Chin. Phys. B Vol. 27, No. 2 (2018) 020200

Soliton–cnoidal interactional wave solutions for the reduced Maxwell–Bloch equations*

Li-Li Huang(黄丽丽)[1,3], Zhi-Jun Qiao(乔志军)[2], and Yong Chen(陈勇)[1,3,4,†]

[1] *Shanghai Key Laboratory of Trustworthy Computing, East China Normal University, Shanghai* 200062, *China*
[2] *School of Mathematical and Statistical Sciences, The University of Texs Rio Grande Valley, Edinburg, TX* 78539, *USA*
[3] *MOE International Joint Laboratory of Trustworthy Software, East China Normal University, Shanghai* 200062, *China*
[4] *Department of Physics, Zhejiang Normal University, Jinhua* 321004, *China*

(Received 20 October 2017; revised manuscript received 17 November 2017; published online 00 00 0000)

In this paper, we study soliton–cnoidal wave solutions for the reduced Maxwell–Bloch equations. The truncated Painlevé analysis is utilized to generate a consistent Riccati expansion, which leads to solving the reduced Maxwell–Bloch equations with solitary wave, cnoidal periodic wave, and soliton–cnoidal interactional wave solutions in an explicit form. Particularly, the soliton–cnoidal interactional wave solution is obtained for the first time for the reduced Maxwell–Bloch equations. Finally, we present some figures to show properties of the explicit soliton–cnoidal interactional wave solutions as well as some new dynamical phenomena.

Keywords: reduced Maxwell–Bloch equations, consistent Riccati expansion, soliton–cnoidal interactional wave solutions

PACS: 02.30.IK, 05.45.Yv, 04.20.Jb **DOI:** 10.1088/1674-1056/27/2/020200

1. Introduction

It is well known that the Maxwell–Bloch equations[1] describe the propagation of short ($< 10^{-9}$ s) optical pulses in a resonant two-level media. The propagation of ultra-short optical pulses is usually governed by the following reduced Maxwell–Bloch (RMB) equations[2]

$$\begin{aligned} &E'_{\xi\xi}(\xi,\tau)-c^{-2}E'_{\tau\tau}(\xi,\tau)=4\pi c^{-2}np\langle u_{\tau\tau}(\xi,\tau)\rangle,\\ &u_\tau(\xi,\tau)=-\mu' v(\xi,\tau),\\ &v_\tau(\xi,\tau)=\mu' u(\xi,\tau)+2p\hbar^{-1}E'(\xi,\tau)\omega(\xi,\tau),\\ &\omega_\tau(\xi,\tau)=-2p\hbar^{-1}E'(\xi,\tau)v(\xi,\tau), \end{aligned} \tag{1}$$

where u is microscopic polarization, v is phase information, ω is the atomic inversion, E' is the electric field, μ' is the atomic resonant frequency, p is the dipole operator, c is the speed of light in vacuum, n is the atomic dipole density, the angular bracket $\langle\,\rangle$ represents the summation over all the media characterized by the frequency, and $\hbar\mu'$ is the energy separation for the two levels.

Apparently, adopting the following transformation

$$x=\tau-c^{-1}\xi,\ \ t=-4\pi np^2(c\hbar)^{-1}\mu\xi,\ \ E=2p\hbar^{-1}E', \tag{2}$$

sends the RMB equations (1) to the following system

$$\begin{aligned} &u_x=-\mu v,\ \ v_x=E\omega+\mu u,\\ &\omega_x=-Ev,\ \ E_t=-v. \end{aligned} \tag{3}$$

The research of the RMB equations was initiated by Eilbeck *et al.*[2] in 1973, where slowly varying envelope approximation was avoided and the backscattered wave was neglected due to a weaker assumption. The RMB equations have a wide usage in describing phenomena in nonlinear optics, including the theory of optical self-induced transparency.[3,4] One of their significant features is the propagation of a short laser pulse in a rarefied medium of two level atoms. Other important applications include electrical engineering experiments with essential algorithms and simulations.[5–7] During the past few decades, many effective methods, such as the inverse scattering transform (IST),[8,9] the Hirota bilinear method,[10–12] and the Darboux transformation (DT),[13–15] have been utilized to investigate the explicit N-soliton solutions of the RMB equations. The RMB equations are one of integrable systems as shown in Refs. [16] and [17] and of course admit other integrable properties including Hamitonian structure and recursion operator[18] as well as the N-degenerate periodic, N-rational solutions, and rogue waves.[19]

To find interactional solutions of nonlinear systems is a difficult and tedious but very meaningful and important work. Fortunately, the truncated Painlevé expansion and the tanh function expansion methods, are effective to derive interac-

*Project supported by the Global Change Research Program of China (Grant No. 2015CB953904), the National Natural Science Foundation of China (Grant Nos. 11675054 and 11435005), the Outstanding Doctoral Dissertation Cultivation Plan of Action (Grant No. YB2016039), and Shanghai Collaborative Innovation Center of Trustworthy Software for Internet of Things (Grant No. ZF1213). Some work was done during the third author (Chen) visited the University of Texas–Rio Grande Valley (UTRGV). The second author (Qiao) thanks the UTRGV President Endowed Professorship (Grant No. 450000123), and the UTRGV College of Science Seed Grant (Grant No. 240000013) for partial support.

†Corresponding author. E-mail: ychen@sei.ecnu.edu.cn

 http://iopscience.iop.org/cpb http://cpb.iphy.ac.cn

tional solutions between solitary waves and other kinds of nonlinear waves, such as Airy waves, cnoidal waves, and Bessel waves. Recently, the consistent Riccati expansion (CRE) method and consistent tanh expansion (CTE) method are proposed to investigate the interactional wave solutions of the nonlinear systems.[20] These methods play an important role in solving nonlinear systems, whether integrable system or not. Then many nonlinear systems are proved CRE solvable and own interactional wave solutions between a solitary wave and a cnoidal wave.[21–30]

In this paper, we concentrate on constructing soliton-cnoidal interactional wave solutions of the RMB equations. The truncated Painlevé analysis is utilized to generate a CRE and the RMB equations are proved to be CRE solvable and CTE solvable. By CTE method, solitary wave, cnoidal periodic wave, and soliton–cnoidal interactional wave solutions of the RMB equations are obtained in an explicit form. Finally, we present some figures to show properties of the explicit soliton–cnoidal interactional wave solutions as well as some new dynamical phenomena, which could be helpful to better understand the dynamic properties of the interactional wave solutions in optical and electromagnetic fields.

The paper is constructed as follows. In Section 2, the RMB equations are verified CRE solvable and CTE solvable. In Section 3, the solitary wave, cnoidal periodic wave, and soliton–cnoidal wave solutions are constructed. The last section gives a short summary and discussion.

2. CRE solvable and CTE solvable

2.1. CRE solvable of the RMB equations

Based on the CRE method, the CRE solution for the RMB equations (3) can be mapped into

$$\begin{aligned}&u = u_0 + u_1 R(z),\ \ v = v_0 + v_1 R(z) + v_2 R(z)^2,\\ &\omega = \omega_0 + \omega_1 R(z) + \omega_2 R(z)^2,\ \ E = E_0 + E_1 R(z),\end{aligned} \tag{4}$$

where u_0, u_1, v_0, v_1, v_2, ω_0, ω_1, ω_2, E_0, E_1, and z are functions of x and t to be determined. The function $R(z)$ satisfies the following Riccati equation

$$R_z = b_0 + b_1 R + b_2 R^2,\ \ R \equiv R(z), \tag{5}$$

and b_0, b_1, and b_2 are arbitrary constants. After substituting Eq. (4) with Eq. (5) into Eqs. (3), all the expansion coefficients of $R(z)$ are needed to be equal to zero. Fourteen overdetermined equations are obtained that only eleven undetermined functions u_0, u_1, v_0, v_1, v_2, ω_0, ω_1, ω_2, E_0, E_1, and z need to be satisfied. Fortunately, these overdetermined functions can be consistent. From the coefficients of $R(z)^3$, $R(z)^2$, and $R(z)$, u_0, u_1, v_0, v_1, v_2, ω_0, ω_1, ω_2, E_0, and E_1 can be solved as,

$$\begin{aligned}&u_0 = I\mu b_1 z_t + I\mu z_x^{-1} z_{xt},\ \ u_1 = 2I\mu b_2 z_t,\\ &v_0 = -2Ib_0 b_2 z_x z_t - Ib_1 z_{xt} - Iz_x^{-1} z_{xxt} + Iz_x^{-2} z_{xx} z_{xt},\\ &v_1 = -2Ib_1 b_2 z_x z_t - 2Ib_2 z_{xt},\ \ v_2 = -2Ib_2^2 z_x z_t,\\ &\omega_1 = -2b_1 b_2 z_x z_t - 2b_2 z_{xt},\ \ \omega_2 = -2b_2^2 z_x z_t,\\ &\omega_0 = -2b_0 b_2 z_x z_t - b_1 z_{xt} - \mu^2 z_x^{-1} z_t - z_x^{-1} z_{xxt} + z_x^{-2} z_{xx} z_{xt},\\ &E_0 = Ib_1 z_x + Iz_x^{-1} z_{xx},\ \ E_1 = 2Ib_2 z_x.\end{aligned} \tag{6}$$

Then from the overdetermined functions, the associated compatibility condition of function z is

$$\delta z_x(z_x C_{1x} + z_{xx} C_1) + \mu^2 C_{1x} + S_{1t} = 0,\ \ (\delta = 4b_0 b_2 - b_1^2), \tag{7}$$

with $C_1 = z_t/z_x$ and $S_1 = z_{xxx}/z_x - (3/2)(z_{xx}/z_x)^2$. Finally, it can be verified that all the coefficients of the overdetermined equations are identically satisfied by using Eqs. (6) and (7).

From the above results, it is clear that the RMB equations (3) really possesses the truncated Painlevé expansion solutions, which is corresponding to the Riccati equation (5). Therefore, the RMB equations (3) have CRE solvable property.[20] It should be point out that equation (7) is a generalization of the Schwarzian form for the RMB equations, as equation (7) is nothing but the standard Schwarzian form if dropping out δ term. In order to make the result clear, we arrive at the following non-auto BT theorem.

Theorem 1 (Non-auto BT) If z is a solution to

$$\delta z_x(z_x C_{1x} + z_{xx} C_1) + \mu^2 C_{1x} + S_{1t} = 0, \tag{8}$$

then

$$\begin{aligned}&u = u_0 + 2I\mu b_2 z_t R(z),\ \ E = E_0 + 2Ib_2 z_x R(z),\\ &v = v_0 - 2I(b_1 b_2 z_x z_t + b_2 z_{xt})R(z) - 2Ib_2^2 z_x z_t R(z)^2,\\ &\omega = \omega_0 - 2(b_1 b_2 z_x z_t + b_2 z_{xt})R(z) - 2b_2^2 z_x z_t R(z)^2,\end{aligned} \tag{9}$$

is a solution to Eqs. (3), where $R \equiv R(z)$ is a solution to the Riccati equation (5).

2.2. CTE solvable of the RMB equations

By using a dependent variable transformation

$$R(z) = -\frac{1}{2b_2}\left(b_1 - \sqrt{\delta}\tanh\left(\frac{1}{2}\sqrt{\delta} z\right)\right),$$

it is obvious that $R(z)$ satisfies Riccati equation (5). With the parameters $b_0 = 1$, $b_1 = 0$, and $b_2 = -1$, equation (4) can be mapped into

$$\begin{aligned}&u = u_0 + u_1 \tanh(z),\\ &v = v_0 + v_1 \tanh(z) + v_2 \tanh(z)^2,\\ &\omega = \omega_0 + \omega_1 \tanh(z) + \omega_2 \tanh(z)^2,\\ &E = E_0 + E_1 \tanh(z),\end{aligned} \tag{10}$$

where u_0, u_1, v_0, v_1, v_2, ω_0, ω_1, ω_2, E_0, E_1, and z are decided by Eqs. (5), (6), and (7).

It is evident that the solutions (10) are consistent with the above Theorem 1. Since CTE (10) is an exceptional situation of CRE, CTE solvable is equivalent to CRE solvable. Based on the CTE solvable property, we can obtain some interesting interactional solitary wave solutions, such as the solitary wave and the interactions between a solitary wave and a elliptic cosine wave, whose dynamical characteristics can be shown visually via the following expressions. Through the non-auto BT theorem below, we can clearly understand the above relation and take advantage of it to derive exact solutions.

Theorem 2 (Non-auto BT) If z is a solution to Eqs. (8), then

$$\begin{aligned} &u=u_0+2I\mu b_2 z_t\tanh(z),\\ &v=v_0-2I(b_1b_2z_xz_t+b_2z_{xt})\tanh(z)-2Ib_2^2z_xz_t\tanh(z)^2,\\ &\omega=\omega_0-2(b_1b_2z_xz_t+b_2z_{xt})\tanh(z)-2b_2^2z_xz_t\tanh(z)^2,\\ &E=E_0+2Ib_2z_x\tanh(z) \end{aligned} \tag{11}$$

are solutions of the RMB equations (3), with u_0, v_0, ω_0, and E_0 being decided by Eq. (6).

3. Soliton–cnoidal interactional wave solutions

To search for the explicit solutions of the RMB equations (3), z is assumed by the following form

$$z=kx+dt+g, \tag{12}$$

where g is an arbitrary function with x and t. The interactional solutions between solitary waves and other nonlinear waves will be obtained. According to Theorem 2, solitary wave solutions, cnoidal wave solutions, and soliton–cnoidal interactional wave solutions can be derived as follows.

Example 1 Solitary wave solutions In Theorem 2, we choose a trivial line solution to z, that is

$$z=Ikx+Idt+c, \tag{13}$$

where k, d, and c are arbitrary constants. When substituting Eq. (13) with line solution (12) into Theorem 2, the following solitary wave solutions of the RMB equations (3) can be derived:

$$\begin{aligned} &u=2d\mu\tanh(z),\\ &v=2I\tanh(z)+2d\tanh(z)^2,\\ &\omega=-\mu^2k^{-1}d+2kd\tanh(z)^2,\\ &E=-2I\tanh(z), \end{aligned} \tag{14}$$

Example 2 Cnoidal periodic wave solutions For the sack of cnoidal periodic wave solutions, assuming z has the following shape

$$z=Z\equiv Z(Ik_2x+Id_2t)\equiv Z(X). \tag{15}$$

Substituting Eq. (15) into Eq. (8), it yields the elliptic equation as follows:

$$Z_{1X}^2=a_1Z_1+a_2Z_1^2+a_3Z_1^3+4Z_1^4, \tag{16}$$

where $Z_1\equiv Z_1(X)=Z_X$, a_1, a_2, and a_3 are arbitrary constants. It is apparent that equation (16) has the general Jacobian elliptic functions solutions. To show this type of solutions clearly, we choose solutions of Eq. (16) to be expressed by

$$Z_1=\frac{1}{2}m+\frac{1}{2}mn\,\mathrm{sn}(mX,n), \tag{17}$$

Substituting Eq. (17) into Eq. (16), the following relations can be obtained

$$a_1=(n^2-1)m^3,\quad a_2=(5-n^2)m^2,\quad a_3=-8m. \tag{18}$$

Then the cnoidal periodic wave solutions can be derived as

$$\begin{aligned} u={}&md_2\mu(nS+1)T-\frac{mnd_2\mu CD}{nS+1},\\ v={}&-\frac{1}{16}m^5nk_2d_2^4S(nS+1)^4+\frac{1}{4}m^3k_2d_2^2(nS+1)^2\\ &+\frac{1}{2}Im^2d_2k_2(nS+1)^2T^2-\frac{Im^2n^2k_2d_2C^2D^2}{(nS+1)^2}\\ &-\frac{Im^2nk_2d_2S(nC^2+D^2)}{nS+1}-Im^2nk_2d_2CDT,\\ \omega={}&\frac{1}{2}m^2k_2d_2(nS+1)^2(T^2-1)-\frac{m^2n^2k_2d_2C^2D^2}{(nS+1)^2)}\\ &-\frac{m^2nk_2d_2S(nC^2+D^2)}{nS+1}-m^2nk_2d_2CDT-\mu^2\frac{d_2}{k_2},\\ E={}&mk_2(nS+1)T-\frac{mnk_2CD}{nS+1}, \end{aligned} \tag{19}$$

where $X=Ik_2x+Id_2t$, S, C, and D respectively express Jacobi elliptic functions sn, cn, and dn with modulus n and

$$T=\tanh(\frac{1}{2}(mX+\ln(-nC+D))).$$

Example 3 Soliton–cnoidal interactional wave solutions In order to construct the interactional solutions between solitary waves and cnoidal periodic waves, assuming z possesses the following form

$$z=k_1x+d_1t+Z(X),\ \ (X\equiv k_2x+d_2t), \tag{20}$$

where $Z_1\equiv Z_1(X)=Z_X$ meets

$$Z_{1X}^2=c_0+c_1Z_1+c_2Z_1^2+c_3Z_1^3+c_4Z_1^4, \tag{21}$$

with c_0, c_1, c_2, c_3, c_4 being constants. Putting Eq. (20) with Eq. (21) into Theorem 2, it requires

$$c_0=\frac{c_1k_1}{k_2}-\frac{c_2k_1^2}{k_2^2}+\frac{c_3k_1^3}{k_2^3}-\frac{4k_1^4}{k_2^4},\ \ c_4=4,$$

$$d_2=\frac{\mu^2 k_2 d_1}{c_1k_2^3-2c_2k_1k_2^2+3c_3k_1^2k_2-16k_1^3+\mu^2 k_1},\qquad(22)$$

which results in the following exact solutions of the RMB equations (3) expressed as

$$\begin{aligned}u=&\frac{I\mu k_2d_2Z_{1X}}{k_1+k_2Z_1}-2I\mu(d_1+d_2Z_1)\tanh(k_1x+d_1t+Z),\\ v=&2I(k_1+k_2)Z_1(d_1+d_2Z_1)^2-\frac{Ik_2^2d_2Z_{1XX}}{k_1+k_2Z_1}+\frac{Ik_2^3d_2Z_{1X}^2}{(k_1+k_2Z_1)^2}\\ &+2Ik_2d_2Z_{1X}\tanh(k_1x+d_1t+Z)-2I(k_1+k_2Z_1)(d_1\\ &+d_2Z_1)\tanh(k_1x+d_1t+Z)^2,\\ \omega=&2(k_1+k_2Z_1)(d_1+d_2Z_1)-\frac{\mu^2(d_1+d_2Z_1)}{k_1+k_2Z_1}-\frac{k_2^2d_2Z_{1XX}}{k_1+k_2Z_1}\\ &+\frac{k_2^3d_2Z_{1X}^2}{(k_1+k_2Z_1)^2}+2k_2d_2Z_{1X}\tanh(k_1x+d_1t+Z)-2(k_1\\ &+k_2Z_1)(d_1+d_2Z_1)\tanh(k_1x+d_1t+Z)^2,\\ E=&\frac{Ik_2^2Z_{1X}}{k_1+k_2Z_1}-2I(k_1+k_2Z_1)\tanh(k_1x+d_1t+Z).\end{aligned}\qquad(23)$$

According to the definition of elliptic function, the explicit solution of an equation can be expressed by the terms of Jacobian elliptic function. It is indicated that equation (23) demonstrates interactional solutions between solitary waves and other nonlinear waves. For a more intuitive demonstration of the above soliton–cnoidal interactional solutions, we give the following ansatz for solutions of Eq. (21)

$$Z_1=\eta_0+\eta_1\,\mathrm{sn}(mX,n),\qquad(24)$$

where $\mathrm{sn}(mX,n)$ is the general Jacobian elliptic sine function. The modulus n satisfies the condition: $0\le n\le 1$. If $n\to 1$, $\mathrm{sn}(mX,n)$ degenerates into hyperbolic function $\tanh(mX)$. If $n\to 0$, it degenerates into trigonometric function $\sin(mX,n)$. If substituting Eq. (24) with Eq. (22) into Eq. (21), gathering the coefficients of $\mathrm{cn}(mX,n)$, $\mathrm{dn}(mX,n)$, $\mathrm{sn}(mX,n)$ and setting them to zero, results in the following relations

$$\begin{aligned}&c_1=m^3-n^2m^3+\frac{2(5-n^2)k_1m^2}{k_2}+\frac{24mk_1^2}{k_2^2}+\frac{16k_1^3}{k_2^3},\\ &c_2=5m^2-n^2m^2+\frac{24mk_1}{k_2}+\frac{24k_1^2}{k_2^2},\quad c_3=8m+\frac{16k_1}{k_2},\\ &\eta_0=-\frac{m}{2}-\frac{k_1}{k_2},\quad \eta_1=\frac{mn}{2}.\end{aligned}\qquad(25)$$

Finally, substituting Eq. (24) and

$$Z=\eta_0X+\eta_1\int_{X_0}^{X}\mathrm{sn}(m\xi,n)\,\mathrm{d}\xi,\qquad(26)$$

with the parameters satisfied by Eq. (25) into the general solutions (23), special interactional solutions with respect to solitary waves and cnoidal waves are obtained.

The solutions of Eq. (23) with Eq. (22), namely the interactional solutions between the solitary waves and the cnoidal waves. In Figs. 1, 2, 4, and 6, the interactional solutions of u, v, ω, and E are visually shown with the condition modulus $n\neq 1$. This type of solutions has a wide application value in analyzing some interesting physical phenomena.

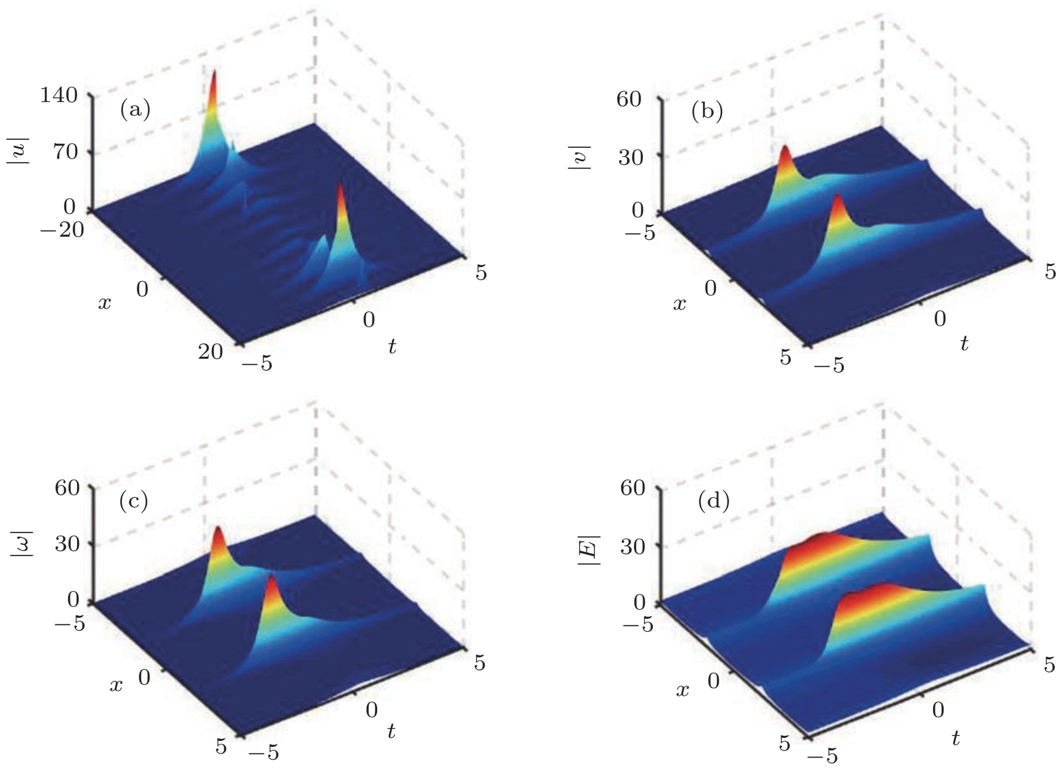

Fig. 1. (color online) The soliton–cnoidal wave interactional solution of the RMB equations for the components u, v, ω, and E expressed by Eq. (23). The parameters are $m=-1$, $n=1/2$, $k_1=-I/4$, $k_2=-I$, $\mu=1/5$, and $d_1=1/2$.

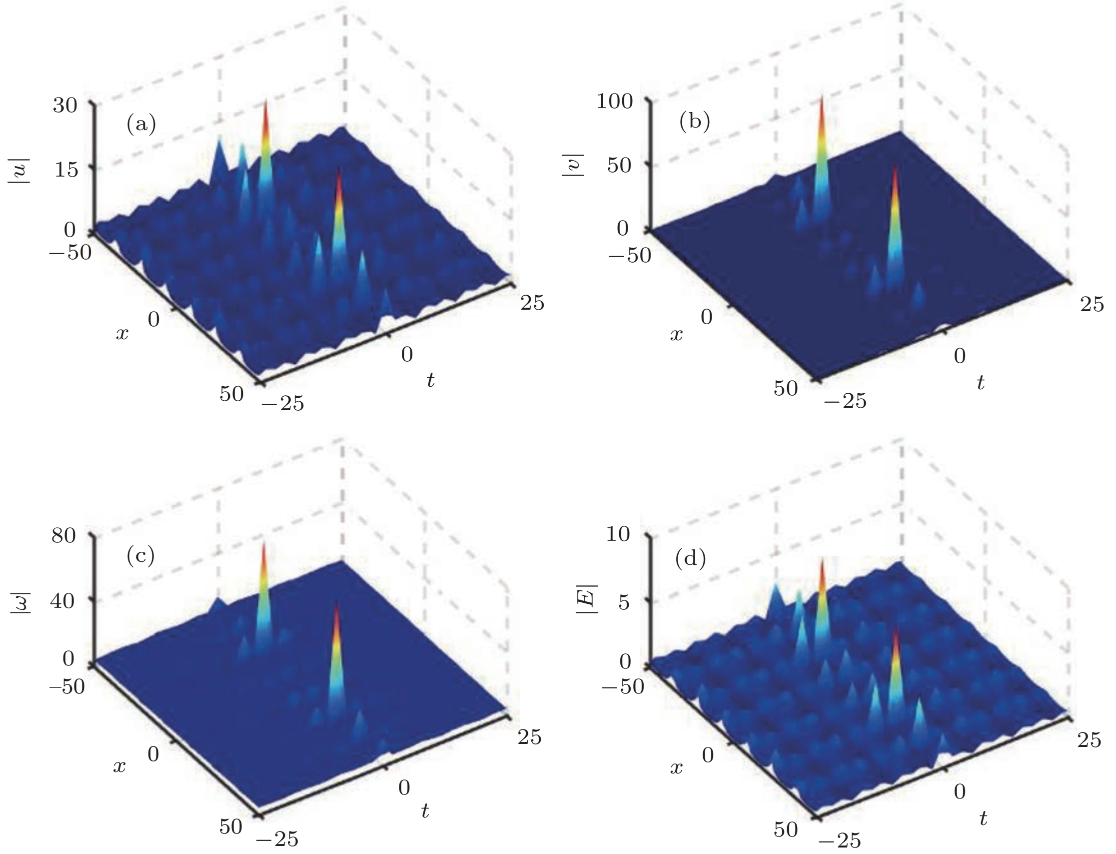

Fig. 2. (color online) The soliton–cnoidal wave interactional solution of the RMB equations for the components u, v, ω, and E expressed by Eq. (23). The parameters are $m=1$, $n=1/2$, $k_1=-I/4$, $k_2=I/4$, $\mu=1$, and $d_1=1$.

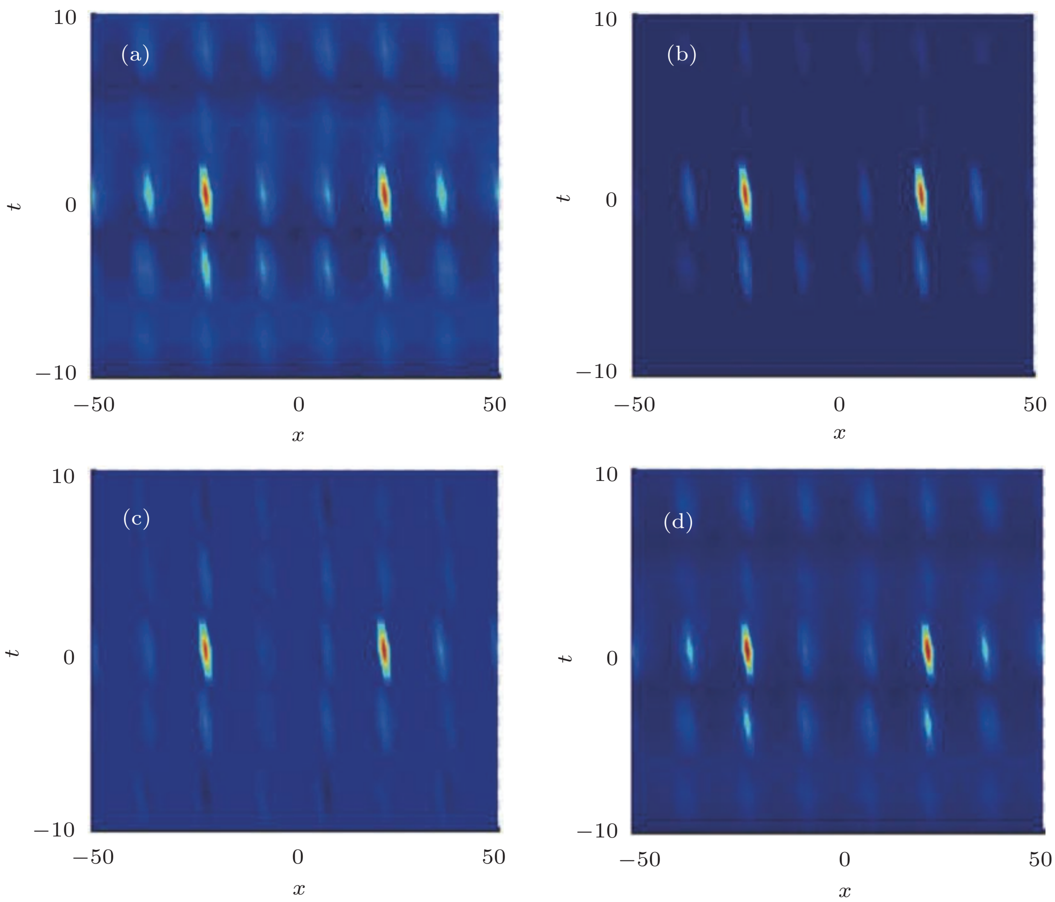

Fig. 3. (color online) Density plots of the components u, v, ω, and E for the values described in Fig. 2.

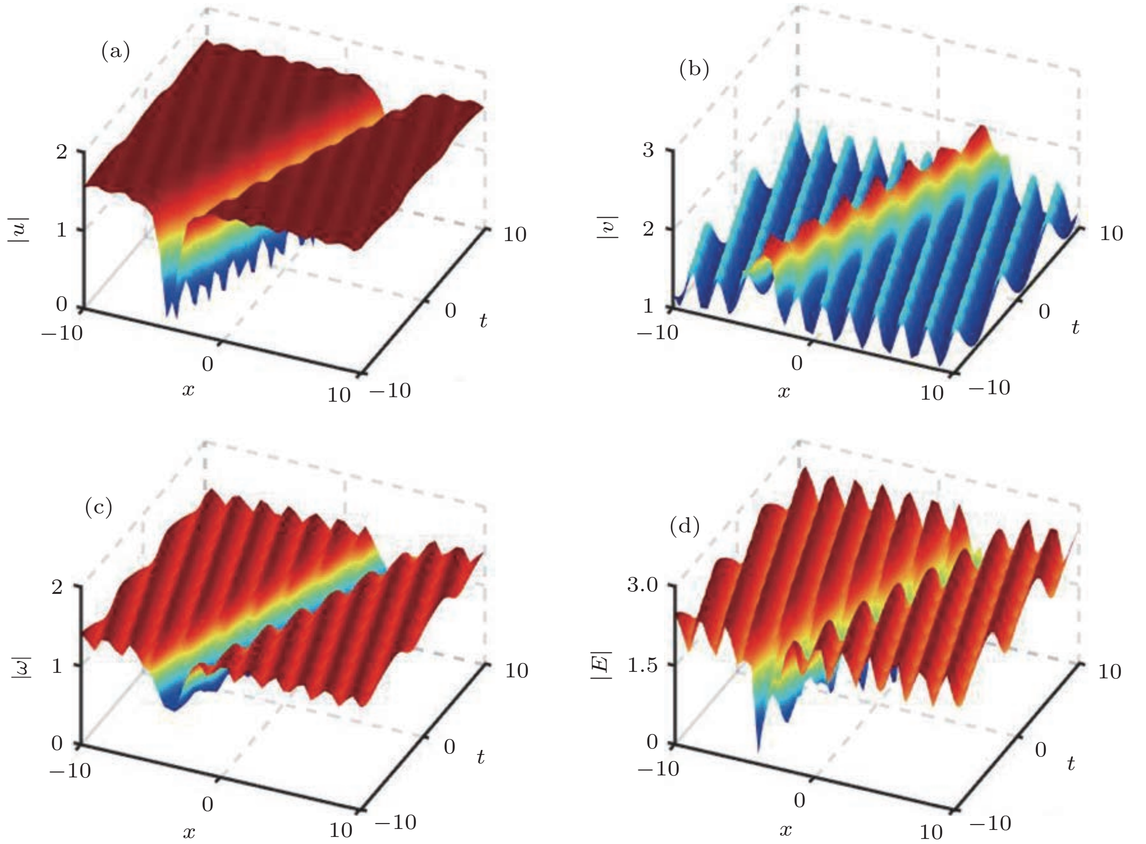

Fig. 4. (color online) The soliton–cnoidal wave interactional solution of the RMB equations for the components u, v, ω, and E expressed by Eq. (23. The parameters are $m = 1/2$, $n = 1/2$, $k_1 = -1/2$, $k_2 = 4$, $\mu = 2$, and $d_1 = 1/2$.

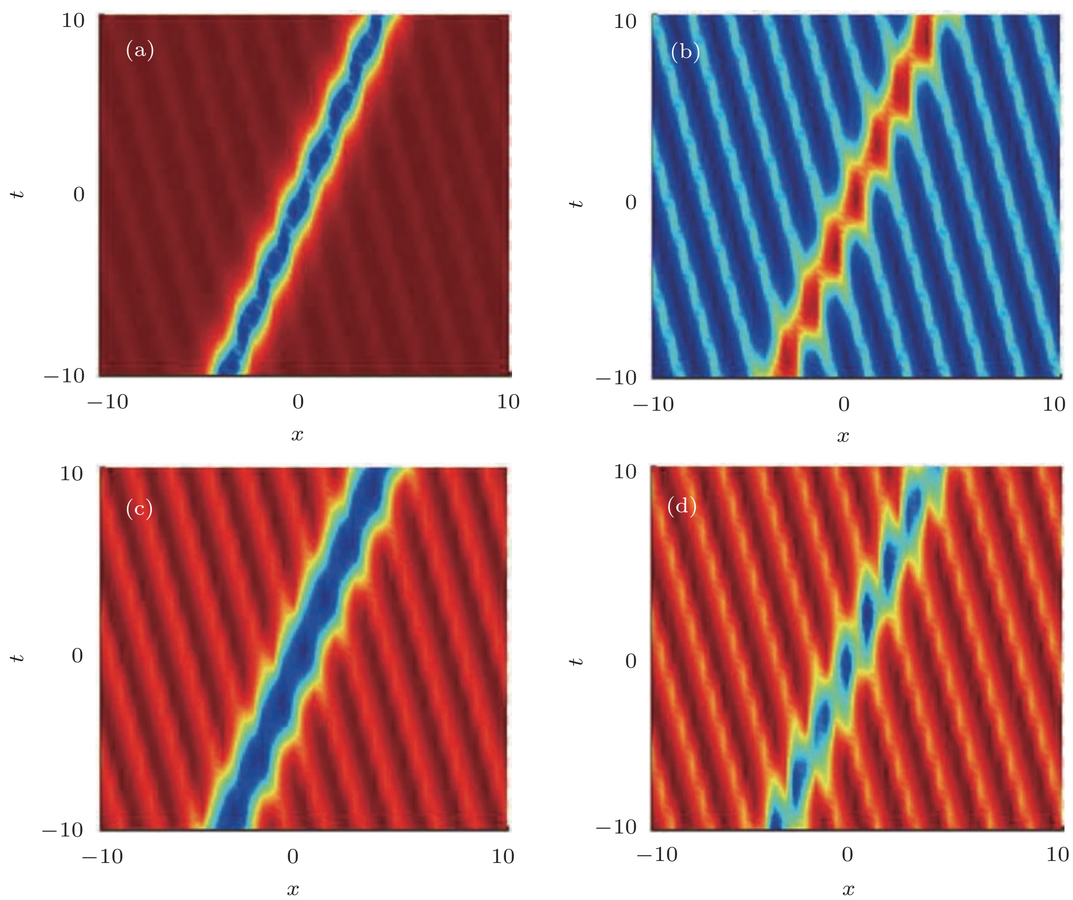

Fig. 5. (color online) Density plots of the components u, v, ω, and E for the values described in Fig. 4.

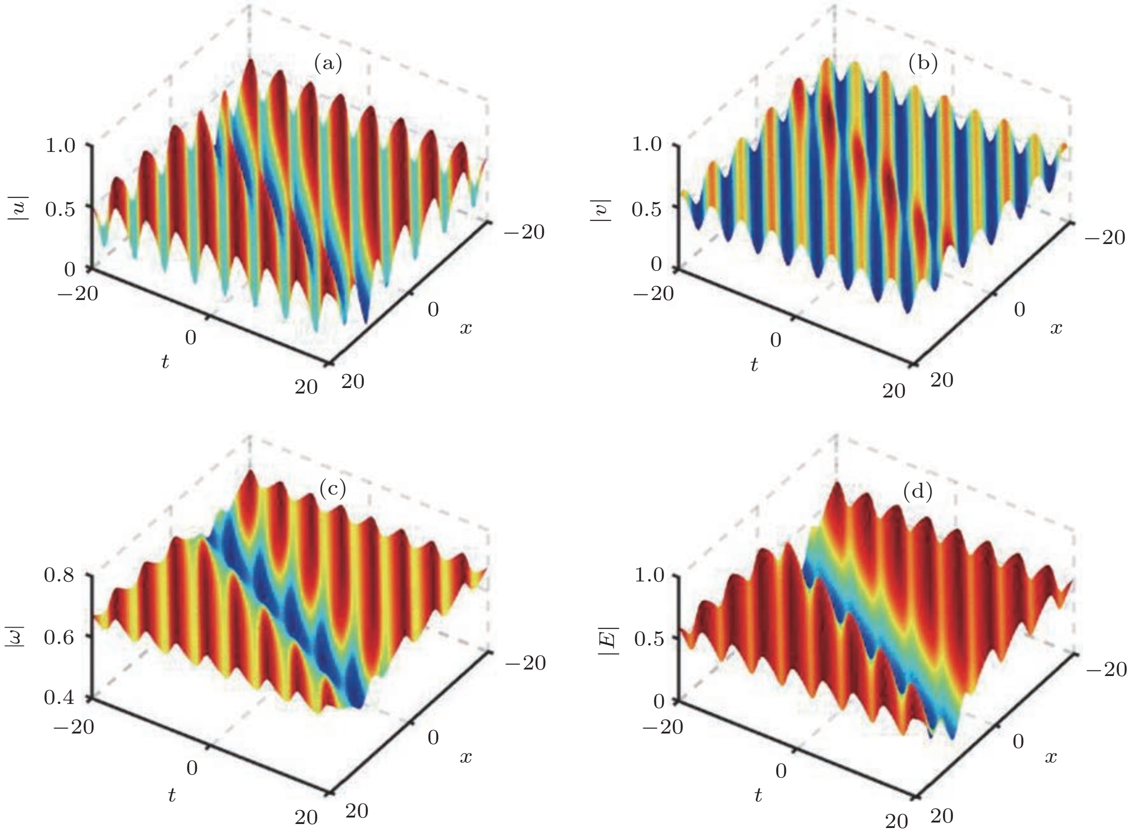

Fig. 6. (color online) The soliton–cnoidal wave interactional solution of the RMB equations for the components u, v, ω, and E expressed by Eq. (23). The parameters are $m = 1/2$, $n = 1/2$, $k_1 = -I/2$, $k_2 = 1$, $\mu = 1$, and $d_1 = 1/2$.

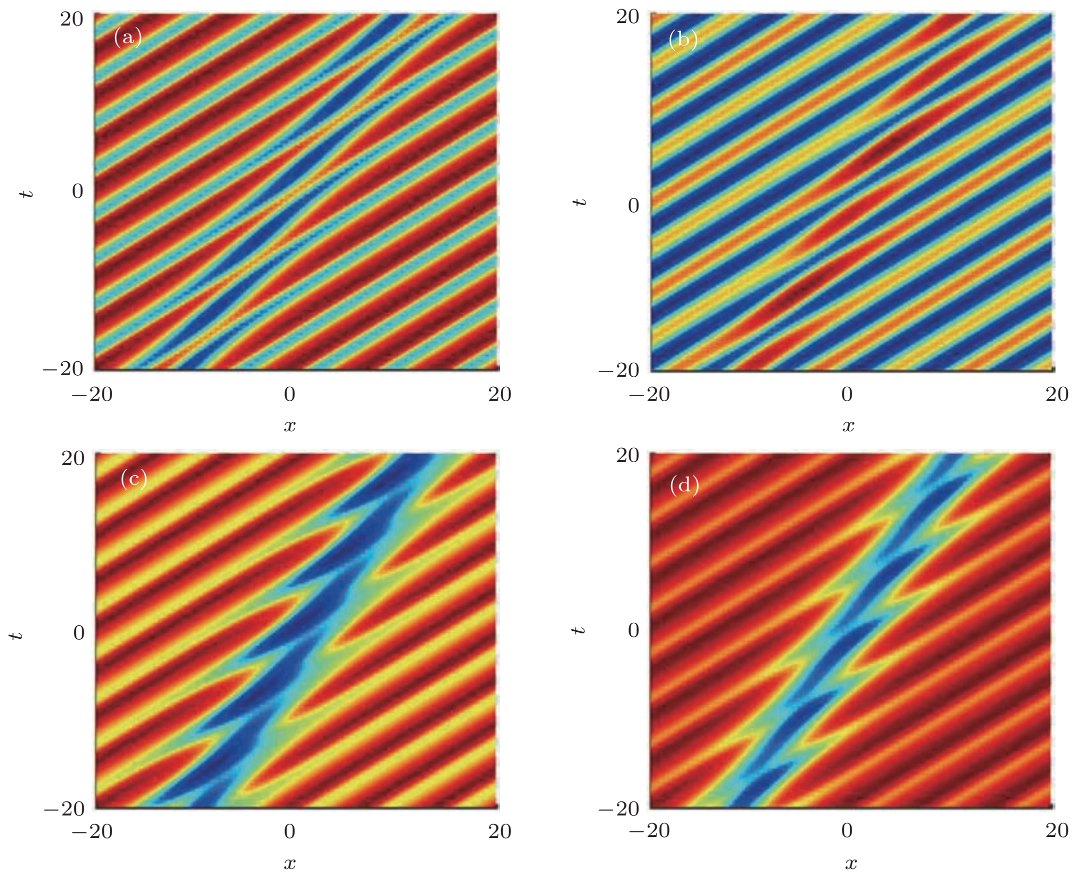

Fig. 7. (color online) Density plots of the components u, v, ω, and E for the values described in Fig. 6.

As a matter of fact, plenty of such interactional solutions in our real physics world.

(i) For Figs. 1 and 2, the waves generate in pairs, and the solutions are symmetrical at $x=0$. The periodic waves are periodic in x directions and localized in t directions. The background waves can be clearly presented from the components u and E in Fig. 2, while the background periodic waves of the components v and ω are not clearly presented.

(ii) For Figs. 4(a), 4(c), 4(d), 6(a), 6(c), and 6(d), dark solitary waves residing on cnoidal waves instead of on constant backgrounds.

(iii) For Figs. 4(b) and 6(b), a bright solitary wave travelling on a cnoidal wave.

(iv) From analysis of the density graph between Figs. 5 and 7, the direction of the cnoidal wave propagation is inconsistent, while direction of the solitary wave is consistent.

These dynamical behaviors of the RMB equations for the components u, v, ω, and E have not shown before, and the physical significance of these solutions are well worth further study. Our research results may play a significant contribution to investigate the dynamics of the distinct nonlinear waves, such as rogue waves, breather solutions, and dark solitary waves for nonlinear systems in optics, electromagnetic field, plasma physics, and Bose–Einstein condensates.

4. Summary and discussions

In summary, the RMB equations (3) are studied through utilizing the truncated Painlevé expansion and the consistent Riccati expansion method. Based on the CRE approach, the soliton–cnoidal wave interactional solutions of the RMB equations are explicitly expressed in terms of the Jacobi elliptic and the corresponding elliptic integral. Some interesting physical phenomena with respect to the soliton–cnoidal interactional solutions are shown in above figures. Compared with other traditional methods, the CRE method and CTE method are more effective in obtaining the above soliton–cnoidal wave interactional solutions. Through the mathematical formulations, we plot the soliton–cnoidal wave interactional solutions, and discover some new dynamical phenomena. Those dynamical behaviors (as shown in Figs. 1, 2, 4, and 6 for the components u, v, ω, and E were not investigated before.

Due to the important physics significance of the RMB equations, the physical properties of these new dynamical behaviors with interactions are needed to do a further investigation. In addition, from the viewpoint of mathematics it is worth of studying the relationship between the CRE method and the associated Schwarzian z functions.

Acknowledgment

We would like to express our sincere thanks to other members of our research group for their valuable suggestions.

References

[1] Maimistov A I and Basharov A M 1999 *Nonlinear optical waves* (Berlin: Springer Press)

[2] Eilbeck J C, Gibbon J D, Caudrey P J and Bullough R K 1973 *J. Phys. A: Math. Nucl. Gen.* **6** 1337

[3] McCall S L and Hahn E L 1969 *Phys. Rev.* **183** 457

[4] McCall S L and Hahn E L 1967 *Phys. Rev. Lett.* **18** 908

[5] Jen S, Mandel B C L and Wolf E 1978 *Coherence and quantum optics*, Vol. 4 (New York: Plenum Press)

[6] Sun B, Qiao Z J and Chen J 2012 *J. Appl. Remote Sens.* **6** 063547

[7] Sun B, Gu H C, Hu M Q and Qiao Z J 2015 *Proc. SPIE, Compressive Sensing IV* **9484** 948402

[8] Gibbon J D, Caudrey P J, Bullough R K and Eilbeck J C 1973 *Lett. Nuovo Cimento* **8** 775

[9] Ablowitz M J and Clarkson P A 1991 *Solitons, Nonlinear Evolution Equations and* **Inverse** Scattering

[10] Caudrey P J, Eilbeck J C and Gibbon J D 1974 *IMA J. Appl. Math.* **14** 375

[11] Caudrey P J 2011 *Phil. Trans. R. Soc. A* **369** 1215

[12] Hirota R 2004 *The Direct Method in Soliton Theory* (Cambridge: Cambridge University Press)

[13] Guo R, Tian B and Wang L 2012 *Nonlinear Dyn.* **69** 2009

[14] Shi G H and Fan F 2017 *Superlattices and Microstructures* **101** 488

[15] Matveev V B and Salle M A 1991 *Darboux transformations and solitons* (Berlin: Springer)

[16] Grauel A 1986 *J. Phys. A: Math. Gen.* **19** 479

[17] Grauel A 1984 *Lett. Nuovo Cimento* **41** 263

[18] Aiyer R N 1983 *J. Phys. A: Math. Nucl. Gen.* **16** 1809

[19] Wei J, Wang X and Geng X G 2017 arXiv: 1705.09881

[20] Lou S Y 2015 *Stud. Appl. Math.* **134** 372

[21] Wang Y H and Wang H 2017 *Nonlinear Dyn.* **89** 235

[22] Cheng X P, Chen C L and Lou S Y 2014 *Wave Motion* **51** 1298

[23] Hu X R and Chen Y 2015 *Z. Naturforsch.* **70** 729

[24] Ren B 2015 *Phys Scr.* **90** 065206

[25] Hu X R and Li Y Q 2016 *Appl. Math. Lett.* **51** 20

[26] Chen J C and Ma Z Y 2017 *Appl. Math. Lett.* **64** 87

[27] Wang Q, Chen Y and Zhang H Q 2005 *Chaos, Solitons and Fractals* **25** 1019

[28] Wang Y H and Wang H 2014 *Phys. Scr.* **89** 125203

[29] Huang L L and Chen Y 2016 *Chin. Phys. B* **25** 060201

[30] Huang L L and Chen Y 2017 *Appl. Math. Lett.* **64** 177