$$

$$\lambda^{-k}:\Gamma_2^{(k+1)}=-\frac{i}{2}\left[-\frac{Q_{2x}}{Q_2}\Gamma_2^{(k)}+\Gamma_{2x}^{(k)}+\mu\sum_{j=1}^{k-1}\Gamma_2^{(j)}(\Gamma_1^{(k-j)}+\Gamma_2^{(k-j)})\right]\ (k=3,4,5,\dots). \tag{7d}$$

In order to use the associated evolution equation Eq. (3), we consider the compatibility condition $(\frac{\psi_{1x}}{\psi_1})_t = (\frac{\psi_{1t}}{\psi_1})_x$. Using Eqs. (6) and (7), then equating the coefficients of the same power of λ again, the infinitely-many conservation laws for Eq. (1) are given as

$$\frac{\partial U_j}{\partial t} = \frac{\partial F_j}{\partial x} \quad (j = 1, 2, 3, \dots), \tag{8}$$

where U_j's and F_j's refer to conserved densities and conserved fluxes respectively.

The first conservation law is

$$U_1 = \Gamma_1^{(1)} + \Gamma_2^{(1)} = -\frac{i\mu}{2}(|Q_1|^2 + |Q_2|^2), \tag{9a}$$

$$\begin{aligned} F_1 &= 2(\Gamma_1^{(2)} + \Gamma_2^{(2)}) + i\left(\frac{Q_{1x}}{Q_1}\Gamma_1^{(1)} + \frac{Q_{2x}}{Q_2}\Gamma_2^{(1)}\right) - 2\beta x(\Gamma_1^{(1)} + \Gamma_2^{(1)}) \\ &= \frac{\mu}{2}[Q_1^* Q_{1x} + Q_2^* Q_{2x} - Q_1 Q_{1x}^* - Q_2 Q_{2x}^* + 2i\beta x(|Q_1|^2 + |Q_2|^2)]. \end{aligned} \tag{9b}$$

The second conservation law is

$$U_2 = \Gamma_1^{(2)} + \Gamma_2^{(2)} = -\frac{\mu}{4}(Q_1 Q_{1x}^* + Q_2 Q_{2x}^*), \tag{10a}$$

$$\begin{aligned} F_2 &= 2(\Gamma_1^{(3)} + \Gamma_2^{(3)}) + i\left(\frac{Q_{1x}}{Q_1}\Gamma_1^{(2)} + \frac{Q_{2x}}{Q_2}\Gamma_2^{(2)}\right) - 2\beta x(\Gamma_1^{(2)} + \Gamma_2^{(2)}) \\ &= \frac{i\mu}{4}\left[Q_1 Q_{1xx}^* + Q_2 Q_{2xx}^* + \mu^2(|Q_1|^2 + |Q_2|^2)^2 - \frac{i}{2}\beta x(Q_1 Q_{1x}^* + Q_2 Q_{2x}^*) - Q_{1x} Q_{1x}^* - Q_{2x} Q_{2x}^*\right]. \end{aligned} \tag{10b}$$

The following conservation laws hold

$$U_k = \Gamma_1^{(k)} + \Gamma_2^{(k)} \quad (k \geq 3), \tag{11a}$$

$$\begin{aligned} F_k &= 2(\Gamma_1^{(k+1)} + \Gamma_2^{(k+1)}) + i\left(\frac{Q_{1x}}{Q_1}\Gamma_1^{(k)} + \frac{Q_{2x}}{Q_2}\Gamma_2^{(k)}\right) - 2\beta x(\Gamma_1^{(k)} + \Gamma_2^{(k)}) \\ &= i\left[2\left(\frac{Q_{1x}}{Q_1}\Gamma_1^{(k)} + \frac{Q_{2x}}{Q_2}\Gamma_2^{(k)}\right) - (\Gamma_{1x}^{(k)} + \Gamma_{2x}^{(k)}) - \mu\sum_{j=1}^{k-1}(\Gamma_1^{(j)}\Gamma_1^{(k-j)} + 2\Gamma_1^{(j)}\Gamma_2^{(k-j)} + \Gamma_2^{(j)}\Gamma_2^{(k-j)}) + 2i\beta x(\Gamma_1^{(k)} + \Gamma_2^{(k)})\right], \end{aligned} \tag{11b}$$

where $\Gamma_1^{(k)}$ and $\Gamma_2^{(k)}$ can be directly derived from Eqs. (6d) and (7d).

3. Darboux transformation and nonautonomous soliton solutions

In this section, we will construct DT of the coupled GP Eq. (1) with nonisospectral parameter λ. Gu [30] found a unified approach to construct DT for the isospectral AKNS hierarchy, and this method had been employed in many integrable equations [9,36–38]. However, there are also other nonisospectral integrable systems [19,26]. Ciéslínski [39] gave the method to construct DT for a class of nonisospectral cases, which is greatly different from Gu's method. After that, Zhou [31,32] generalized Gu's formula to the nonisospectral AKNS hierarchy. Here, we use Zhou's method to construct DT for the coupled GP system (1).

Let $\Psi = (\psi[1](\xi_j), \phi[1](\xi_j), \chi[1](\xi_j))^T$ be a special vector solution of Lax pair (2)–(3) at $q_1 = q_1[0],\ q_2 = q_2[0]$ and $\lambda = \lambda_j = \frac{\alpha}{4\beta} + \xi_j e^{-2\beta t}$ $(j = 1, 2, 3, \dots, N)$.

Then, we give the first-step DT of the coupled nonisospectral GP system (1)

$$\Psi[1] = T[1]\Psi, \quad T[1] = \rho_1(\lambda)(\lambda I - H[1]\Lambda_1 H[1]^{-1}), \tag{12}$$

$$q_1[1] = q_1[0] + \frac{2i(\lambda_1^* - \lambda_1)\psi[1](\xi_1)\phi[1](\xi_1)^*}{\mu(|\psi[1](\xi_1)|^2 + |\phi[1](\xi_1)|^2 + |\chi[1](\xi_1)|^2)} e^{\frac{i\beta x^2}{2}}, \tag{13}$$

$$q_2[1] = q_2[0] + \frac{2i(\lambda_1^* - \lambda_1)\psi[1](\xi_1)\chi[1](\xi_1)^*}{\mu(|\psi[1](\xi_1)|^2 + |\phi[1](\xi_1)|^2 + |\chi[1](\xi_1)|^2)} e^{\frac{i\beta x^2}{2}}, \tag{14}$$

where I is the 3×3 identity matrix,

$$\begin{aligned}\rho_1(\lambda) &= [det(\lambda I - H[1]\Lambda_1 H[1]^{-1})]^{-\frac{1}{3}} \\ &= [(\lambda-\lambda_1)(\lambda-\lambda_1^*)^2]^{-\frac{1}{3}},\end{aligned} \tag{15}$$

$$H[1]=\begin{bmatrix}\psi[1](\xi_1) & \phi[1](\xi_1)^* & 0\\ \phi[1](\xi_1) & -\psi[1](\xi_1)^* & \chi[1](\xi_1)^*\\ \chi[1](\xi_1) & 0 & -\phi[1](\xi_1)^*\end{bmatrix},\quad \Lambda_1=\begin{bmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_1^* & 0\\ 0 & 0 & \lambda_1^*\end{bmatrix}. \tag{16}$$

The above DT should hold the two following relations $T[1]_x + T[1]U = U[1]T[1]$ and $T[1]_t + T[1]V = V[1]T[1]$, where $U[1]$ and $V[1]$ enjoys the same forms as U and V except that $(q_1[0], q_2[0])$ is replaced with $(q_1[1], q_2[1])$. The validity of the above two relations have been verified by us though Maple software.

Similarly, the first-step DT for the one-component GP equation can be expressed as

$$\widetilde{\Psi} = T\Psi_0,\quad T=\rho(\lambda)(\lambda I - H\Lambda_1 H^{-1}), \tag{17}$$

$$q[1]=q[0]+\frac{2i(\lambda_1^*-\lambda_1)\psi(\xi_1)\phi(\xi_1)^*}{\mu(|\psi(\xi_1)|^2+|\phi(\xi_1)|^2}e^{\frac{i\beta x^2}{2}}, \tag{18}$$

where I is the 2×2 identity matrix and $\Psi_0 = (\psi(\xi_1), \phi(\xi_1))^T$,

$$\rho(\lambda)=[(\lambda-\lambda_1)(\lambda-\lambda_1^*)]^{-\frac{1}{2}}, \tag{19}$$

$$H=\begin{bmatrix}\psi(\xi_1) & \phi(\xi_1)^*\\ \phi(\xi_1) & -\psi(\xi_1)^*\end{bmatrix},\quad \Lambda_1=\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_1^*\end{bmatrix}. \tag{20}$$

In [19], $\rho(\lambda)$ was not included in the DT of the one-component GP equation, which made their DT can not admit the t-part of the Lax pair. Without iterating the DT, the expressions of the first-step solutions $q[1]$ are right, such as one-soliton and one-breather. In [19], based on their incorrect DT, the two-soliton is not correct, besides, the second-order rogue wave constructed by the generalized DT [38] is not reasonable. It is necessary for us to reconstruct the DTs of the coupled system (1) and its multi-component generalization.

The second-step DT is

$$\Psi[2]=T[2]T[1]\Psi,\quad T[2]=\rho_2(\lambda)(\lambda I - H[2]\Lambda_2 H[2]^{-1}), \tag{21}$$

$$q_1[2]=q_1[1]+\frac{2i(\lambda_2^*-\lambda_2)\psi[2](\xi_2)\phi[2](\xi_2)^*}{\mu(|\psi[2](\xi_2)|^2+|\phi[2](\xi_2)|^2+|\chi[2](\xi_2)|^2)}e^{\frac{i\beta x^2}{2}}, \tag{22}$$

$$q_2[2]=q_2[1]+\frac{2i(\lambda_2^*-\lambda_2)\psi[2](\xi_2)\chi[2](\xi_2)^*}{\mu(|\psi[2](\xi_2)|^2+|\phi[2](\xi_2)|^2+|\chi[2](\xi_2)|^2)}e^{\frac{i\beta x^2}{2}}, \tag{23}$$

with

$$T[1]|_{\lambda=\lambda_2}(\psi[1](\xi_2), \phi[1](\xi_2), \chi[1](\xi_2))^T = (\psi[2](\xi_2), \phi[2](\xi_2), \chi[2](\xi_2))^T, \tag{24}$$

$$\rho_2(\lambda)=[(\lambda-\lambda_2)(\lambda-\lambda_2^*)^2]^{-\frac{1}{3}}, \tag{25}$$

$$H[1]=\begin{bmatrix}\psi[1](\xi_2) & \phi[1](\xi_2)^* & 0\\ \phi[1](\xi_2) & -\psi[1](\xi_2)^* & \chi[1](\xi_2)^*\\ \chi[1](\xi_2) & 0 & -\phi[1](\xi_2)^*\end{bmatrix},\quad \Lambda_2=\begin{bmatrix}\lambda_2 & 0 & 0\\ 0 & \lambda_2^* & 0\\ 0 & 0 & \lambda_2^*\end{bmatrix}. \tag{26}$$

The N-step DT is

$$\Psi[N]=T[N]T[N-1]\cdots T[2]T[1]\Psi,\quad T[N]=\rho_N(\lambda)(\lambda I - H[N]\Lambda_N H[N]^{-1}), \tag{27}$$

$$q_1[N]=q_1[N-1]+\frac{2i(\lambda_N^*-\lambda_N)\psi[N-1](\xi_{N-1})\phi[N-1](\xi_{N-1})^*}{\mu(|\psi[N-1](\xi_{N-1})|^2+|\phi[N-1](\xi_{N-1})|^2+|\chi[N-1](\xi_{N-1})|^2)}e^{\frac{i\beta x^2}{2}}, \tag{28}$$

$$q_2[N] = q_2[N-1] + \frac{2i(\lambda_N^* - \lambda_N)\psi[N-1](\xi_{N-1})\chi[N-1](\xi_{N-1})^*}{\mu(|\psi[N-1](\xi_{N-1})|^2 + |\phi[N-1](\xi_{N-1})|^2 + |\chi[N-1](\xi_{N-1})|^2)} e^{\frac{i\beta x^2}{2}}, \tag{29}$$

with

$$\begin{aligned} &T[N-2]|_{\lambda=\lambda_{N-1}} T[N-3]|_{\lambda=\lambda_{N-2}} \cdots T[2]|_{\lambda=\lambda_3} T[1]|_{\lambda=\lambda_2} (\psi[1](\xi_{N-1}), \phi[1](\xi_{N-1}), \chi[1](\xi_{N-1}))^T \\ &\quad = (\psi[N-1](\xi_{N-1}), \phi[N-1](\xi_{N-1}), \chi[N-1](\xi_{N-1}))^T, \end{aligned} \tag{30}$$

$$\rho_j(\lambda) = [(\lambda - \lambda_j)(\lambda - \lambda_j^*)^2]^{-\frac{1}{3}}, \tag{31}$$

$$H[j] = \begin{bmatrix} \psi[1](\xi_j) & \phi[1](\xi_j)^* & 0 \\ \phi[1](\xi_j) & -\psi[1](\xi_j)^* & \chi[1](\xi_j)^* \\ \chi[1](\xi_j) & 0 & -\phi[1](\xi_j)^* \end{bmatrix}, \quad \Lambda_j = \begin{bmatrix} \lambda_j & 0 & 0 \\ 0 & \lambda_j^* & 0 \\ 0 & 0 & \lambda_j^* \end{bmatrix} (3 \le j \le N), \tag{32}$$

where $\lambda_i \neq \lambda_j (i \neq j)$. Owing to the $\rho_j = [(\lambda - \lambda_j)(\lambda - \lambda_j^*)^2]^{-\frac{1}{3}}$ $(j = 1, 2, 3, \ldots, N)$ exist in the j-step DT of Eq. (1), here, the determinant representations of the above DT and the solutions of Eq. (1) can not be given directly by Cramer's rule.

In the following contents, we will give nonautonomous solitons of the coupled system (1) though the above DT. For convenience, we chose the trivial seed solution $q_1[0] = 0$ and $q_2[0] = 0$ at $\xi = \xi_j$ $(j = 1, 2, 3, \ldots, N)$. Thus, the special solutions of Lax pair (2)–(3) can be given as

$$\psi[1](\xi_j) = e^{\theta_j}, \quad \phi[1](\xi_j) = e^{-\theta_j}, \quad \chi[1](\xi_j) = -2e^{-\theta_j}, \tag{33}$$

where

$$\theta_j = -i(\frac{\alpha}{4\beta} + \xi_j e^{-2\beta t})x + \frac{i}{8\beta^2}(-\alpha^2 t + 4\xi_j^2 \beta e^{-4\beta t} + 4\alpha \xi_j e^{-2\beta t}).$$

In order to get nonautonomous one-soliton for Eq. (1), we substitute Eq. (33) into Eqs. (13) and (14). The expressions of one-soliton can be presented as follows

$$q_1[1] = \frac{2i(\lambda_1^* - \lambda_1)\psi[1](\xi_1)\phi[1](\xi_1)^*}{\mu(|\psi[1](\xi_1)|^2 + |\phi[1](\xi_1)|^2 + |\chi[1](\xi_1)|^2)} e^{\frac{i\beta x^2}{2}} = \frac{2i(\xi_1^* - \xi_1)e^{\theta_1 - \theta_1^*}}{\mu(e^{\theta_1 + \theta_1^*} + 5e^{-\theta_1 - \theta_1^*})} e^{\frac{i\beta x^2}{2} - 2\beta t}, \tag{34}$$

$$q_2[1] = \frac{2i(\lambda_1^* - \lambda_1)\psi[1](\xi_1)\chi[1](\xi_1)^*}{\mu(|\psi[1](\xi_1)|^2 + |\phi[1](\xi_1)|^2 + |\chi[1](\xi_1)|^2)} e^{\frac{i\beta x^2}{2}} = -\frac{4i(\xi_1^* - \xi_1)e^{\theta_1 - \theta_1^*}}{\mu(e^{\theta_1 + \theta_1^*} + 5e^{-\theta_1 - \theta_1^*})} e^{\frac{i\beta x^2}{2} - 2\beta t}. \tag{35}$$

Then the modules of $q_1[1]$ and $q_2[1]$ hold

$$|q_1[1]| = \frac{4e^{-2\beta t}|Im(\xi_1)|}{|\mu|\left(e^{-\frac{e^{-2\beta t}Im(\xi_1)(2e^{-2\beta t}Re(\xi_1)\beta - 2x\beta^2 + \alpha)}{\beta^2}} + 5e^{\frac{e^{-2\beta t}Im(\xi_1)(2e^{-2\beta t}Re(\xi_1)\beta - 2x\beta^2 + \alpha)}{\beta^2}}\right)}, \tag{36}$$

$$|q_2[1]| = \frac{8e^{-2\beta t}|Im(\xi_1)|}{|\mu|\left(e^{-\frac{e^{-2\beta t}Im(\xi_1)(2e^{-2\beta t}Re(\xi_1)\beta - 2x\beta^2 + \alpha)}{\beta^2}} + 5e^{\frac{e^{-2\beta t}Im(\xi_1)(2e^{-2\beta t}Re(\xi_1)\beta - 2x\beta^2 + \alpha)}{\beta^2}}\right)}, \tag{37}$$

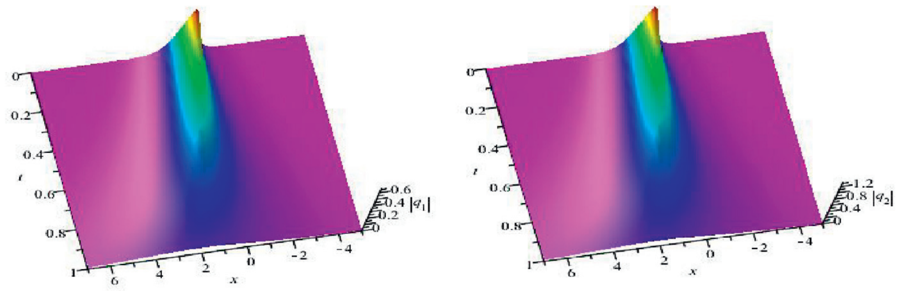
where $\xi_1 = Re(\xi_1) + iIm(\xi_1)$ ($Re(\xi_1)$ and $Im(\xi_1)$ are all real numbers). Then we establish the following equality

$$2e^{-2\beta t}Re(\xi_1)\beta - 2x\beta^2 + \alpha = 0. \tag{38}$$

From Eq. (38), the propagation velocity of one-soliton holds

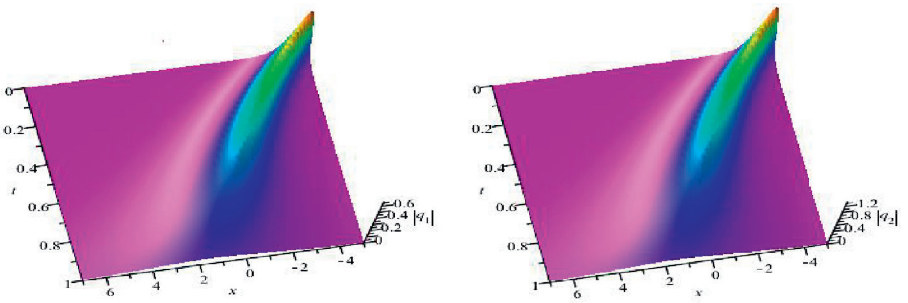
$$v = \frac{dx}{dt} = -2Re(\xi_1)e^{-2\beta t}.$$

It can be found that v is dependent of time variable t and $Re(\xi_1)$, when $\beta > 0$, the absolute value of v decays exponentially as t increases, while increases exponentially as t increases if $\beta < 0$. Additionally, $Re(\xi_1)$ determines both the direction of propagation of one-soliton and the value of v. When $Re(\xi_1)$=0, the propagation velocity of one-soliton is zero, which indicates



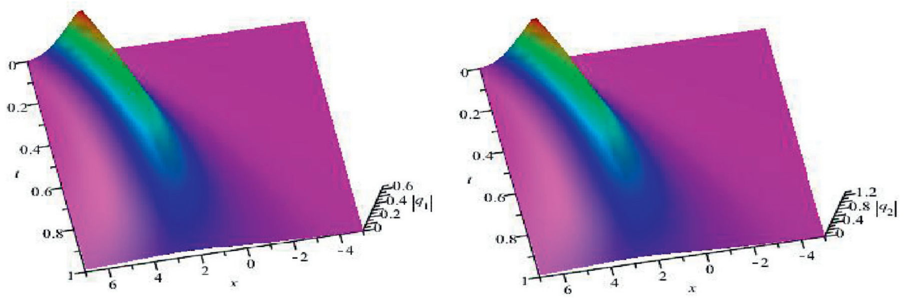
(a) (b)

Fig. 1. Evolution plot of the nonautonomous one-soliton with parameters chosen by $\alpha = \frac{1}{2},\ \beta = 1,\ \mu = 2,\ Re(\xi_1) = 0,\ Im(\xi_1) = \frac{3}{2}$, (a) q_1, (b) q_2.



(a) (b)

Fig. 2. Evolution plot of the nonautonomous one-soliton with parameters chosen by $\alpha = \frac{1}{2},\ \beta = 1,\ \mu = 2,\ Re(\xi_1) = -5,\ Im(\xi_1) = \frac{3}{2}$, (a) q_1, (b) q_2.



(a) (b)

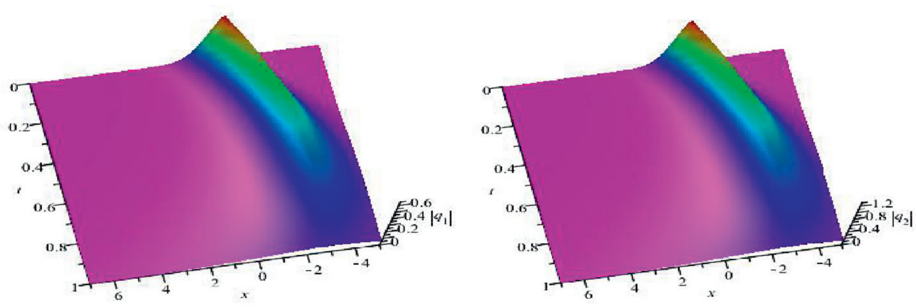
Fig. 3. Evolution plot of the nonautonomous one-soliton with parameters chosen by $\alpha = \frac{1}{2},\ \beta = 1,\ \mu = 2,\ Re(\xi_1) = 5,\ Im(\xi_1) = \frac{3}{2}$, (a) q_1, (b) q_2.

this soliton is stationary, and this phenomenon can be shown in Fig. 1. When $Re(\xi_1) > 0$, the nonautonomous one-soliton propagates along the negative direction of x-axis; if $Re(\xi_1) < 0$, it propagates along the positive direction of x-axis. These interesting results are shown in Figs. 2 and 3. With the value of $|\mu|$ increasing, the amplitude of one-soliton decreases. From Figs. 3 and 4, we can observe that the parameter α affects the localization of one-soliton.

Through Eqs. (12), (22) and (23), the nonautonomous two-soliton of the coupled GP system (1) can be written as

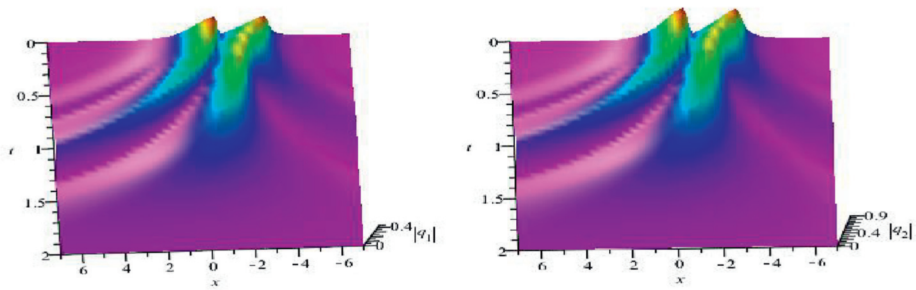
$$q_1[2] = \frac{2i}{\mu}\left(\frac{(\xi_1^* - \xi_1)e^{\theta_1 - \theta_1^*}}{e^{\theta_1 + \theta_1^*} + 5e^{-\theta_1 - \theta_1^*}} + \frac{(\xi_2^* - \xi_2)G_1}{F}\right)e^{\frac{i\beta x^2}{2} - 2\beta t}, \tag{39}$$

$$q_2[2] = \frac{2i}{\mu}\left(-2\frac{(\xi_1^* - \xi_1)e^{\theta_1 - \theta_1^*}}{e^{\theta_1 + \theta_1^*} + 5e^{-\theta_1 - \theta_1^*}} + \frac{(\xi_2^* - \xi_2)G_2}{F}\right)e^{\frac{i\beta x^2}{2} - 2\beta t}, \tag{40}$$



(a) (b)

Fig. 4. Evolution plot of the nonautonomous one-soliton with parameters chosen by $\alpha = -11,\ \beta = 1,\ \mu = 2,\ Re(\xi_1) = 5,\ Im(\xi_1) = \frac{3}{2}$, (a) q_1, (b) q_2.



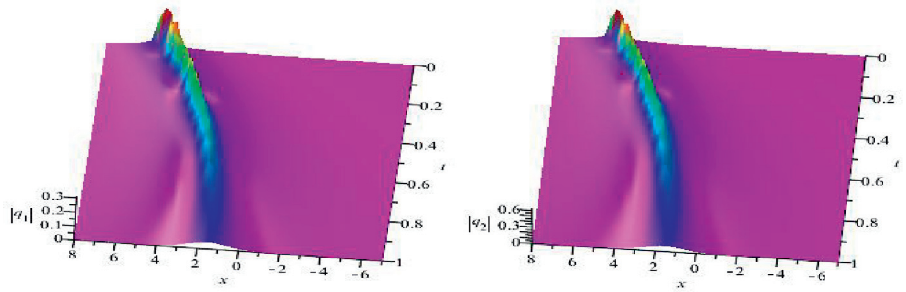
(a) (b)

Fig. 5. Evolution plot of the nonautonomous two-soliton with parameters chosen by $\alpha = -3,\ \beta = 1,\ \mu = 2,\ Re(\xi_1) = 1,\ Im(\xi_1) = 1,\ Re(\xi_2) = -1,$ $Im(\xi_2) = 1$, (a) q_1, (b) q_2.

where

$$
\begin{aligned}
G_1 = {} & 5(-\xi_1^{*2} + \xi_1^*\xi_1 + \xi_1^*\xi_2 - \xi_1\xi_2)e^{-2\theta_1^*+\theta_2^*+\theta_2} + 5(\xi_1^*\xi_2^* - 2\xi_1^*\xi_1 - \xi_1^*\xi_2 + \xi_2^*\xi_1 + \xi_1\xi_2)e^{-\theta_2^*+\theta_2} \\
& +25(\xi_1^{*2} - \xi_1^*\xi_2^* - \xi_1^*\xi_1 + \xi_2^*\xi_1)e^{-\theta_2^*-2\theta_1^*-\theta_2} + (-\xi_1^*\xi_1 - \xi_1^*\xi_2 + \xi_1^2 + \xi_1\xi_2)e^{2\theta_1+\theta_2^*+\theta_2} \\
& +5(\xi_1^{*2} - 2\xi_1^*\xi_1 + \xi_1^2)e^{2\theta_1-2\theta_1^*+\theta_2^*-\theta_2} + 25(-\xi_1^{*2} + \xi_1^*\xi_2^* + \xi_1^*\xi_2 - \xi_2^*\xi_2)e^{-2\theta_1-\theta_2^*-2\theta_1^*+\theta_2} \\
& +(\xi_2^*\xi_1 + \xi_2^*\xi_2 - \xi_1^2 - \xi_1\xi_2)e^{2\theta_1+2\theta_1^*-\theta_2^*+\theta_2} + 5(-\xi_1^*\xi_2^* + \xi_1^*\xi_1 + \xi_2^*\xi_1 - \xi_1^2)e^{2\theta_1-\theta_2^*-\theta_2}, \\
G_2 = {} & 10(\xi_1^{*2} - \xi_1^*\xi_1 - \xi_1^*\xi_2 + \xi_1\xi_2)e^{-2\theta_1^*+\theta_2^*+\theta_2} + 2(\xi_1^*\xi_1 + \xi_1^*\xi_2 - \xi_1^2 - \xi_1\xi_2)e^{2\theta_1+\theta_2^*+\theta_2} \\
& +10(-\xi_1^{*2} + 2\xi_1^*\xi_1 - \xi_1^2)e^{2\theta_1-2\theta_1^*+\theta_2^*-\theta_2} + 50(-\xi_1^{*2} + \xi_1^*\xi_2^* + \xi_1^*\xi_1 - \xi_2^*\xi_1)e^{-\theta_2^*-2\theta_1^*-\theta_2} \\
& +50(\xi_1^{*2} - \xi_1^*\xi_2^* - \xi_1^*\xi_2 + \xi_2^*\xi_2)e^{-2\theta_1-\theta_2^*-2\theta_1^*+\theta_2} + 10(-\xi_1^*\xi_1 + \xi_1^*\xi_2 + \xi_1^2 - \xi_1\xi_2)e^{2\theta_1-\theta_2^*-\theta_2} \\
& +10(2\xi_1^*\xi_1 - \xi_2^*\xi_1 - \xi_2^*\xi_2 - \xi_1\xi_2 + \xi_2^2)e^{-\theta_2^*+\theta_2} + 2(\xi_1^2 - \xi_2^2)e^{2\theta_1+2\theta_1^*-\theta_2^*+\theta_2}, \\
F = {} & 25(-\xi_1^*\xi_2^* + \xi_1^*\xi_2 + \xi_2^*\xi_1 - \xi_1\xi_2)e^{-2\theta_1^*-\theta_2+\theta_2^*} - (\xi_1^*\xi_1 + \xi_1^*\xi_2 + \xi_2^*\xi_1 + \xi_2^*\xi_2)e^{2\theta_1+2\theta_1^*+\theta_2+\theta_2^*} \\
& +125(-\xi_1^*\xi_1 + \xi_1^*\xi_2 + \xi_2^*\xi_1 - \xi_2^*\xi_2)e^{-2\theta_1-\theta_2^*-2\theta_1^*-\theta_2} + (\xi_1^*\xi_2^* - 5\xi_1^*\xi_1 + 4\xi_1^*\xi_2 - \xi_2^*\xi_2 + 5\xi_1\xi_2 - 4\xi_2^2)e^{2\theta_1+2\theta_1^*-\theta_2^*-\theta_2} \\
& +5(5\xi_1\xi_2 - 10\xi_1^*\xi_1 + 5\xi_1^*\xi_2 + \xi_1^*\xi_1 - 6\xi_2^*\xi_2 + 9\xi_1\xi_2 - 4\xi_2^2)e^{-\theta_2-\theta_2^*} + 25(-\xi_1^*\xi_2^* + \xi_1^*\xi_2 + \xi_2^*\xi_1 - \xi_1\xi_2)e^{-2\theta_1+\theta_2-\theta_2^*} \\
& +(-\xi_1^*\xi_2^* - 9\xi_1^*\xi_2 + \xi_2^*\xi_1 + 9\xi_1\xi_2)e^{2\theta_1^*+\theta_2-\theta_2^*} + 5(\xi_1^*\xi_2^* + \xi_1^*\xi_2 - \xi_2^*\xi_1 - \xi_1\xi_2)e^{2\theta_1-\theta_2+\theta_2^*} + 25(\xi_1^*\xi_2^* - \xi_1^*\xi_1 - \xi_2^*\xi_2 \\
& +\xi_1\xi_2)e^{-2\theta_1-2\theta_1^*+\theta_2+\theta_2^*} + 5(-\xi_1^*\xi_2^* - 2\xi_1^*\xi_1 + \xi_1^*\xi_2 + \xi_2^*\xi_1 + 2\xi_2^*\xi_2 - \xi_1\xi_2)e^{\theta_2+\theta_2^*}.
\end{aligned}
$$

From the above two Eqs. (39) and (40), we can find that the parameter μ affects the amplitudes of the nonautonomous two-soliton. Similarly, the velocities of two solitons can be written as $v_j = -2Re(\xi_j)e^{-2\beta t}$ $(j = 1, 2)$. When $v_1 \neq v_2\,(Re(\xi_1) \neq Re(\xi_2))$, these two solitons will interact with each other, which include overtaking and head-on interactions, see Fig. 5. Setting $v_1 = v_2$ $(Re(\xi_1) = Re(\xi_2))$, the bound state [40] of the two-solition is obtained and shown in Fig. 6. Meanwhile, by iterating N times of the above DT, we can get nonautonomous N-soliton of Eq. (1).



(a) (b)

Fig. 6. Evolution plot of the two-soliton bound state with parameters chosen by $\alpha = \frac{1}{2},\ \beta = 1,\ \mu = 10,\ Re(\xi_1) = 5,\ Im(\xi_1) = 4,\ Re(\xi_2) = 5,\ Im(\xi_2) = \frac{1}{2}$, (a) q_1, (b) q_2.

4. Nonautonomous breather and rogue wave

In this section, we will give breather and rogue wave solutions for the coupled nonisospectral GP system (1) through the above DT. In order to obtain this kind of rational solutions of Eq. (1), we consider the following seed solutions with nonzero background and the amplitude of background wave is the function of t,

$$q_1[0] = d_1 e^{-2\beta t + i(\frac{\beta}{2}x^2+\theta)}, \quad q_2[0] = d_2 e^{-2\beta t + i(\frac{\beta}{2}x^2+\theta)}, \tag{41}$$

where

$$\theta = (-\frac{\alpha}{2\beta} + ce^{-2\beta t})x - \frac{1}{4\beta^2}e^{-4\beta t}[2\beta\mu^2(d_1^2+d_2^2) - \beta c^2 + 2\alpha c e^{2\beta t} + \alpha^2 e^{4\beta t} t], \tag{42}$$

here, $d_1, d_2,$ and c are all real constants ($d_1 \neq d_2$). By choosing the spectral parameter $\lambda = \lambda_1 = \frac{\alpha}{4\beta} + \xi_1 e^{-2\beta t}$ and substituting (41) into Lax pair (2)-(3), the special vector solution of the Lax pair can be expressed as

$$\Psi_1 = \begin{pmatrix} \frac{i}{2}[c_2(\beta c + 2\xi_1\beta - \eta)e^{m_2} + c_3(\beta c + 2\xi\beta + \eta)e^{m_3}]e^{\frac{2i}{3}\theta} \\ [-c_1 d_2 e^{m_1} + \mu d_1 \beta(c_2 e^{m_2} + c_3 e^{m_3})]e^{-\frac{i}{3}\theta} \\ [c_1 d_1 e^{m_1} + \mu d_2 \beta(c_2 e^{m_2} + c_3 e^{m_3})]e^{-\frac{i}{3}\theta} \end{pmatrix}, \tag{43}$$

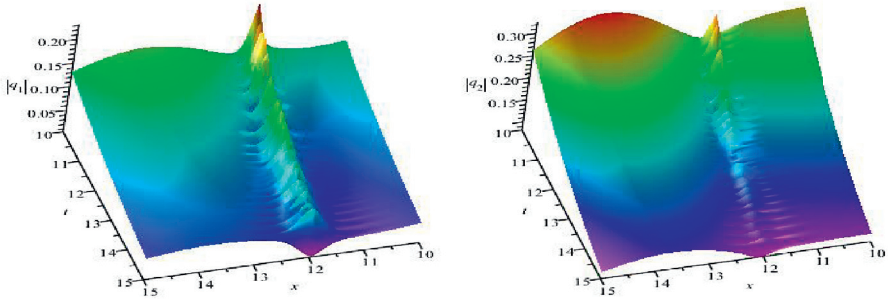
where

$$\begin{aligned}
\eta &= \sqrt{\beta^2[4\mu^2(d_1^2+d_2^2) + (c+2c\xi_1)^2]}, \\
m_1 &= [(\frac{i}{3}c + i\xi_1)e^{-2\beta t} + \frac{i\alpha}{12\beta}]x - \frac{i\alpha}{6\beta^2}(3\xi_1 + c)e^{-2\beta t} + \frac{i}{12\beta}[-2\mu^2(d_1^2+d_2^2) + c^2 - 6\xi_1^2]e^{-4\beta t} + \frac{i\alpha^2}{24\beta^2}t, \\
m_2 &= [-\frac{i}{6}(c - \frac{3\eta}{\beta})e^{-2\beta t} + \frac{i\alpha}{12\beta}]x + \frac{i\alpha}{12\beta^2}(\beta c - 3\eta)e^{-2\beta t} + \frac{i}{24\beta^2}[2\beta\mu^2(d_1^2+d_2^2) - \beta c^2 + 3\eta c - 6\eta\xi_1]e^{-4\beta t} \\
&\quad + \frac{i\alpha^2}{24\beta^2}t, \\
m_3 &= [-\frac{i}{6}(c + \frac{3\eta}{\beta})e^{-2\beta t} + \frac{i\alpha}{12\beta}]x + \frac{i\alpha}{12\beta^2}(\beta c + 3\eta)e^{-2\beta t} + \frac{i}{24\beta^2}[2\beta\mu^2(d_1^2+d_2^2) - \beta c^2 - 3\eta c + 6\eta\xi_1]e^{-4\beta t} \\
&\quad + \frac{i\alpha^2}{24\beta^2}t,
\end{aligned}$$

with $c_1, c_2,$ and c_3 being all arbitrary real constants. In order to construct nonautonomous breather, we should make η be a pure imaginary number. Through choosing $\xi_1 = -\frac{c}{2} + i\mu Im(\xi_1)$ with $Im(\xi_1)$ being a real constant, the expression of η can be simplified as $\eta = 2\sqrt{\beta^2\mu^2(d_1^2 + d_2^2 - Im(\xi_1)^2)}$ ($Im(\xi_1) > d_1^2 + d_2^2$). Substituting Eqs. (41) and (43) into Eqs. (13) and (14) respectively, the compact expressions of one-breather can be written as

$$q_1[1] = (d_1 + \frac{4Im(\xi_1)H_1}{H_3})e^{-2\beta t + i(\frac{\beta}{2}x^2+\theta)}, \tag{44}$$

$$q_2[1] = (d_2 + \frac{4Im(\xi_1)H_2}{H_3})e^{-2\beta t + i(\frac{\beta}{2}x^2+\theta)}, \tag{45}$$



(a) (b)

Fig. 7. Evolution plot of the nonautonomous one-breather with parameters chosen by $\alpha = \frac{1}{2},\ \beta = \frac{1}{10},\ \mu = 10,\ c_1 = 1,\ c_2 = 1,\ c_3 = 1,\ c = 1,\ \alpha = \frac{1}{4},\ Im(\xi_1) = 5$, (a) q_1, (b) q_2.

where

$$
\begin{aligned}
H_1 &= -(\mu\beta Im(\xi_1) + \eta_0)(\mu d_1\beta c_2 c_3 e^{m_2+m_3} + \mu\beta d_1 c_3^2 e^{m_3+m_3^*} - d_2 c_1 c_3 e^{m_1^*+m_3}) + (\eta_0 - \mu\beta Im(\xi_1))(\mu d_1\beta c_2 c_3 e^{m_2+m_3^*} \\
&\quad + \mu d_1\beta c_2^2 e^{2m_2} - d_2 c_1 c_2 e^{m_1^*+m_2}),\\
H_2 &= -(\mu\beta Im(\xi_1) + \eta_0)(\mu d_2\beta c_2 c_3 e^{m_2^*+m_3} + \mu d_2\beta c_3^2 e^{m_3+m_3^*} + d_1 c_1 c_3 e^{m_1^*+m_3}) + (\eta_0 - \mu\beta Im(\xi_1))(\mu d_2\beta c_2 c_3 e^{m_3+m_3^*} \\
&\quad + \mu d_2\beta c_2^2 e^{m_2+m_2^*} + d_1 c_1 c_2 e^{m_1^*+m_2}),\\
H_3 &= c_1^2(d_1^2 + d_2^2)e^{m_1+m_1^*} + \mu c_2^2\beta(2\mu\beta Im(\xi_1)^2 - \mu d_1^2\beta - 2Im(\xi_1)\eta_0)e^{m_2+m_2^*} + 2\mu\beta c_3^2 Im(\xi_1)(\mu\beta Im(\xi_1) + \eta_0)e^{m_3+m_3^*} \\
&\quad + 2\mu^2\beta^2 c_2 c_3(d_1^2 + d_2^2)e^{m_2+m_3^*} + \mu^2\beta^2 c_2 c_3(d_1^2 + 2d_2^2)e^{m_2^*+m_3} + \mu^2 d_1^2\beta^2 c_2^2 e^{2m_2} - \mu\beta d_1 d_2 c_1 c_2 e^{m_1+m_2} \\
&\quad + \mu^2 d_1^2\beta^2 c_2 c_3 e^{m_2+m_3} + \mu\beta d_1 d_2 c_1 c_2 e^{m_1+m_2^*},
\end{aligned}
$$

with $\eta_0 = \sqrt{\beta^2\mu^2(Im(\xi_1)^2 - d_1^2 - d_2^2)}$.

From Eqs. (44) and (45), we find that the breather of this coupled GP equations has some deformations along the direction of t and its amplitude becomes small till being zero as t increases, see Fig. 7. Furthermore, the breather in this couple GP system has a curved background which is greatly determined by the parameter β. This type of breather is very different from the one in the autonomous system, such as NLS equation [4] and [5]. Besides, this kind of breather was also constructed in [20].

In order to obtain the rogue wave solutions of Eq. (1), we should choose an appropriate spectral ξ_1 and set $\eta \to 0$. For convenience, we choose $\beta > 0,\ \mu > 0,\ c = 0,\ c_1 = 0,\ \xi_1 = ih$ (h is a real constant) in Eq. (43) and get $\eta = 2i\beta\sqrt{h^2 - \mu^2(d_1^2 + d_2^2)}$ ($h > \sqrt{\mu^2(d_1^2 + d_2^2)}$). By fixing appropriate values of c_2 and c_3, the special vector solution of Lax pair (2)–(3) holds

$$
\Psi_2 = \begin{pmatrix} \left(r_2 e^{A} - r_1 e^{-A}\right)e^{\frac{2i}{3}\theta_0 + B} \\ \rho_1\left(r_2 e^{-A} - r_1 e^{A}\right)e^{-\frac{i}{3}\theta_0 + B} \\ \rho_2\left(r_2 e^{-A} - r_1 e^{A}\right)e^{-\frac{i}{3}\theta_0 + B} \end{pmatrix}, \tag{46}
$$

where

$$
\begin{aligned}
r_1 &= \frac{\sqrt{h - \sqrt{h^2 - \mu^2(d_1^2 + d_2^2)}}}{\sqrt{h^2 - \mu^2(d_1^2 + d_2^2)}}, \quad r_2 = \frac{\sqrt{h + \sqrt{h^2 - \mu^2(d_1^2 + d_2^2)}}}{\sqrt{h^2 - \mu^2(d_1^2 + d_2^2)}},\\
\rho_1 &= \frac{d_1}{\sqrt{d_1^2 + d_2^2}}, \quad \rho_2 = \frac{d_2}{\sqrt{d_1^2 + d_2^2}}, \quad A = \sqrt{h^2 - \mu^2(d_1^2 + d_2^2)}e^{-2\beta t}\left(x - \frac{\alpha + ih\beta e^{-2\beta t}}{2\beta^2}\right),\\
B &= \frac{i}{24\beta^2}\left[2\alpha x + \alpha^2 t + 2\mu^2(d_1^2 + d_2^2)e^{-4\beta t}\right], \quad \theta_0 = -\frac{\alpha}{2\beta}x - \frac{e^{-4\beta t}}{4\beta^2}\left[2\beta\mu^2(d_1^2 + d_2^2) + \alpha^2 e^{4\beta t} t\right].
\end{aligned}
$$

In Eq. (46), taking $h \to \sqrt{\mu^2(d_1^2+d_2^2)}$, a new vector solution of Lax pair (2)–(3) can be given as

$$\Psi_3 = \begin{pmatrix} \left(\frac{1}{\sqrt{\mu\sqrt{d_1^2+d_2^2}}} + \eta_1\right)e^{\theta_1} \\ \frac{1}{\sqrt{d_1^2+d_2^2}}\left(d_1\eta_1 + \frac{1}{\sqrt{\mu\sqrt{d_1^2+d_2^2}}}\right)e^{\theta_2} \\ \frac{1}{\sqrt{d_1^2+d_2^2}}\left(d_2\eta_1 + \frac{1}{\sqrt{\mu\sqrt{d_1^2+d_2^2}}}\right)e^{\theta_2} \end{pmatrix}, \tag{47}$$

where

$$\begin{aligned} \eta_1 &= 2\sqrt{\mu\sqrt{d_1^2+d_2^2}}e^{-2\beta t}\left(x - \frac{\alpha + i\sqrt{d_1^2+d_2^2}\mu\beta e^{-2\beta t}}{2\beta^2}\right), \\ \theta_1 &= -\frac{i}{24\beta^2}[2\mu^2(4\beta-1)(d_1^2+d_2^2)e^{-4\beta t} + 3\alpha^2 t + 8\alpha\beta x - 2\alpha x], \\ \theta_2 &= \frac{i}{24\beta^2}[2\mu^2(2\beta+1)(d_1^2+d_2^2)e^{-4\beta t} + 3\alpha^2 t + 4\alpha\beta x + 2\alpha x]. \end{aligned}$$

Finally, substituting the eigenfunction (47) into Eqs. (13) and (14), the nonautonomous first-order rogue wave solution can be derived in the following form

$$q_1[1] = \frac{L_1}{L}e^{i\theta_3}, \tag{48}$$

$$q_2[1] = \frac{L_2}{L}e^{i\theta_3}, \tag{49}$$

with

$$\begin{aligned} \theta_3 &= \frac{\beta^2}{2}x^2 - \frac{\alpha}{2\beta}x - \frac{e^{-4\beta t}}{4\beta^2}[2\beta\mu^2(d_1^2+d_2^2) + \alpha^2 e^{4\beta t}t], \\ L &= (4\beta^4\mu^2x^2d_1^2 + 4\beta^4\mu^2x^2d_2^2 - 4\alpha\beta^2\mu^2xd_1^2 - 4\alpha\beta^2\mu^2xd_2^2 + \alpha^2\mu^2d_1^2 + \alpha^2\mu^2d_2^2)e^{4\beta t} + \mu^4d_1^4\beta^2 + 2\mu^4d_1^2d_2^2\beta^2 \\ &\quad + \mu^4d_2^4\beta^2 + e^{8\beta t}\beta^4, \\ L_1 &= (4\beta^4\mu^2x^2d_1^3 + 4\beta^4\mu^2x^2d_1d_2^2 - 4\alpha\beta^2\mu^2xd_1^3 - 4\alpha\beta^2\mu^2xd_1d_2^2 + \alpha^2\mu^2d_1^3 + \alpha^2\mu^2d_1d_2^2)e^{4\beta t} + (-8d_1^3\beta^4\mu^3x^2 \\ &\quad - 8d_1\beta^4\mu^3x^2d_2^2 - 4id_1^3\beta^3\mu^3 - 4id_1\beta^3\mu^3d_2^2 + 8d_1^3\alpha\beta^2\mu^3x + 8d_1\alpha\beta^2\mu^3xd_2^2 - 2d_1^3\alpha^2\mu^3 - 2d_1\alpha^2\mu^3d_2^2)e^{2\beta t} \\ &\quad - (2\beta^2\mu^5d_1^5 + 4\beta^2\mu^5d_1^3d_2^2 + 2\beta^2\mu^5d_1d_2^4)e^{-2\beta t} + 2d_1\mu\beta^4e^{6\beta t} + d_1\beta^4e^{8\beta t} + d_1\mu^4d_2^4\beta^2 + d_1^5\mu^4\beta^2 + 2d_1^3\mu^4d_2^2\beta^2, \\ L_2 &= (4\beta^4\mu^2x^2d_1^2d_2 + 4\beta^4\mu^2x^2d_2^3 - 4\alpha\beta^2\mu^2xd_1^2d_2 - 4\alpha\beta^2\mu^2xd_2^3 + \alpha^2\mu^2d_1^2d_2 + \alpha^2\mu^2d_2^3)e^{4\beta t} + (-8d_2\beta^4\mu^3x^2d_1^2 \\ &\quad - 8d_2^3\beta^4\mu^3x^2 - 4id_2\beta^3\mu^3d_1^2 - 4id_2^3\beta^3\mu^3 + 8d_2\alpha\beta^2\mu^3xd_1^2 + 8d_2^3\alpha\beta^2\mu^3x - 2d_2\alpha^2\mu^3d_1^2 - 2d_2^3\alpha^2\mu^3)e^{2\beta t} \\ &\quad - (2\beta^2\mu^5d_1^4d_2 + 4\beta^2\mu^5d_1^2d_2^3 + 2\beta^2\mu^5d_2^5)e^{-2\beta t} + 2d_2\mu\beta^4e^{6\beta t} + d_2\beta^4e^{8\beta t} + d_2^5\mu^4\beta^2 + d_2\mu^4d_1^4\beta^2 + 2d_2^3\mu^4d_1^2\beta^2. \end{aligned}$$

From the above expressions (48) and (49), we can find that the first-order rogue wave of the coupled GP equations also has a curved background. Additionally, the parameter β determines the degree of this curved background, namely, the background will become steeper as β increases and it will become flatter as β decreases. This phenomenon can be observed in Figs. 8 and 9. Owning to $\rho_j = [(\lambda-\lambda_j)(\lambda-\lambda_j^*)^2]^{-\frac{1}{3}}$ occurs in the above DT, the determinant representations of DT and its general solutions ($q_1[N]$, $q_2[N]$) are not given directly. Thus, we can not utilize He's method [41–43] to construct higher-order rogue waves of the coupled GP system (1), meanwhile, the generalized DT [38] is also not used directly. Up to now, we haven't found an appropriate method to give the higher-order rogue waves of this coupled GP equations.

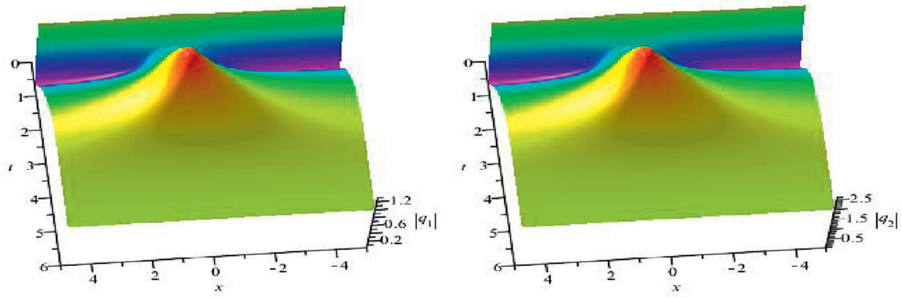
5. Multi-component coupled G-P system and its Darboux transformation

If these waves propagate in more than two fields in plasma, we need to generalize the two-component coupled G-P Eqs. (1) to the multi-component system [24] to determine this case. This kind of multi-component coupled G-P equations can be written as

$$iq_{jt} + q_{jxx} + 2\mu^2\left(\sum_{i=1}^{N}|q_i|^2\right)q_j + (i\beta - \alpha x + \beta^2x^2)q_j = 0 \ (j = 1, 2, 3, \cdots, N), \tag{50}$$

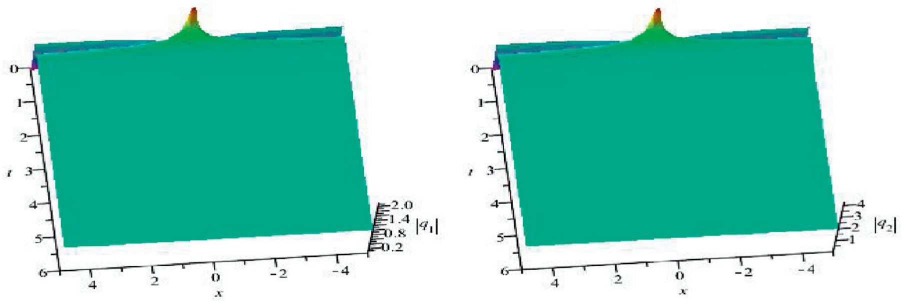
where α, μ, β are all real constants, Eq. (50) admits the following Lax pair with $(N+1)\times(N+1)$ matrices

$$\Psi_x = U\Psi = (i\lambda U_0 + \mu U_1)\Psi, \tag{51}$$



(a) (b)

Fig. 8. Evolution plot of the first-order rogue wave with parameters chosen by $d_1 = 1,\ d_2 = -2,\ \mu = 1,\ \alpha = 0.2,\ \beta = 0.5$, (a) q_1, (b) q_2.



(a) (b)

Fig. 9. Evolution plot of the first-order rogue wave with parameters chosen by $d_1 = 1,\ d_2 = -2,\ \mu = 1,\ \alpha = 0.2,\ \beta = 5$, (a) q_1, (b) q_2.

$$\Psi_t = V\Psi = (2i\lambda^2 U_0 + 2\lambda(-i\beta x U_0 + \mu U_1) + iV_1)\Psi, \tag{52}$$

where

$$U_0 = \begin{pmatrix} -1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & \ddots & & \\ & & & & 1 & \\ & & & & & 1 \end{pmatrix}, \quad U_1 = \begin{pmatrix} 0 & Q_1 & Q_2 & \cdots & Q_{N-1} & Q_N \\ -Q_1 & 0 & & & & \\ -Q_2 & & 0 & & & \\ \vdots & & & \ddots & & \\ -Q_{N-1}^* & & & & 0 & \\ -Q_N^* & & & & & 0 \end{pmatrix},$$

$$V_1 = \begin{pmatrix} \mu^2\sum_{i=1}^{N}|Q_i|^2 - \frac{\alpha x}{2} & \mu Q_{1x} + 2i\mu\beta x Q_1 & \mu Q_{2x} + 2i\mu\beta x Q_2 & \cdots & \mu Q_{(N-1)x} + 2i\mu\beta x Q_{(N-1)} & \mu Q_{Nx} + 2i\mu\beta x Q_N \\ \mu Q_{1x}^* - 2i\mu\beta x Q_1^* & -\mu^2|Q_1|^2 + \frac{\alpha x}{2} & -\mu^2 Q_2 Q_1^* & \cdots & -\mu^2 Q_{(N-1)} Q_1^* & -\mu^2 Q_N Q_1^* \\ \mu Q_{2x}^* - 2i\mu\beta x Q_2^* & -Q_1 Q_2^* & -\mu^2|Q_2|^2 + \frac{\alpha x}{2} & \cdots & -\mu^2 Q_{(N-1)} Q_2^* & -\mu^2 Q_N Q_2^* \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mu Q_{(N-1)x}^* - 2i\mu\beta x Q_{(N-1)}^* & -\mu^2 Q_1 Q_{(N-1)}^* & -\mu^2 Q_2 Q_{(N-1)}^* & \cdots & -\mu^2|Q_{(N-1)}|^2 + \frac{\alpha x}{2} & -\mu^2 Q_N Q_{(N-1)}^* \\ \mu Q_{Nx}^* - 2i\mu\beta x Q_N^* & -\mu^2 Q_1 Q_N^* & -\mu^2 Q_2 Q_N^* & \cdots & -\mu^2 Q_{(N-1)} Q_N^* & -\mu^2|Q_N|^2 + \frac{\alpha x}{2} \end{pmatrix},$$

with $\Psi = (\psi_1, \psi_2, \psi_3, \cdots, \psi_{(N+1)})^T$, $Q_j = q_j e^{-\frac{i\beta x^2}{2}}$ $(j = 1, 2, 3, \cdots, N)$.

Similarly, let $\Psi(\xi_1) = (\psi_1(\xi_1), \psi_2(\xi_1), \psi_3(\xi_1), \cdots, \psi_{(N+1)}(\xi_1))^T$ be a special vector solution of Lax pair (51)-(52) at $q_j = q_j[0]$ $(j = 1, 2, 3, \cdots, N)$ and $\lambda = \lambda_1 = \frac{\alpha}{4\beta} + \xi_1 e^{-2\beta t}$. Then, the DT of the multi-component coupled GP system (50) holds

$$\widetilde{\Psi} = T\Psi, \quad T = \rho_1(\lambda)(\lambda I - H[1]\Lambda_1 H[1]^{-1}), \tag{53}$$

$$\widetilde{q_j[1]} = q_j[0] + \frac{2i(\lambda_1^* - \lambda_1)\psi_1(\xi_1)\psi_{j+1}(\xi_1)^*}{\sum_{j=1}^{N+1}|\psi_j(\xi_1)|^2} e^{\frac{i\beta x^2}{2}}, \tag{54}$$

where

$$\begin{aligned}\rho_1(\lambda) &= [det(\lambda I - H[1]\Lambda_1 H[1]^{-1})]^{-\frac{1}{N+1}} \\ &= [(\lambda - \lambda_1)(\lambda - \lambda_1^*)^N]^{-\frac{1}{N+1}},\end{aligned}$$

$$\Lambda_1 = \begin{pmatrix} \lambda_1 & & & & & \\ & \lambda_1^* & & & & \\ & & \lambda_1^* & & & \\ & & & \ddots & & \\ & & & & \lambda_1^* & \\ & & & & & \lambda_1^* \end{pmatrix},$$

$$H[1] = \begin{pmatrix} \psi_1(\xi_1) & \psi_2^*(\xi_1) & \psi_3^*(\xi_1) & \cdots & \psi_N^*(\xi_1) & \psi_{(N+1)}^*(\xi_1) \\ \psi_2(\xi_1) & -\psi_1^*(\xi_1) & 0 & \cdots & 0 & 0 \\ \psi_3(\xi_1) & 0 & -\psi_1^*(\xi_1) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \psi_N(\xi_1) & 0 & 0 & \cdots & -\psi_1^*(\xi_1) & 0 \\ \psi_{(N+1)}(\xi_1) & 0 & 0 & \cdots & 0 & -\psi_1^*(\xi_1) \end{pmatrix}.$$

Here, I is the $(N+1)\times(N+1)$ identity matrix, besides, both Λ_1 and $H[1]$ are all $(N+1)\times(N+1)$ matrices. Through the similar procedures in the two coupled GP equations, the N-step DT of the multi-component coupled GP equations can be also given. Choosing $N=1$ in Eqs. (53) and (54), we can get the first-step DT for the one-component GP equation in [19].

6. Conclusion

We test the DT constructed in [19] by Maple software and find that their DT is incorrect, because it doesn't hold the t-part of the Lax pair. Actually, the Lax pair of the one-component GP equation in [19] is nonisospectral and the spectral parameter holds $\lambda_t \neq 0$. In [19], Su et al. constructed their DT based on the standard AKNS hierarchy with isospectral Lax pair. However, the DT of the nonisospectral system is essentially different from the one of the isospectral system.

In our paper, we reconstruct the DT of the multi-component coupled GP equations, especially, the two-component couple GP equations are discussed in detail. In the Lax pair (2)–(3), the spectral parameter λ holds $\lambda = \frac{\alpha}{4\beta} + \xi e^{-2\beta t}$ with ξ being an arbitrary constant and we take it as a new spectral parameter. To make sure integrability of the two-component coupled equations, the infinitely-many conservation laws are demonstrated. Utilizing the DT constructed by us, nonautonomous solitons, breather and first-order rogue wave have been presented. The parameter μ affects the amplitudes of solitons and β changes the values of velocities $v_i = -2Re(\xi_j)e^{-2\beta t}$ $(i = 1, 2, 3, \cdots, N)$. Both the directions of solitons' propagation and the values of v_i are all determined by the real parts of the new spectral parameters ξ_i $(i = 1, 2, 3, \cdots, N)$. Here, some dynamics of nonautonomous one-soliton are discussed in detail. By choosing $Re(\xi_1) \neq Re(\xi_2)$ in Eqs. (39) and (40), the two interactional solitons are obtained and shown in Fig. 5. When $Re(\xi_1) = Re(\xi_2)$, the two-soliton bound state is also exhibited in Fig. 6. In order to get breather and rogue wave solutions, the seed solution of Eq. (1) should be selected as a nonzero curved background. Beginning with this kind of curved background, the nonautonomous one-breather and first-order rogue wave are all presented. Here, the amplitude of the breather becomes small till being zero as t increases. The parameter β determines the degree of this curved background in the first-order rogue wave, see Figs. 8 and 9.

These results further reveal the striking dynamic structures of analytical solutions in the nonautonomous coupled system, and we hope they will be verified in physical experiments in the future. Owing to $\rho_j = [(\lambda - \lambda_j)(\lambda - \lambda_j^*)^2]^{-\frac{1}{3}}$ $(j = 1, 2, 3, \ldots, N)$ exists in the j-step DT of the two-component couple GP equations, it isn't possible to obtain the determinant representations of DT and general solutions of Eq. (1) directly by Cramers rule. Here, it can not be utilized that the method to construct higher-order rogue waves of nonlinear models proposed by He [41–43], meanwhile, the generalized DT [38] is also not used directly. Up to now, we haven't found an appropriate method to give the higher-order rogue waves of this coupled GP system. We will attempt to work out this problem in our future work.

Acknowledgment

We would like to express our sincere thanks to S Y Lou, Z Y Yan, X Wang and other members of our discussion group for their valuable comments. The project is supported by the Global Change Research Program of China (No. 2015CB953904), National Natural Science Foundation of China (No. 11675054, 11435005), Natural Science Foundation of Hebei Province (No. A2014210140) and Shanghai Collaborative Innovation Center of Trustworthy Software for Internet of Things (No. ZF1213).

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